# Green Patents in an Oligopolistic Market with Green Consumers

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**Abstract**

We develop a theoretical framework to investigate the impact of patent policies and emission taxes on green innovation that reduces the emission-output ratio and on the emission level. We allow strategic interaction of firms in a duopolistic market. A key finding is that the greater the proportion of green-conscious consumers, the less likely firms are to license green innovation, which results in higher emissions levels. Therefore, policymakers may consider implementing technology standards to force licensing effectively. Increasing the emission tax beyond a certain threshold induces licensing in equilibrium for sufficiently large proportions of green-conscious consumers, thereby causing emissions to fall discretely. Finally, we find that there exists a second threshold level of the tax beyond which increasing the emission tax leads to increasing the emission level. This paradox can be mitigated by decreasing patenting costs and/or making the patentability requirement stricter.

**Keywords**: Patent, Green Innovation, Pollution

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# 1 Introduction

International organizations such as the UN and the G8, national governments, as well as the private sector have invested heavily in the development of 'green', or less polluting, technologies in recent years.[1] While patent policies[2] and firms' licensing decisions play a key role in affecting private sector investment decisions for green innovations, these have received little attention in the related literature. An analysis of patent policy instruments in the context of green innovations is complicated due to the potential interactions between knowledge and environmental externalities. In this paper, therefore, we analyze specific tools of patent policy, that is, changing patentability requirements and patenting costs, and how these work in conjunction with an emission tax to impact firms' investment and licensing decisions and emission levels. While undertaking our analysis, we allow for strategic behaviour of firms within an oligopolistic setting, and for heterogeneity across consumers in terms of the degree to which they care about the environment.

The literature on green technologies has evolved around the work of Porter (1991) and Porter and van der Linde (1995), referred to as the Porter Hypothesis, and examines whether stricter environmental regulations increase green innovation. The empirical evidence surrounding the

[1] In the U.S., the Department of Energy's Loans Program Office has more than $40 billion in remaining loans to help finance innovative technologies that can reduce carbon emissions. See https://www.bloomberg.com/news/articles/2021-10-10/u-s-to-invest-billions-in-clean-energy-innovations

The U.S. government is investing heavily in clean technologies, targeting 100 percent carbon pollution-free electricity by 2030, and 100 percent zero-emission vehicle acquisitions by 2035. See https://www.whitehouse.gov/briefing-room/statements-releases/2021/12/08/fact-sheet-president-biden-signs-executive-order-catalyzing-americas-clean-energy-economy-through-federal-sustainability/

In 2018, the Canadian federal government announced an unprecedented $2.3 billion investment in clean technologies. See https://www.bdc.ca/en/about/mediaroom/news-releases/government-canada-investing-clean-technology

See also Barrett (2009) and Galiana and Green (2009).

[2] In Canada and the U.S. respectively, about 2500 and 18500 patents for green technologies are issued annually. In OECD countries, green patent applications increased by 78% whereas all patent applications grew by 3.9%. In BRICS countries, green patent applications increased by 528% whereas all patent applications grew by 363%. For further details, see https://www.oecd.org/env/indicators-modelling-outlooks/green-patents.htm

Porter Hypothesis is mixed (Ambec et al., 2013; Dechezleprêtre and Sato, 2017; Cohen and Tubb, 2018). Our paper is more closely related to a second stream of the literature, which argues that given the combination of environmental externalities and knowledge market failures facing regulators, environmental policies are not sufficient to achieve the first best social outcome, and need to be combined with policies addressing the relevant knowledge market failure (Carraro and Siniscalco, 1994; Carraro and Soubeyran, 1996; Katsoulacos and Xepapadeas, 1996; Popp, 2006, 2019; Fischer and Newell, 2008; Gerlagh et al., 2009; Acemoglu et al., 2012; Hepburn et al., 2018; Lehmann and Söderholm, 2018). Most of these papers model R&D subsidies and other policy tools that correct for knowledge market failure, while implicitly assuming the existence of intellectual property rights. By contrast, in this paper we explicitly model different aspects of patent systems in an effort to compare different patent policy regimes while endogenizing firms' licensing decisions. This analysis is important in light of the lack of consensus among policy-makers and academics regarding the role of patents in promoting R&D in general and green technologies in particular.[3] On the one hand, international organizations advocate royalty-free compulsory licensing of green technologies, excluding green technologies from patenting, and even revoking existing patent rights on them (UNFCCC, 2009).[4] On the other hand, many countries actively lower the cost of obtaining patents for green innovations.

In this paper, we focus on the role played by two aspects of patent policies, patenting costs and patentability requirements, and how they interact with emission taxes in fostering green innovation.[5] We introduce a lump sum cost associated with obtaining and implementing patents, and examine the impact of lowering this cost.[6] In practice, policy makers use different means

[3]Patents address the problem due to the externality that results from imperfect appropriability of knowledge by endowing innovators with property rights on their inventions. A patent confers its owner a temporary right to exclude others from exploiting the innovation. In exchange for the exclusionary right, the patent holder must disclose his innovation. For surveys of the patent literature, see Langinier and Moschini (2002), Rockett (2010), Eckert and Langinier (2014).

[4]Such provisions are also incorporated in the Agreement on Trade Related Aspects of Intellectual Property Rights (TRIPS) (Derclaye, 2008; Rimmer, 2011).

[5]Gerlagh et al., (2014) is another paper to examine patent policies for green technologies. It focuses on analyzing the impact of changing the lifetime of patents issued to green innovations.

[6]A number of components constitute the overall cost of obtaining and implementing patents. First, the

to reduce patenting costs. One such method which has been frequently used is fast-tracking, or expediting the review process, of green patent applications. Fast-tracking has been implemented by several countries including Australia, Brazil, Canada,[7] China, UK, U.S., Japan and Korea. Evidence shows that fast-tracking programs have accelerated the diffusion of knowledge in green technologies in the short run, and reduced the time from application to grant by up to 75% (Dechezleprêtre, 2013).

In order to be patentable an innovation must be sufficiently novel (not already in the public domain), non-obvious (to a person with ordinary skills in the particular field), and useful (to have at least one application). The relevant requirements vary across jurisdictions and are currently stricter in the EU than in the U.S. (Eckert and Langinier, 2014). In the spirit of Crampes and Langinier (2009), we model patentability requirement as a minimum investment threshold level that must be satisfied in order to obtain a patent. We then vary this investment threshold to examine whether a stricter patentability requirement fosters more green innovation.[8]

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monetary fees associated with the application process range on average between \$5000-\$15000. Second, the average waiting time for patents to be granted is about three years, implying that the opportunity cost in terms of lost profits during the waiting period can be significant. See Eckert and Langinier (2014).

Third, the potential litigation costs of enforcing a patent may be large enough to deter small firms from obtaining patents in several industries. According to the American Intellectual Property Law Association, the cost of an average patent lawsuit, where \$1 million to \$25 million is at risk, is \$1.6 million through the end of discovery and \$2.8 million through final disposition.

Fourth, there exist renewal costs which are monetary fees payable on a regular basis during the life time of the patent. For instance, in the U.S. a patent must be renewed three times at age 3, 7 and 14 years in order to be kept in force, whereas in Europe and in Canada the renewal fee is charged annually for 20 years.

[7] For further details, see https://www.ic.gc.ca/eic/site/cipointernet-internetopic.nsf/eng/wr02462.html

[8] Some studies have illustrated that strong Intellectual Property Rights may not necessarily enhance innovation (Green and Scotchmer, 1995; Gallini, 2002; Bessen and Maskin, 2009). Even though in a static world (single innovation), patents of appropriate scope can encourage innovations (Klemperer, 1990; Gilbert and Shapiro, 1990), this is no longer the case when the cumulative nature of innovation is accounted for. In the case of cumulative innovations, strict patentability requirement may even discourage follow-on innovations (Scotchmer, 1991). The prospect of being imitated inhibits inventors in a static world but, in a dynamic world, imitators can benefit both the original inventor and society (Bessen and Maskin, 2009). In our paper, we abstract away from these issues and present an alternative mechanism through which stronger patentability requirements affect innovation.

Another paper that examines these patent policy tools in the context of green innovations is Langinier and Ray Chaudhuri (2020). While Langinier and Ray Chaudhuri (2020) consider an incumbent monopolist which faces potential entry if it does not innovate, this paper generates novel insights by considering a Bertrand duopoly. This key difference in the market structure enables us to endogenize the licensing decision within a framework where multiple firms behave strategically, whereas Langinier and Ray Chaudhuri (2020) does not allow for licensing of the green innovation.[9] Thus, in our setting, while the patent holding firm decides whether to license its innovation, the rival firm evaluates whether to purchase the license and compete in a homogenous cleaner product (i.e., a product made by using the cleaner innovation) or to differentiate its product from the patent holder by producing the dirtier product (i.e., a product made by using the more polluting technology). This decision is impacted by the existence of environmentally friendly consumers that are heterogeneous in their preferences.

It is important to take into account environmentally friendly consumers given the increasing environmental consciousness of citizens globally which is, for instance, reflected in widely used eco-labeling schemes internationally.[10] Papers that study optimal environmental policies in the presence of environmentally friendly consumers do not address green innovation (see e.g. Arora and Gangopadhyay, 1995; Cremer and Thisse, 1999; Moraga-Gonzalez and Padron-Fumero, 2002; Bansal and Gangopadhyay, 2003; Lombardini-Riipinen, 2005; Bansal, 2008).[11]

Our framework analyzes the market of a product, the production of which causes pollution. The implementation of a cleaner technology is assumed to reduce the emission per unit of output ratio (similar to, for example, Benchekroun and Ray Chaudhuri, 2014, 2015). We further assume that the product is vertically differentiated in terms of its emission-output ratio with green conscious consumers preferring products with lower emission-output ratios (similar to Bansal

[9] Another difference between this paper and Langinier and Ray Chaudhuri (2020) is that while we model investment in R&D to generate green innovation as a discrete variable, Langinier and Ray Chaudhuri (2020) models investment as a continuous variable.

[10] For example, in countries like Sweden about 50% of the market share for certain products consists of the environmentally friendly variant. Green marketing is also frequently used to influence consumer behavior in transportation and electricity markets (Kraftborsen, 2001).

[11] An exception is Langinier and Ray Chaudhuri (2020). Gil-Moltó and Varvarigos (2013) examine adoption of clean technologies in the presence of green consumers.

and Gangopadhyay, 2003; Ibañez and Grolleau, 2008). We incorporate heterogeneity across consumers in terms of their degree of environmental friendliness. Our model is applicable to a wide variety of products, the production processes of which are becoming cleaner. Consider for example, products such as toys, furniture and packaging, the "greenness" of which may depend on the proportion of inputs used in the production process that are recycled.[12] Another example is energy production, the "greenness" of which may depend on the proportion of renewable energy sources used in the production process.

Our framework has two stages where, in the first stage, each firm decides its level of investment in R&D. If an innovation has been discovered by a firm, it decides whether to obtain a patent. If both firms invest the same amount and apply for patents, only one of them obtains the patent with probability 1/2. The patent holder then decides whether to sell a license to the other firm for the use of the green innovation. The firm that does not obtain the patent may decide to purchase the license and produce the cleaner product, to produce the dirty product, or to not produce. In the second stage, the firms engage in price competition in the product market.

We find that in the benchmark scenario where there are no environmentally friendly consumers, licensing always occurs in equilibrium. At the same time, increasing a per unit emission tax beyond a certain threshold has the paradoxical effect of increasing emissions by making investment in green innovation unprofitable. Decreasing patenting costs or making patentability requirements stricter mitigate this paradox by increasing the tax threshold beyond which the paradox occurs. Making the patentability requirement stricter has the added benefit of yielding a lower level of emissions as long as the tax is below the threshold, as compared to reducing patenting costs.

In the presence of environmentally friendly consumers, a key difference arises in terms of licensing. In contrast to the benchmark scenario without environmentally friendly consumers, when the proportion of environmentally friendly consumers is sufficiently high, licensing does not occur in equilibrium, and the firms differentiate their products by using production technologies with different emission-output ratios. Therefore, when the proportion of environmentally

[12] Over the period 2005-2016, the volume of recycling in Europe increased by 34%.

friendly consumers is high, policy makers may consider setting a technology standard which would reduce emissions by effectively forcing licensing to occur.

For sufficiently large proportions of environmentally friendly consumers, we obtain a useful result. Increasing the emission tax beyond a certain threshold induces licensing in equilibrium, thereby causing emissions to fall discretely. At the same time, we note that, similar to the benchmark scenario without environmentally friendly consumers, there exists a second threshold level of the tax beyond which the paradoxical effect of taxation applies, which can be avoided by decreasing patenting costs and/or making patentability requirements stricter.

In sum, the impact of emission tax depends on the proportion of consumers who are environmentally friendly. If this proportion is low, we obtain a paradox such that increasing emission taxes could lead to an increase in emissions. For sufficiently large proportions of consumers who are environmentally friendly, increasing the emission tax from a low to an intermediate level leads to a discrete fall in emission by inducing licensing to occur in equilibrium, while further increases in the tax could lead to the paradox. In these cases, decreasing patenting costs and/or making the patentability requirements stricter may help to mitigate this paradox and make emission taxes more effective in inducing innovation.

The paper is organized as follows. In Section 2, we present our model. In Section 3, we present the benchmark case without green consumers. In Section 4, we derive the equilibrium and policy implications for the case with green consumers. Section 5 presents our concluding remarks. All proofs have been relegated to the Appendix.

## 2 Model Setting

We consider a two-stage model in which two firms, firms 1 and 2, sell a final good to consumers, the production of which is polluting and has a marginal cost, $c$. For each firm $i$, $i = 1, 2$, the emission of the pollutant generated per unit of production is given by:

$$\gamma_i \equiv \frac{e_i}{q_i}, \tag{1}$$

where $e_i$ denotes emission and $q_i$ denotes output. Firm $i$ can invest $I_i$ to reduce its emission-output ratio, $\gamma_i$. For simplicity, we assume that $I_i$ is a discrete variable that can take the following values: 0, and $I_P > 0$. Thus, the emission-output ratio is given by:

$$\gamma_i = \begin{cases} \gamma_H & \text{if } I_i = 0 \\ \gamma_P & \text{if } I_i = I_P \end{cases} \tag{2}$$

with $\gamma_H \equiv \gamma_P + \Delta$, where $\Delta > 0$, and where $\gamma_H$ represents the emission-output ratio of the dirty product (i.e. the product that both firms produce without any innovation), and $\gamma_P$ represents the emission-output ratio of the cleaner product (i.e. the product that is made using the green innovation). We note that $I_P$ is a function of $\Delta$ with $I'_P(\Delta) > 0$, i.e., a higher level of investment is required to obtain a larger decrease in emission-output ratio. We also assume that $I''_P(\Delta) > 0$. Henceforth, for notational convenience we do not include the argument of the function $I_P$ unless it plays a role in our analysis.

## 2.1 The demand side

The demand side consists of a continuum of $N$ consumers. Each of them buys either 0 or 1 unit of the good. There exists a fraction $\lambda$ of 'green conscious' consumers, whose utility is increasing in the "greenness" of the product, that is, decreasing in $\gamma_i$, and a fraction $(1 - \lambda)$ of 'non-green conscious' consumers, whose utility is independent of the greenness of the product.

Let $G$ denote the degree of environmental friendliness of a consumer, with $G$ being uniformly distributed over the interval $\left[\underline{G}, \overline{G}\right]$ with $\underline{G} > 0$. We assume that consumers can observe how green a product is. Within this context, this is equivalent to assuming that consumers can observe $\gamma$.[13] We normalize $N$ such that $N = 1$, $\overline{G} - \underline{G} = 1$. Let $P(e)$ denote the pollution damage to each consumer, which is a function of total emissions, $e = e_1 + e_2$. Following Bansal and Gangopadhyay (2003), Ibañez and Grolleau (2008) and Langinier and Ray Chaudhuri (2020), we assume that the pollution level generated by total production is exogenous to each consumer, regardless of her/his consumption level. Let $p_1$ and $p_2$ denote the product prices set by firm 1 and firm 2, respectively.

A green conscious consumer has the following utility function

$$U_G = \begin{cases} v - \gamma_i G - p_i - P(e) & \text{from buying the product from firm } i \\ -P(e) & \text{from not buying} \end{cases} \tag{3}$$

[13] This is a relevant scenario to consider in the presence of effective eco-labeling programs.

for $i = 1, 2$. The term $-\gamma_i G$ in (3) reflects that the greener the product from firm $i$, that is, the lower is $\gamma_i$, the better off the green conscious consumer. Also, $v$ represents the gross utility of consuming one unit of the good. A green conscious consumer does not buy the product if $v - \gamma_i G - p_i < 0$. Note that if $\gamma_1 = \gamma_2 = \gamma$, $U_G = v - \gamma G - \min\{p_1, p_2\} - P(e)$ from buying the product.

A non-green conscious consumer has the following utility function

$$U_{NG} = \begin{cases} v - \min\{p_1, p_2\} - P(e) & \text{from buying the product} \\ -P(e) & \text{from not buying} \end{cases} \tag{4}$$

A non-green consumer buys the product only if $v \geq \min\{p_1, p_2\}$. Henceforth, we assume that $P(e) = e$.

If firm 1 sells the greener product, such that $\gamma_1 < \gamma_2$, a green conscious consumer $G$ buys the greener product from firm 1 rather than the dirty product from firm 2 as long as $v - \gamma_1 G - p_1 - P(e) > v - \gamma_2 G - p_2 - P(e)$, that is, as long as $G \in \left(\frac{p_1 - p_2}{\gamma_2 - \gamma_1}, \overline{G}\right]$. Also, a green conscious consumer buys the green product from firm 1 rather than not buying anything as long as $v - \gamma_1 G - p_1 - P(e) > -P(e)$, that is, $G < (v - p_1)/\gamma_1$.

There exists a green conscious consumer, $\widetilde{G}_1$, who is indifferent between buying the greener product and the dirty product such that $v - \gamma_1 G - p_1 - P(e) = v - \gamma_2 G - p_2 - P(e)$, or $\widetilde{G}_1 \equiv \frac{p_1 - p_2}{\gamma_2 - \gamma_1}$. There exists a green conscious consumer, $\widetilde{G}_2$, who is indifferent between buying the greener product and not buying anything such that $v - \gamma_1 G - p_1 - P(e) > -P(e)$, or $\widetilde{G}_2 \equiv \frac{v - p_1}{\gamma_1}$. As long as $\underline{G} < \widetilde{G}_1 < \widetilde{G}_2 < \overline{G}$, some green conscious consumers buy the dirty product, some buy the greener product, and others buy nothing.

If both firms sell the same product such that $\gamma_1 = \gamma_2 = \gamma$, a green conscious consumer buys the good if $v - \gamma G - \min\{p_1, p_2\} - P(e) > -P(e)$, or $G < (v - \min\{p_1, p_2\})/\gamma$. There exists a green conscious consumer, $\widetilde{G}_3$, who is indifferent between buying the product and not buying anything such that $v - \gamma G - \min\{p_1, p_2\} - P(e) = -P(e)$, or $\widetilde{G}_3 \equiv \frac{v - \min\{p_1, p_2\}}{\gamma}$. As long as $\underline{G} < \widetilde{G}_3 < \overline{G}$, some, but not all, green conscious consumers buy the product.

Next, we derive the demand function for each of the two firms when $\underline{G} < \widetilde{G}_1 < \widetilde{G}_2 < \overline{G}$ and $\underline{G} < \widetilde{G}_3 < \overline{G}$. If $\gamma_1 = \gamma_2 = \gamma$, we have the following demand function for firm $i$, for $i, j = 1, 2$

and $i \neq j$:

$$D_i(p_i, p_j; \gamma) = \begin{cases} 0 & \text{if } p_i > p_j \\ \frac{1}{2}\lambda(\frac{v-p_i}{\gamma} - \underline{G}) + \frac{1}{2}(1-\lambda) & \text{if } p_i = p_j \leq v \\ \lambda(\frac{v-p_i}{\gamma} - \underline{G}) + (1-\lambda) & \text{if } p_i < p_j \leq v \end{cases} \tag{5}$$

When $\gamma_1 = \gamma_2$, (5) shows that if $p_i > p_j$, then firm $i$ does not attract any consumers as only prices matter. If $p_i = p_j$, firms split the market equally, with $(1-\lambda)/2$ non-green consumers and $\lambda(\frac{v-p_i}{\gamma} - \underline{G})/2$ green consumers buying the product from firm $i$. If $p_i < p_j$, firm $i$ gets the entire demand.

If $\gamma_i = \gamma_P < \gamma_j = \gamma_H$, such that firm $i$ has the cleaner product, we have the following demand functions:

$$D_i(p_i, p_j; \gamma_P) = \begin{cases} \lambda(\frac{v-p_i}{\gamma_P} - \frac{p_i-p_j}{\Delta}) & \text{if } p_i > p_j \\ \lambda(\frac{v-p_i}{\gamma_P} - \underline{G}) + \frac{1}{2}(1-\lambda) & \text{if } p_i = p_j \leq v \\ \lambda(\frac{v-p_i}{\gamma_P} - \underline{G}) + (1-\lambda) & \text{if } p_i < p_j \leq v \end{cases} \tag{6}$$

and

$$D_j(p_i, p_j; \gamma_H) = \begin{cases} \lambda(\frac{p_i-p_j}{\Delta} - \underline{G}) + (1-\lambda) & \text{if } p_i > p_j \\ \frac{1}{2}(1-\lambda) & \text{if } p_i = p_j \leq v \\ 0 & \text{if } p_i < p_j \leq v \end{cases} \tag{7}$$

Recall that $\Delta = \gamma_H - \gamma_P$. If $p_i > p_j$, firm $j$ with the lower price attracts some green and some non-green consumers, whereas firm $i$ with the higher price only attracts a fraction of the green consumers. If $p_i = p_j$, each firm attracts half of the demand from non-green consumers and firm $i$ also captures the demand from the green consumers. That is, if $p_i = p_j$ and firm $i$ produces a greener product, green consumers only buy from firm $i$. If $p_i < p_j$, given that firm $i$'s product is cleaner and cheaper, nobody buys from firm $j$.

For completeness, we need to consider the cases where, in equilibrium, the conditions $\underline{G} < \widetilde{G}_1 < \widetilde{G}_2 < \overline{G}$ and/or $\underline{G} < \widetilde{G}_3 < \overline{G}$ are not satisfied. If $\underline{G} < \widetilde{G}_1 < \overline{G} < \widetilde{G}_2$, the market is entirely covered and both firms have part of the demand from green consumers. If $\widetilde{G}_1 < \underline{G} < \widetilde{G}_2 < \overline{G}$, only the firm that is producing the cleaner product will get some demand from green consumers, and the other firm does not get any demand. If $\widetilde{G}_1 < \underline{G} < \overline{G} < \widetilde{G}_2$, the firm producing the cleaner product gets the entire demand from green consumers while the other firm gets no demand from green consumers. Similarly, if $\underline{G} < \overline{G} < \widetilde{G}_3$, the entire market for green consumers is covered, while if $\widetilde{G}_3 < \underline{G} < \overline{G}$, none of the green consumers by the only (cleaner or dirty) product that is offered. These demands are presented in the appendix.

Furthermore, in order to have some demand from the green-consumers we assume that

$$\tau < \frac{v-c}{\gamma_H} - \frac{\gamma_P}{\gamma_H}\underline{G}.$$

## 2.2 Policy Tools

We consider a combination of policy tools. On the R&D side, we model a patenting policy for green innovations. On the environmental side, we assume that the firms must pay a tax, $\tau$, per unit of emission. Thus, the tax bill faced by each firm $i$ is given by $\tau\gamma_i D_i(.)$, where, by (1), $\gamma_i D_i(.)$ represents the emissions generated by the firm.

The patent policy is such that firm $i$ must discover a sufficiently novel innovation to be able to obtain a patent. We assume that novelty of the innovation is increasing in the investment level. In (2), $\Delta$ represents the inventive step, that is, the difference between the level of emission-output ratio before and after innovation. In order to obtain a patent, we assume that firm $i$ must reduce $\gamma_i$ by at least $\Delta$. A weak patentability requirement corresponds to a small value of $\Delta$, whereas a stronger patentability requirement corresponds to a larger value of $\Delta$. Henceforth, we hold $\gamma_H$ constant and allow $\Delta$ to vary in order to reflect changes in the patentability requirement. Effectively, a stronger patentability requirement corresponds to a lower value of $\gamma_P$.

Initially, both firms produce the dirty product with $\gamma_H$. As per (2), a firm must invest $I_P$ in order to reduce the emission-output ratio from $\gamma_H$ to $\gamma_P \equiv \gamma_H - \Delta$. However, if both firms invest the same amount and thereby obtain $\gamma_P$, only one of them obtains the patent with probability 1/2. The firm that does not obtain the patent may decide to produce the dirty product or not to produce.

In order to obtain a patent, each firm must also incur an exogenously given cost $C_P$, which is broadly defined to include a monetary application fee payable by the firm to the patent office, the opportunity cost in terms of lost profits incurred while waiting for the patent to be granted, renewal costs which are monetary fees payable on a regular basis during the life time,[14] as well

[14] For instance, in the U.S. a patent must be renewed three times at age 3.5, 7.5 and 11.5 years in order to be

potential litigation costs for enforcing the patent. There are several ways in which policy makers may reduce $C_P$, including by implementing a fast-track patent system for green technologies that reduces the patent application processing time for green innovations. We assume that the firm that does not obtain the patent does not incur the cost $C_P$, regardless of its choice of investment level. This is to reflect that typically patenting costs are higher for the patent holder than for other firms that apply but do not get a patent since some of the elements of $C_P$, such as renewal costs and potential litigation costs, are only incurred by the firm that obtains the patent. Therefore, although any firm that applies for a patent must pay the application fee, for simplicity, we assume that the patenting cost for the firm that does not obtain the patent is 0.[15]

## 2.3 Timing

The timing of the game is as follows. There are two stages. In the first stage of the game, each firm $i$, $i = 1, 2$, decides the level of investment in the green technology, $I_i \in \{0, I_P\}$. In the second stage, if an innovation has been discovered by firm $i$, it decides whether to obtain a patent.[16] Once firm $i$ has obtained a patent, firm $i$ decides whether to license the patent. If a license is offered, firm $j$ decides whether to accept it or refuse it. Given the licensing decision, each firm chooses its price of the product, $p_1$ and $p_2$.

We solve for the equilibrium investment levels, patenting and licensing decisions and prices through backward induction. For given levels of investment, patenting and licensing decisions, we first determine the pricing strategy of the firms. Then, we determine the licensing decision, the patenting decisions, and finally the levels of investment in equilibrium.

As a benchmark case, we first consider the scenario where there exist only non-green conscious consumers (i.e., $\lambda = 0$) in Section 3. Next, we enrich our analysis by including both green and

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kept in force, whereas in Europe and in Canada the renewal fee is charged annually for 20 years. In the U.S. the average renewal fees are US$2000, US$3760 and US$7700 (USPTO website).

[15] Adding a positive application fee for the firm that applies for but does not obtain the patent would not generate any new insights.

[16] We make the simplifying assumption that an innovation will be discovered with probability 1. If we alternatively assumed that an innovation will be discovered with an exogenously given probability less than 1, this would make the notation and analysis more cumbersome without generating any new insights.

non-green conscious consumers (i.e., $0 < \lambda \leq 1$), in Section 4.

# 3 Benchmark Case: Non-Green Conscious Consumers

## 3.1 Second Stage: Price Competition

Let us begin with the second stage of the game. The non-green consumers care only about prices, and buy from the firm that offers the lower price. From (7), the demand for firm $i$ with $i \neq j$ for $i, j = 1, 2$ is given by:

$$D_i(p_i, p_j) = \begin{cases} 1 & \text{if} \quad p_i < p_j \text{ and } p_i \leq v \\ \frac{1}{2} & \text{if} \quad p_i = p_j \leq v \\ 0 & \text{if} \quad p_i > \min\{p_j, v\} \end{cases} \tag{8}$$

For any combination of investment levels made by both firms in the first stage of the game, we determine their pricing strategy in the second stage. Recall that firms choose from two levels of investment: 0, in which case the dirty good is produced, and $I_P$, which is the requirement to patent the innovation of size $\Delta = \gamma_H - \gamma_P$. There are three possible cases that may arise: the first where neither firm invests, the second where one firm invests and the other one does not, and the third where both firms invest.

In the first case, when none of the firms invest ($I_1 = I_2 = 0$), we obtain the classical finding of Bertrand competition; both firms set prices at marginal cost, $p_1 = p_2 = c + \tau\gamma_H$, and, thus, their payoffs are zero. They both produce the dirty good.

In the second case, when firm $i$ decides not to invest and firm $j$ decides to invest $I_P$, and apply for a patent, firm $j$ must decide whether to license its innovation to firm $i$. If firm $j$ does not license, or if firm $i$ refuses the license agreement, firm $i$ produces the dirty good corresponding to $\gamma_H$, and firm $j$ produces the cleaner good corresponding to $\gamma_P$. The payoff of firm $i$ is

$$D_i(p_i, p_{jP})(p_i - c - \tau\gamma_H),$$

and the payoff of firm $j$ is

$$D_j(p_i, p_{jP})(p_{jP} - c - \tau\gamma_P).$$

Each firm chooses the price that maximizes its payoff. The Nash equilibrium is $p_{jP} = c + \tau\gamma_H - \varepsilon$ such that firm $i$ gets a payoff of zero (and does not produce) while firm $j$ obtains $\Delta\tau - C_P$, and produces the cleaner product, for $i, j = 1, 2$ and $i \neq j$. In the case where only firm $j$ invests $I_P$, it patents if $\tau \geq C_P/\Delta$, and, thus, firm $j$ produces a cleaner product while firm $i$ does not produce. However, if $\tau < C_P/\Delta$, firm $j$ does not patent, both firms produce the dirty product and get zero payoffs.

If firm $j$ offers a license to firm $i$ and firm $i$ accepts it, both firms produce the cleaner good $\gamma_P$. Firm $j$ offers a license $(r, F)$ where $r$ is the per-unit royalty rate, and $F$ a fixed fee. For any license $(r, F)$ accepted by firm $i$, firm $j$ obtains the following payoff:

$$D_j(p_i, p_{jP})(p_{jP} - c - \tau\gamma_P) + rD_i(p_i, p_{jP}) + F.$$

Firm $i$ obtains the following payoff:

$$D_i(p_i, p_{jP})(p_i - c - \tau\gamma_P - r) - F.$$

We find that for a given $(r, F)$, there is a unique Nash equilibrium in prices such that $p_i^* = p_{jP}^* = c + \tau\gamma_P + r$, and firm $j$ obtains the payoff $(r + F)$ while firm $i$ obtains the payoff $(-F)$.

Firm $j$ will choose $(r, F)$ that maximize its profit $(r + F)$ subject to the constraint that $-F \geq 0$ and $c + \tau\gamma_P + r \leq v$. Thus, $F^* = 0$, and $r^* = v - c - \tau\gamma_P$. The Subgame Perfect Nash equilibrium is such that firm $j$ offers a license $(r^* = v - c - \tau\gamma_P, F^* = 0)$, and both firms choose prices $p_i^* = p_{jP}^* = v$, so that the payoff of firm $j$ is $v - c - \tau\gamma_P$, while the payoff of firm $i$ is null. Firm $j$ decides to patent as long as $C_P \leq v - c - \tau\gamma_P$.

Firm $j$ decides to license if $v - c - \tau\gamma_P \geq \Delta\tau$, which is always satisfied as, by assumption, we have $\tau < (v - c)/\gamma_H$.[17] Therefore, in the case where one firm invests and the other one does not, if the firm that invests patents its innovation, it will always license it to the other firm.

Let us consider the third case where both firms decide to invest $I_P$, and both firms decide to apply for a patent. In this case, each firm gets the patent (and pays the cost $C_P$) with probability 1/2. Then, as we have seen above, the firm with the patent will always prefer to

[17] For firms to make non-negative profits in equilibrium, we must have that $v - c - \tau\gamma_H \geq 0$.

license its innovation to the other firm. Thus, the expected payoff of firm $i$ is given by

$$\frac{1}{2}\underbrace{[D_i(p_{iP}, p_j)(p_{iP} - c - \tau\gamma_P) - C_P]}_{(i)} + \frac{1}{2}\underbrace{D_i(p_i, p_{jP})(p_i - c - \tau\gamma_P)}_{(ii)}, \tag{9}$$

for $i = 1, 2$, where $p_i^* = p_{jP}^* = v$. Thus, $(i)$ in (9) represents the payoff of firm $i$ if it gets the patent, and $(ii)$ represents the payoff if it does not get the patent. Therefore, if both firms invest, only one of them obtains a patent and licenses its technology to the other. Thus, each firm gets

$$\frac{1}{2}(v - c - \tau\gamma_P - C_P),$$

such that the condition to apply for a patent is given by $C_P \leq v - c - \tau\gamma_P$.

### 3.2 First stage: Investment Decisions

We now turn to the investment decisions of the firms. Since for $C_P > v - c - \tau\gamma_P$, none of the firms apply for a patent, they would earn zero payoff even if they were to invest. For this reason, firms have no incentive to invest as long as $C_P > v - c - \tau\gamma_P$. To summarize the analysis of the previous subsection, if none of the firms invest, they each obtain a null payoff. If one firm invests $I_P$ and the other does not, the one that invests, licenses its innovation and has a payoff of $v - c - \tau\gamma_P - C_P - I_P$, while the other has a null payoff. If both of them invest $I_P$, each of them obtains $\frac{1}{2}(v - c - \tau\gamma_P - C_P) - I_P$.

Let

$$C_1(\tau) \equiv v - c - \tau\gamma_P - I_P, \tag{10}$$

and

$$C_2(\tau) \equiv v - c - \tau\gamma_P - 2I_P, \tag{11}$$

with $C_1(\tau) > C_2(\tau)$ for any $\tau$.

If $C_P \in [0, C_2(\tau)[$, the best response of firm $i$ to any investment $I_j$ by firm $j$ is $BR_i(I_j) = I_P$. If $C_P \in [C_2(\tau), C_1(\tau)[$, the best response of firm $i$ to any investment $I_j$ by firm $j$ is

$$BR_i(I_j) = \begin{cases} I_P & \text{if } I_j = 0 \\ 0 & \text{if } I_j = I_P \end{cases}$$

If $C_P \geq C_1(\tau)$, the best response of firm $i$ to any investment $I_j$ by firm $j$ is $BR_i(I_j) = 0$.

Thus, for $C_P \in [0, C_2(\tau)[$, the unique Nash equilibrium is $(I_P, I_P)$, that is, both firms invest. For $C_P \in [C_2(\tau), C_1(\tau)[$, there exists two Nash equilibria $(0, I_P)$ and $(I_P, 0)$ such that one firm does not invest while the other invests $I_P$. For $C_P \geq C_1(\tau)$, there exists a unique Nash equilibrium $(0, 0)$ in which none of the firms invest.

In Figure 1, we represent the functions $C_1(\tau)$ and $C_2(\tau)$ in a graph where the horizontal axis represents the emission tax $\tau$, and the vertical axis represents the patenting cost $C_P$.

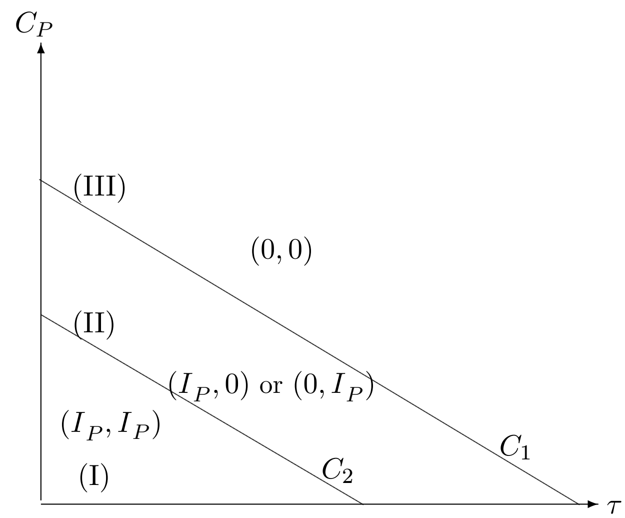


Figure 1: Equilibrium Investments with Non-Green Consumers

For a given patenting cost, we obtain a paradoxical result whereby increasing the emission tax beyond a certain threshold leads to less innovation as the tax bill increases. Moreover, the higher the patenting cost, the greater the range of taxes for which the paradox occurs.

The Subgame Perfect Nash Equilibrium is presented in the following Proposition.

**Proposition 1** *For a given $\tau$, the Subgame Perfect Nash Equilibrium is*

1. *if $C_P \in [0, C_2(\tau)[$, both firms invest $(I_1^*, I_2^*) = (I_P, I_P)$, but only one of them gets a patent and licenses it to the other firm ($r^* = v - c - \tau\gamma_P, F^* = 0$); both firms produce the cleaner product at price $p_1^* = p_2^* = v$;*

2. *if $C_P \in [C_2(\tau), C_1(\tau)[$, firm $j$ invests while firm $i$ does not $(I_j^*, I_i^*) = (I_P, 0)$ for $i, j = 1, 2$*

*and $i \neq j$; firm $j$ gets a patent and licenses it to firm $i$ ($r^* = v - c - \tau\gamma_P, F^* = 0$); both firm produce the cleaner product at price $p_i^* = p_j^* = v$;*

3. *if $C_P \geq C_1(\tau)$, none of the firms invest $(I_1^*, I_2^*) = (0,0)$, they both produce the dirty product, and set the price $p_1^* = p_2^* = c + \tau\gamma_H$.*

As illustrated by Figure 1, we note that for a given patenting cost $C_P$, as $\tau$ increases firms invest less.

### 3.3 Equilibrium Emission levels

Before presenting the equilibrium emission levels, it is useful to define the following thresholds on $\tau$. Let

$$\tau_1 \equiv \tfrac{v-c-C_P-I_P}{\gamma_P}, \tag{12}$$

and

$$\tau_2 \equiv \tfrac{v-c-C_P-2I_P}{\gamma_P}, \tag{13}$$

with $\tau_1 > \tau_2$, where both $\tau_1$ and $\tau_2$ are decreasing with $C_P$ and $I_P$.

**Proposition 2** *In equilibrium, the total emission level is given by:*

$$e^* = \begin{cases} \gamma_P & \textit{for } \tau \leq \tau_1 \\ \gamma_H & \textit{for } \tau > \tau_1 \end{cases} \tag{14}$$

Figure 2 illustrates the equilibrium emission level. For $\tau > \tau_1$, neither firm invests which results in both firms producing the dirty product. Each firm has half the demand, such that $e^* = \gamma_H$. For $\tau \in [\tau_2, \tau_1[$, only one firm invests, gets a patent and licenses it to the other firm, such that both firms produce the cleaner technology and the emission is $e^* = \gamma_P$. For $\tau \leq \tau_1$, both firms invest, one of them obtains a patent and licenses it to the other firm such that we have $e^* = \gamma_P$.

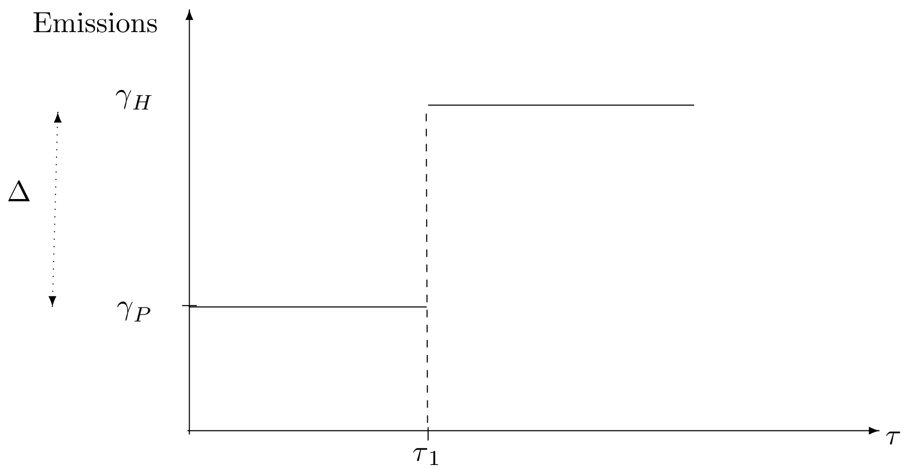


Figure 2: Equilibrium Emission with Non-Green Consumers

By Proposition (2) and as illustrated by Figure 2, for a given patenting cost, we obtain a paradoxical result as highlighted in the following Corollary.

**Corollary 1**: *Increasing the emission tax beyond the threshold $\tau_1$ leads to a discrete increase in the emission level.*

Moreover, as illustrated by Figure 2.1, the higher the patenting cost, the greater the range of taxes for which the paradox occurs as $\tau_1$ is a decreasing function of $C_P$. Thus, reducing the patenting cost $C_P$ increases $\tau_1$, such that a lower emission level is realized for a larger range of taxes.

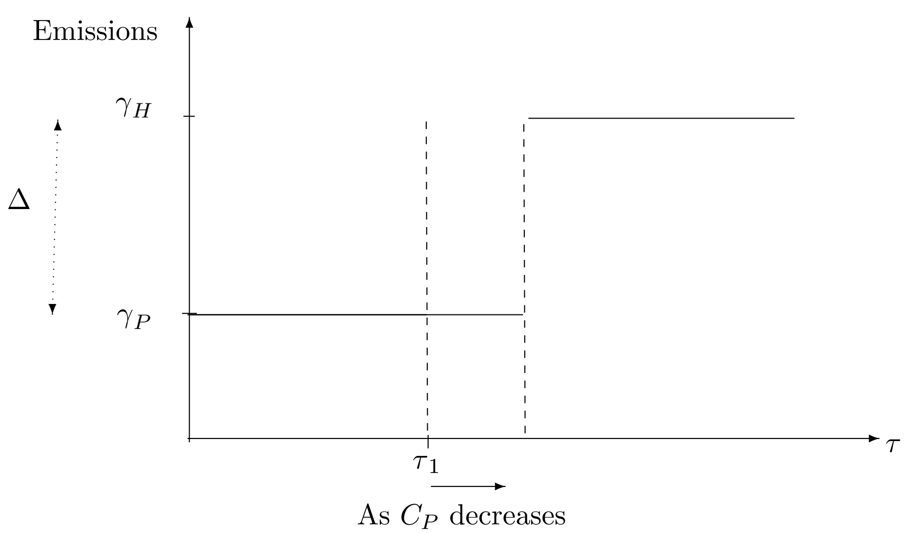


Figure 2.1: Equilibrium Emission with Non-Green Consumers as $C_P$ Decreases

Making the patentability requirement more stringent by reducing the level of $\gamma_P$ also increases $\tau_1$, as shown by Figure 2.2. Moreover, since $\gamma_P$ represents the emission level itself, a lower emission level is achieved for $\tau < \tau_1$ by making the patentability requirement more stringent compared to that obtained by reducing the patenting cost $C_P$.

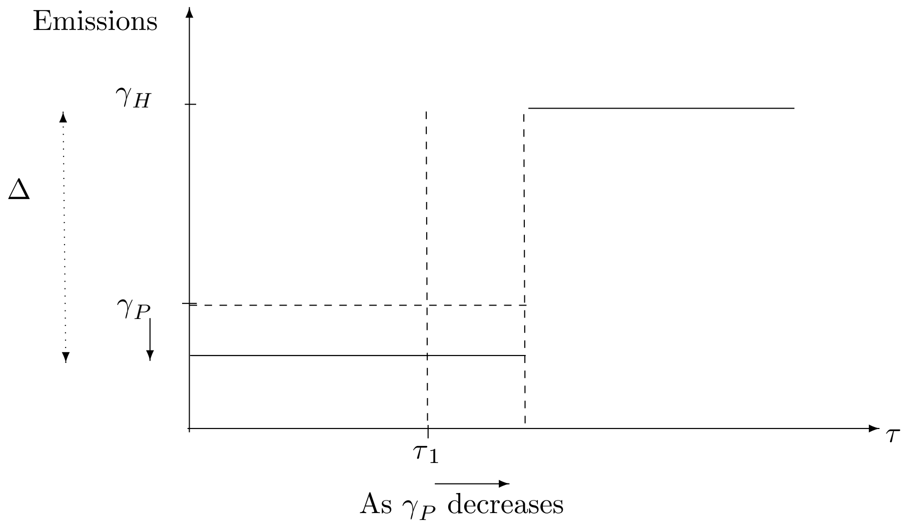


Figure 2.2: Equilibrium Emission with Non-Green Consumers as $\gamma_P$ Decreases

We summarize our results in the following Proposition.

**Proposition 3** *With only non-green consumers, i.e.,* $\lambda = 0$,

*(i) either reducing patenting cost* $C_P$ *or making the patentability requirement more stringent by reducing the level of* $\gamma_P$ *lead to a higher* $\tau_1$, *thereby reducing the emission level from* $\gamma_H$ *to* $\gamma_P$ *for a greater range of emission taxes.*

*(ii) making the patentability requirement more stringent leads to a lower emission level for* $\tau < \tau_1$ *compared to that obtained by reducing the patenting cost* $C_P$.

Proposition (3 i) follows from (12) which shows that $\tau_1$ is decreasing in $C_P$ and $\gamma_P$. It implies that a policy maker may use either instrument, i.e. reducing $C_P$ or making the patentability requirement more stringent, or some combination of both to mitigate the paradoxical result associated with increasing the emission tax. Thus, both these instruments work in conjunction with the emission tax to allow the emission tax to function more smoothly.

At the same time, Figures 2.1-2.2 shows that at a given value of $\tau$ such as $\tau = \tau_1 + \varepsilon$ with $\varepsilon > 0$, by (3 ii), making the patentability requirement more stringent is more effective in terms of reducing emission levels than reducing the patent cost. Moreover, at a given value of $\tau < \tau_1$, only making the patentability requirement more stringent reduces the emission level by reducing $\gamma_P$ whereas reducing $C_P$ is ineffective and does not affect the emission level. This is summarized in the Corollary.

**Corollary 2**: *With only non-green consumers, i.e., $\lambda = 0$, at any given level of $\tau$, making the patentability requirement more stringent is more effective in terms of reducing emission levels than reducing the patent cost, $C_P$.*

It is useful to compare our findings with Langinier and Ray Chaudhuri (2020). Recall that the main differences between this paper and Langinier and Ray Chaudhuri (2020) are the following. First, the market structure is different since Langinier and Ray Chaudhuri (2020) consider an incumbent monopolist which faces potential entry if it does not innovate. Second, Langinier and Ray Chaudhuri (2020) does not allow for licensing of the green innovation. Third, Langinier and Ray Chaudhuri (2020) model investment as a continuous variable while we model investment as a discrete variable.

In the absence of any green consumers, Langinier and Ray Chaudhuri (2020) also obtain the paradoxical result that we have reported in Proposition (2), as well as a similar effect of reducing patenting costs. However, it is important to note that this does not imply that competition (i.e. duopoly versus monopoly) does not matter in determining policy implications. Rather, the reason we obtain these results is that we allow for licensing while Langinier and Ray Chaudhuri (2020) does not. We would not obtain these results in our duopoly setting if we did not allow for licensing. Within a duopolistic setup, licensing effectively allows the innovator to behave as a monopolist.[18] In the absence of licensing, we would obtain very different results as firms would be engaged in a tough price competition. In both papers, as the tax rate increases, the cost to the innovator increases (Effect 1). This is the only effect of tax increase captured by Langinier and Ray Chaudhuri (2020), which results in the innovation becoming unprofitable

[18] Recall that with licensing prices are set to $v$, which corresponds to the monopoly price. Also, the patent holder extracts all the surplus from the licensee, effectively obtaining monopoly profits.

beyond a certain tax rate. In this paper, there is a second effect of a tax increase arising due to duopolistic competition. The price of the cleaner product is set at the marginal cost (which includes the marginal tax, $\tau\gamma_H$) of the firm producing the dirty product. This implies that the price of the cleaner product increases in $\tau$ (Effect 2) in the absence of licensing, which results in equilibrium investment and emission levels that are different from Proposition 1. However, because of the license, Effect 2 vanishes, since prices are set to $v$, so that only Effect 1 matters. For this reason, we obtain results similar to Langinier and Ray Chaudhuri (2020) after allowing for licensing, despite the different market structure.

# 4 Green and Non-Green Consumers

In this section, we continue our analysis with the general model introduced in Section 2 with green consumers, that is, $0 < \lambda \leq 1$.

## 4.1 Second Stage: Price competition

We begin with the second stage and analyze the firms' pricing strategy for any combination of investment levels of both firms in the first stage of the game.

Recall that firms choose from two levels of investment: 0 and $I_P$. As in Section 3, three possible cases may arise: the first where neither firm invests, the second where one firm invests and the other one does not, and the third where both firms invest.

Furthermore, in the second and third cases (if one or both firms invest), once one of the firms is granted a patent, it must decide whether to license its innovation or not. We first consider the case where the patentholder decides not to license. Then, we study the case where the patentholder decides to license.

If none of the firms invest ($I_1 = I_2 = 0$), the demand for each firm is given by (5), both firms set their prices at marginal cost, $p_1 = p_2 = c + \tau\gamma_H$, and, thus, their payoffs are zero.

Let us consider the case where firm $i$ decides to invest $I_P$, while firm $j$ does not, where $i, j = 1, 2$ and $i \neq j$. Once firm $i$ discovers the innovation, it first decides whether to obtain a patent or not. If it does patent its innovation and pays the patenting cost $C_P$, then firm $i$

decides whether to license its innovation to firm $j$ or not.

If firm $i$ decides not to license, its payoff is $D_i(p_{iP}, p_j; \gamma_P)(p_{iP}-c-\tau\gamma_P)$, and the payoff of firm $j$ is $D_j(p_{iP}, p_j; \gamma_H)(p_j - c - \tau\gamma_H)$ where $p_{iP}$ is firm $i$'s price, $p_j$ is firm $j$'s price, $D_i(p_{iP}, p_j; \gamma_P)$ is firm $i$'s demand which is given by (6), and $D_j(p_{iP}, p_j; \gamma_H)$ is firm $j$'s demand which is given by (7) if $\underline{G} < \widetilde{G}_1 < \widetilde{G}_2 < \overline{G}$ is satisfied. Thus, firm $i$ chooses the price $p_{iP}$ that maximizes

$$\lambda(\frac{v - p_{iP}}{\gamma_P} - \frac{p_{iP} - p_j}{\Delta})(p_{iP} - c - \tau\gamma_P),$$

and firm $j$ chooses the price $p_j$ that maximizes

$$(\lambda(\frac{p_{iP} - p_j}{\Delta} - \underline{G}) + (1 - \lambda))(p_j - c - \tau\gamma_H),$$

where $p_{iP} > p_j$. We illustrate later that this condition holds in equilibrium for $\tau < \tau_E(\lambda)$, where

$$\tau_E(\lambda) \equiv \frac{v - c}{2\gamma_H} + \frac{2\gamma_H - \gamma_P}{2\gamma_H}(\underline{G} - \frac{1 - \lambda}{\lambda}), \tag{15}$$

which is increasing with $\lambda$. There is a positive value of $\lambda$, $\lambda_0$, such that $\tau_E(\lambda_0) = 0$, which is

$$\lambda_0 \equiv \frac{2\gamma_H - \gamma_P}{v - c + (2\gamma_H - \gamma_P)(\underline{G} + 1)}. \tag{16}$$

The best response of firm $i$ to any price $p_j$ set by firm $j$ is

$$p_{iP}(p_j) = \frac{1}{2}(c + \tau\gamma_P + v\frac{\Delta}{\gamma_H} + \frac{\gamma_P}{\gamma_H}p_j),$$

and the best response function of firm $j$ to any price $p_{iP}$ set by firm $i$ is

$$p_j(p_{iP}) = \frac{1}{2}(c + \tau\gamma_H - \Delta\underline{G} + \tfrac{1-\lambda}{\lambda}\Delta + p_{iP}).$$

Solving for the equilibrium values for $p_{iP}$ and $p_j$, we find

$$p_{iP}^G = \frac{1}{4\gamma_H - \gamma_P}[c(2\gamma_H + \gamma_P) + 2v\Delta + 3\tau\gamma_H\gamma_P - \gamma_P\Delta\underline{G} + \gamma_P\Delta\frac{1 - \lambda}{\lambda}], \tag{17}$$

and

$$p_j^G = \frac{1}{4\gamma_H - \gamma_P}[3\gamma_H c + v\Delta + \gamma_H\tau(2\gamma_H + \gamma_P) - 2\Delta\gamma_H\underline{G} + 2\Delta\gamma_H\frac{1 - \lambda}{\lambda}]. \tag{18}$$

As $\lambda$ increases, both equilibrium prices decrease, and as $c$ and $v$ increase, both prices increase. We verify that $p_{iP}^G > p_j^G$ only if $\tau < \tau_E(\lambda)$. We also verify under what conditions $\underline{G} < \widetilde{G}_1 < \widetilde{G}_2 < \overline{G}$ is satisfied, which are for $\tau \in [\tau_C(\lambda), \tau_L(\lambda)]$ where

$$\tau_L(\lambda) \equiv \frac{v - c}{2\gamma_H} - \underline{G} - \frac{2\gamma_H - \gamma_P}{2\gamma_H}\frac{1 - \lambda}{\lambda} < \tau_E(\lambda), \tag{19}$$

and

$$\tau_C(\lambda) \equiv \frac{2\gamma_H + \gamma_P}{\gamma_H\gamma_P}\frac{v-c}{3} - \underline{G} - \frac{\gamma_H - \gamma_P}{3\gamma_H}\frac{1-\lambda}{\lambda} - \frac{4\gamma_H - \gamma_P}{3\gamma_H}. \tag{20}$$

Evaluated at prices $p_{iP}^G$ and $p_j^G$ as defined by (17) and (18), the demands for the cleaner and the dirty products are, respectively,

$$D_i(p_{iP}^G, p_j^G; \gamma_P) = \frac{\lambda}{4\gamma_H - \gamma_P}\frac{\gamma_H}{\gamma_P}(2(v-c) - \gamma_P\tau - \gamma_P\underline{G} + \gamma_P\frac{1-\lambda}{\lambda}), \tag{21}$$

and

$$D_j(p_{iP}^G, p_j^G; \gamma_H) = \frac{\lambda}{4\gamma_H - \gamma_P}(v - c - 2\tau\gamma_H - 2\underline{G}\gamma_H + 2\gamma_H\frac{1-\lambda}{\lambda}). \tag{22}$$

As long as $\tau \in [\tau_C(\lambda), \tau_L(\lambda)]$, both demands are strictly positive in equilibrium and the market for green consumers is not covered. Both demands decrease as $\tau$ increases. However, the demand for the dirty product decreases at a faster rate than the demand for the cleaner product.

In the absence of a license, if $\tau \in [\tau_C(\lambda), \tau_L(\lambda)]$ the equilibrium profits for firms $i$ and $j$ are thus

$$\Pi_{iP}^G = \lambda\frac{\gamma_H}{\gamma_P}\frac{\Delta}{(4\gamma_H - \gamma_P)^2}[2(v-c) - \gamma_P\tau - \gamma_P\underline{G} + \gamma_P\frac{1-\lambda}{\lambda}]^2, \tag{23}$$

and

$$\Pi_j^G = \lambda\frac{\Delta}{(4\gamma_H - \gamma_P)^2}[v - c - 2\gamma_H\tau - 2\gamma_H\underline{G} + 2\gamma_H\frac{1-\lambda}{\lambda}]^2. \tag{24}$$

We now determine the equilibrium prices if $\tau \geq \tau_L(\lambda)$, in which case firm $j$ does not have any demand from green consumers. If $\tau > \tau_C(\lambda)$, not all green consumers buy the cleaner product, while if $\tau \leq \tau_C(\lambda)$, all of the green consumers buy the cleaner product. In that case, firm $i$, which is producing the cleaner good, sets its price at $p_{iP} = c + \tau\gamma_H - \varepsilon$, so that it gets all the demand, and firm $j$ does not get any demand. Thus, if $\tau \geq \tau_L(\lambda)$ (and no matter whether $\tau > \tau_C(\lambda)$ or $\tau \leq \tau_C(\lambda)$), the equilibrium payoffs for firms $i$ and $j$ are, respectively,[19]

$$\Pi_{iP}'^G = [\lambda(\frac{v - c - \tau\gamma_H}{\gamma_P} - \underline{G}) + (1-\lambda)]\tau\Delta, \tag{25}$$

and

$$\Pi_j'^G = 0.$$

[19] We omit $\varepsilon$ in the profit function of firm $i$.

If $\tau < \min\{\tau_C(\lambda), \tau_L(\lambda)\}$, the market for the green consumers is covered, and both firms are producing different products (see appendix for calculations). Their prices are thus

$$p_{iP}^{GC} = c + \frac{1}{3}[\tau(2\gamma_P + \gamma_H) + \Delta\underline{G} + 2\Delta + \Delta\frac{1-\lambda}{\lambda}], \tag{26}$$

and

$$p_j^{GC} = c + \frac{1}{3}[\tau(\gamma_P + 2\gamma_H) - \Delta\underline{G} + \Delta + 2\Delta\frac{1-\lambda}{\lambda}]. \tag{27}$$

Demands are

$$D_i(p_i^{GC}, p_j^{GC}; \gamma_P) = \lambda\frac{1}{3}[\underline{G} + \tau + 2 + \frac{1-\lambda}{\lambda}], \tag{28}$$

and

$$D_j(p_{iP}^{GC}, p_j^{GC}; \gamma_H) = \frac{1}{3}\lambda(1 - \tau - \underline{G} + 2\frac{1-\lambda}{\lambda}). \tag{29}$$

The profits for firms $i$ and $j$ are

$$\Pi_{iP}^{GC} = \lambda\frac{\Delta}{9}[\tau + \underline{G} + 2 + \frac{1-\lambda}{\lambda}]^2, \tag{30}$$

and

$$\Pi_j^{GC} = \lambda\frac{\Delta}{9}[1 - 2\tau - \underline{G} + 2\frac{1-\lambda}{\lambda}]^2. \tag{31}$$

Let us now consider the case where firm $i$ decides to offer a licence $(r, F)$ such that firm $j$ must pay a royalty fee $r$ and a fixed fee $F$. For any given $r$ and $F$, firm $j$ always accepts the license if its payoff is at least as much as the payoff it will get from refusing the license, which is $\Pi_j^G$ as defined by (24) or $\Pi_j^{GC}$ as defined by (31) if $\tau < \tau_L(\lambda)$, and 0 if $\tau \geq \tau_L(\lambda)$. If firm $i$ offers a license that is accepted by firm $j$, both firms could produce the cleaner product, in which case each firm faces demand (5). For a pair of prices $(p_i, p_j)$, firm $i$ obtains $D_i(p_i, p_j)(p_i - c - \tau\gamma_P) + rD_j(p_i, p_j) + F$, while firm $j$ gets $D_j(p_i, p_j)(p_j - c - r - \tau\gamma_P) - F$. As firms compete with the same cleaner product, we have Bertrand competition in which both firms set their prices at the marginal cost of firm $j$, $p_i = p_j = c + \tau\gamma_P + r$, and both firms share the demand (we show that firm $j$ has no incentive to reduce its price even further to capture all the demand as it will get the same payoff). Thus, if both firms produce the cleaner product, firm $i$ gets $\Pi_{iP}^{\prime G} + F$ and firm $j$ gets $-F$.

However, firm $j$ could decide to accept the license $(r, F)$ and produce the dirty product, in which case it will only have to pay $F$, and will not have to pay $r$ as it is not producing the patented innovation. If firm $j$ decides not to produce the cleaner product, both firms compete with different products, and we obtain the previous results: if $\tau < \tau_L(\lambda)$, the equilibrium profits for firms $i$ and $j$ are $\Pi_{iP}^G + F$ and $\Pi_j^G - F$ where $\Pi_{iP}^G$ and $\Pi_j^G$ are given by (23) and (24) if $\tau > \tau_C(\lambda)$, and $\Pi_{iP}^{GC} + F$ and $\Pi_j^{GC} - F$ where $\Pi_{iP}^{GC}$ and $\Pi_j^{GC}$ are given by (30) and (31) if $\tau < \tau_C(\lambda)$. If $\tau \geq \tau_L(\lambda)$, the equilibrium profits for firms $i$ and $j$ are $\Pi_j^{\prime G} + F$ and $-F$ where $\Pi_j^{\prime G}$ is given by (25).

Therefore, if $\tau < \tau_L(\lambda)$, firm $j$ always prefers to produce the dirty product, even if it accepted the license, as $\Pi_{iP}^G - F > 0 - F$ and $\Pi_{iP}^{GC} - F > 0 - F$. Thus, unless $F < 0$, firm $j$ will never accept a license as $\Pi_{iP}^G > \Pi_{iP}^G - F$, and $\Pi_{iP}^{GC} > \Pi_{iP}^{GC} - F$ and, in equilibrium, there will be no licensing if $\tau < \tau_L(\lambda)$. Even if firm $i$ were to set up $F < 0$ (which means that firm $i$ would have to pay firm $j$ to use its license), firm $j$ will still prefer to produce the dirty product, and therefore firm $i$ has no incentive to pay firm $j$ to use its license.

If $\tau \geq \tau_L(\lambda)$, firm $j$ is indifferent between producing the cleaner product or the dirty product once the license has been accepted. We assume that firm $j$ will always produce the cleaner good when indifferent. Therefore, firm $j$ accepts the license if $-F \geq 0$. Firm $i$ will choose $(r, F)$ solution of

$$\underset{r,F}{Max} \frac{1}{\gamma_P}\lambda(v - c - \tau\gamma_P - r - \gamma_P \underline{G} + \gamma_P \frac{1-\lambda}{\lambda})r + F,$$

such that $-F \geq 0$. Therefore, firm $i$ chooses $F^* = 0$ and

$$r^* = \frac{1}{2}(v - c - \tau\gamma_P - \gamma_P \underline{G} + \gamma_P \frac{1-\lambda}{\lambda}), \tag{32}$$

and firm $i$ gets the payoff

$$\Pi_{iP}^L = \lambda \frac{1}{4\gamma_P}(v - c - \tau\gamma_P - \gamma_P \underline{G} + \gamma_P \frac{1-\lambda}{\lambda})^2, \tag{33}$$

while firm $j$ gets 0. Evaluated at $r^*$ as determined by (32), the equilibrium prices charged are thus

$$p_{iP}^L = p_j^L = \frac{1}{2}(v + c + \gamma_P(\tau - \underline{G} + \frac{1-\lambda}{\lambda})),$$

and the demand that each firm faces is thus

$$D^L = \frac{1}{4}\frac{\lambda}{\gamma_P}(v - c - \tau\gamma_P - \gamma_P \underline{G} + \gamma_P \frac{1-\lambda}{\lambda}). \tag{34}$$

To summarize, if $\tau \geq \tau_L(\lambda)$, firm $i$ offers the license $(r^* = \frac{1}{2}(v-c-\tau\gamma_P-\gamma_P\underline{G}+\gamma_P\frac{1-\lambda}{\lambda}), F^* = 0)$ that firm $j$ accepts. Firm $j$ produces the cleaner product, but firm $i$ captures all the payoff of firm $j$ through licensing.

We now finish determining under what conditions firm $i$ will decide to license. We have seen that if $\tau < \tau_L(\lambda)$, there will be no licensing, both firms will compete with different products, and they will obtain the payoffs $\Pi^G_{iP}$ and $\Pi^G_j$ as defined by (23) and (24) or $\Pi^{GC}_{iP}$ and $\Pi^{GC}_j$ as defined by (30) and (31). However, if $\tau \geq \tau_L(\lambda)$, firm $i$ will always license as $\Pi^L_{iP} > \Pi'^G_{iP}$ is always satisfied where $\Pi^L_{iP}$ and $\Pi'^G_{iP}$ are defined by (33) and (25) respectively.

Lastly, we consider the case where both firms decide to invest $I_P$, and they both decide to apply for a patent. In that case, each firm gets the patent with probability 1/2. If firm $i$ obtains the patent, the analyses without and with licensing are the ones that have been developed above for $i = 1, 2$. Thus, if $\tau < \tau_L(\lambda)$, there will be no licensing, and once the innovation has been discovered, the expected payoff for each firm $i$ for $i = 1, 2$, is

$$\frac{1}{2}(\Pi^G_{iP} - C_P) + \frac{1}{2}\Pi^G_i, \tag{35}$$

where $\Pi^G_{iP}$ is the equilibrium payoff that firm $i$ will get if it obtains the patent and is defined by (23), and $\Pi^G_i$ is the payoff it will get if it does not obtain the patent and is defined by (24). On the other hand, if $\tau \geq \tau_L(\lambda)$, there will be licensing, and firm $i$ will get

$$\frac{1}{2}(\Pi^L_{iP} - C_P).$$

Figure 3 illustrates these findings where the horizontal axis represents the fraction of green-conscious consumers, and the vertical axis represents the emission tax $\tau$.

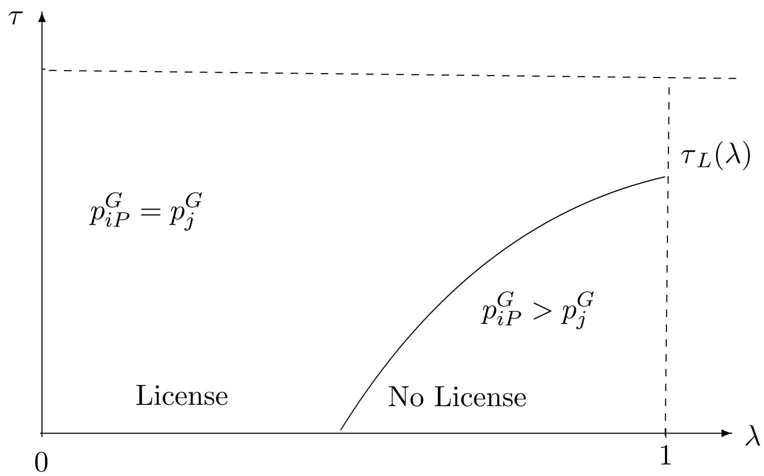


Figure 3: Licensing with Green and Non-Green Consumers

On the right part of Figure 3, for $\tau < \tau_L(\lambda)$, in equilibrium, there is no licensing, and both firms compete with differentiated goods (one dirty good and one cleaner good), where prices are different. On the left part of Figure 3, for $\tau \geq \tau_L(\lambda)$, in equilibrium, the patentholder offers a license to its competitor who accepts it, and both firms set identical prices.

After having decided whether or not to invest, the innovator must decide whether to patent its innovation or not. If $\tau < \tau_L(\lambda)$, firm $i$ will apply for a patent only if $\Pi^G_{1P} - C_P \geq 0$, and if $\tau \geq \tau_L(\lambda)$, if $\Pi^L_{iP} - C_P \geq 0$. We summarize these findings in the following Lemma.

**Lemma 1** *If firm $i$ innovates, it will apply for a patent if*

*(i) $C_P \leq \Pi^G_{iP}$ if $\tau < \tau_L(\lambda)$;*

*(ii) $C_P \leq \Pi^L_{iP}$ if $\tau \geq \tau_L(\lambda)$.*

**Proof**: If $\tau < \tau_L(\lambda)$, and if firm $i$ decides not to apply for a patent, firm $j$ obtains the patent and gets $\Pi^G_{jP} = \Pi^G_{iP}$ (firms are symmetric). Thus, if $\Pi^G_{iP} - C_P < 0$, then $\Pi^G_{jP} - C_P < 0$, and none of the firms decide to apply for a patent, in which case they both get a null payoff due to Bertrand competition and prices at $c + \gamma_P \tau$. Similar proof for $\tau \geq \tau_L(\lambda)$.

## 4.2 First stage: Investment decisions

We now turn to the firms' investment decisions in the presence of green and non-green consumers. We start with $\tau < \tau_L(\lambda)$, in which case, there is no license. The best response of firm $i$ for

$i = 1, 2$ with $i \neq j$ to $I_j = 0$ is

$$BR_i(I_j = 0) = \begin{cases} I_P & \text{if } \Pi_{iP}^G - C_P - I_P \geq 0 \Leftrightarrow \tau \leq \tau_1^{NL}(\lambda, C_P) \\ 0 & \text{if } \Pi_{iP}^G - C_P - I_P < 0 \Leftrightarrow \tau > \tau_1^{NL}(\lambda, C_P) \end{cases}$$

where $\tau_1^{NL}(\lambda, C_P)$ is defined in the appendix. The best response of firm $i$ to $I_j = I_P$ is

$$BR_i(I_j = I_P) = \begin{cases} I_P & \text{if } \frac{1}{2}(\Pi_{iP}^G - C_P) + \frac{1}{2}\Pi_i^G - I_P \geq \Pi_i^G \Leftrightarrow \tau \leq \tau_2^{NL}(\lambda, C_P) \\ 0 & \text{if } \frac{1}{2}(\Pi_{iP}^G - C_P) + \frac{1}{2}\Pi_i^G - I_P < \Pi_i^G \Leftrightarrow \tau > \tau_2^{NL}(\lambda, C_P) \end{cases}$$

where $\tau_2^{NL}(\lambda, C_P)$ is defined in the appendix with $\tau_1^{NL}(\lambda, C_P) > \tau_2^{NL}(\lambda, C_P)$. We show that $\tau_1^{NL}(\lambda, C_P)$ and $\tau_2^{NL}(\lambda, C_P)$ are both increasing in $\lambda$ for the range of parameters considered, and they are both decreasing in $C_P$. Therefore, if $\tau \leq \tau_2^{NL}(\lambda, C_P)$, the best response of firm $i$ to any investment $I_j \in \{0, I_P\}$ by firm $j$ is $BR_i(I_j) = I_P$. If $\tau \in [\tau_2^{NL}(\lambda, C_P), \tau_1^{NL}(\lambda, C_P)[$, the best response of firm $i$ to any investment $I_j$ by firm $j$ is

$$BR_i(I_j) = \begin{cases} I_P & \text{if } I_j = 0 \\ 0 & \text{if } I_j = I_P \end{cases}$$

Lastly, if $\tau \geq \tau_1^{NL}(\lambda, C_P)$, the best response of firm $i$ to any investment $I_j \in \{0, I_P\}$ by firm $j$ is $BR_i(I_j) = 0$.

Thus, for $\tau \geq \tau_1^{NL}(\lambda, C_P)$, the unique Nash equilibrium is $(I_1^*, I_2^*) = (0, 0)$, that is, none of the firms invests. For $\tau \in [\tau_2^{NL}(\lambda, C_P), \tau_1^{NL}(\lambda, C_P)[$, there exist two Nash equilibria $(I_1^*, I_2^*) = (0, I_P)$ and $(I_1^*, I_2^*) = (I_P, 0)$ such that one firm does not invest while the other invests $I_P$. For $\tau \leq \tau_2^{NL}(\lambda, C_P)$, there exists a unique Nash equilibrium $(I_1^*, I_2^*) = (I_P, I_P)$ such that both firms invest $I_P$.

If $\tau \geq \tau_L(\lambda)$, the innovator licenses his patented innovation to his competitor. The best response of firm $i$ for $i = 1, 2$ $i \neq j$ to $I_j = 0$ is

$$BR_i(I_j = 0) = \begin{cases} I_P & \text{if } \Pi_{iP}^L - C_P - I_P \geq 0 \Leftrightarrow \tau \leq \tau_1^L(\lambda, C_P) \\ 0 & \text{if } \Pi_{iP}^L - C_P - I_P < 0 \Leftrightarrow \tau > \tau_1^L(\lambda, C_P) \end{cases}$$

where $\tau_1^L(\lambda, C_P)$ is defined in the appendix. The best response of firm $i$ to $I_j = I_P$ is

$$BR_i(I_j = I_P) = \begin{cases} I_P & \text{if } \Pi_{iP}^L - C_P - 2I_P \geq 0 \Leftrightarrow \tau \leq \tau_2^L(\lambda, C_P) \\ 0 & \text{if } \Pi_{iP}^L - C_P - 2I_P < 0 \Leftrightarrow \tau > \tau_2^L(\lambda, C_P) \end{cases}$$

where $\tau_2^L(\lambda, C_P)$ is defined in the appendix with $\tau_1^L(\lambda, C_P) > \tau_2^L(\lambda, C_P)$. We show that $\tau_1^L(\lambda, C_P)$ and $\tau_2^L(\lambda, C_P)$ are both decreasing in $C_P$.

Thus, for $\tau \geq \tau_1^L(\lambda, C_P)$, the unique Nash equilibrium is $(I_1^*, I_2^*) = (0, 0)$, that is, none of the firms invests. For $\tau \in [\tau_2^L(\lambda, C_P), \tau_1^L(\lambda, C_P)[$, there exist two Nash equilibria $(I_1^*, I_2^*) = (0, I_P)$ and $(I_1^*, I_2^*) = (I_P, 0)$ such that one firm does not invest while the other invests $I_P$. For $\tau \leq \tau_2^L(\lambda, C_P)$, there exists a unique Nash equilibrium $(I_1^*, I_2^*) = (I_P, I_P)$ such that both firms invest $I_P$.

In Figures 4a, 4b, and 4c, we represent the different areas where firms invest in equilibrium in graphs where the horizontal axis represents the fraction of green-conscious consumers $\lambda$, and the vertical axis represents the emission tax $\tau$. Figure 4a represents the different areas for low values of the patenting cost $C_P$.

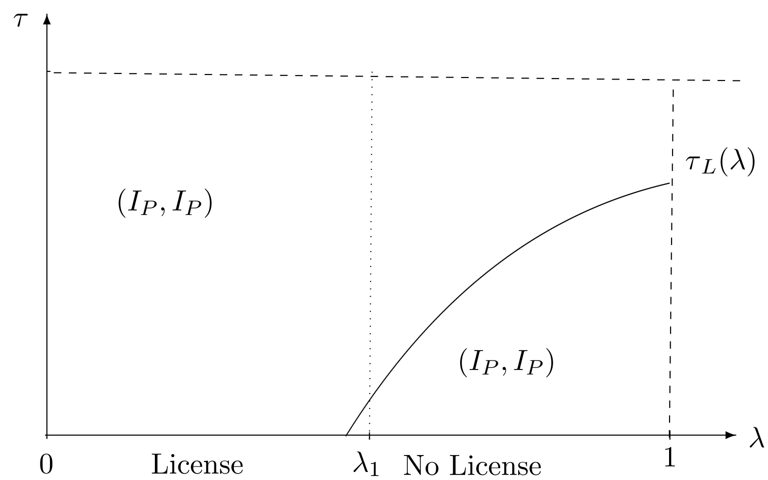


Figure 4a: Investments with Green and Non Green consumers for low values of $C_P$

In Figure 4a, as the patenting cost $C_P$ is small, both firms always invest, no matter whether licensing occurs or not. For a given fraction of green-conscious consumers, let's say at $\lambda_1$ in Figure 4a, as the emission tax $\tau$ increases, both firms invest even though we go from no licensing to licensing.

Figure 4b represents the different areas for intermediate values of the patenting cost $C_P$.

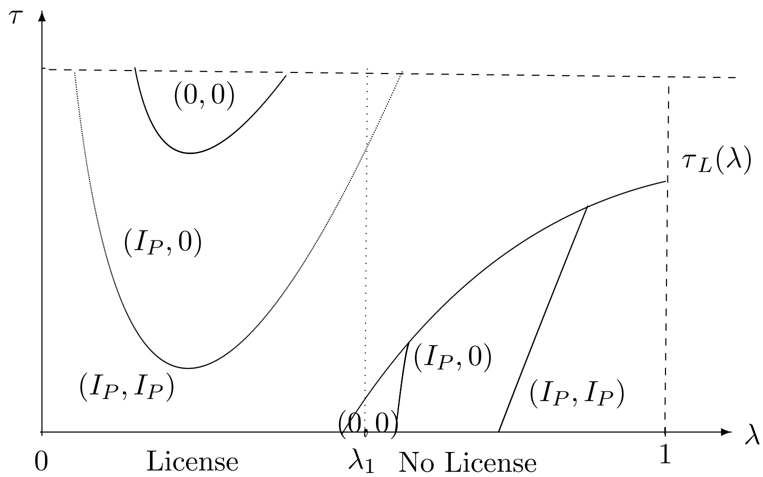


Figure 4b: Investments with Green and Non Green consumers for intermediate values of $C_P$

For intermediate values of $C_P$, we have all three possibilities (both firms invest, one of them invests, none of them invests) in both cases of licensing and not licensing. Here again, for a given value of $\lambda$ at $\lambda_1$, as the emission tax $\tau$ increases, we go from no licensing and no investment to licensing where both firms invest to a situation where only one firm invests.

Figure 4c represents the different areas for large values of the patenting cost $C_P$.

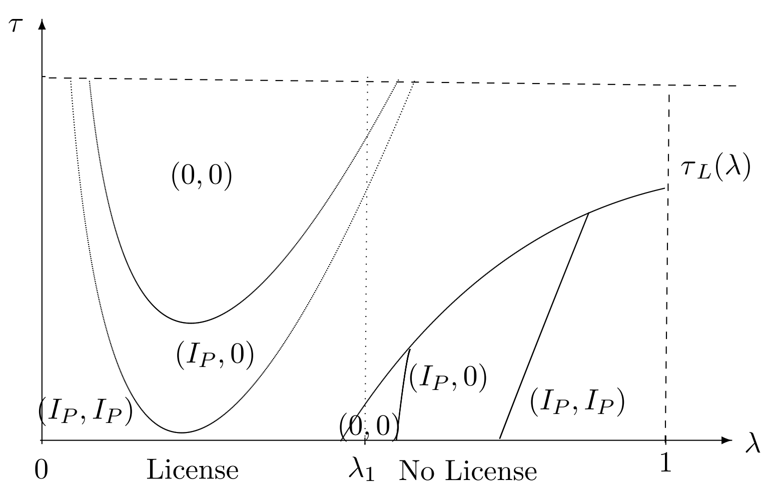


Figure 4c: Investments with Green and Non Green consumers for large values of $C_P$

When the patenting cost is very large, for a given value of $\lambda$ at $\lambda_1$, as the emission tax $\tau$ increases, we go from a situation with no licensing and no investment to licensing where both firms invest to only one firm invests and, eventually, to a situation where none of them invests again.

To compare these findings with the non-green consumers only case, in Figure 5, we represent these different areas where firms invest in equilibrium in a graph where the horizontal axis represents the emission tax $\tau$, and the vertical axis represents the patenting cost $C_P$ when the fraction of green-conscious consumers is given at $\lambda_1$. In this representation, we take the fraction of consumers as given as we want to understand how patenting cost and emission tax interact.

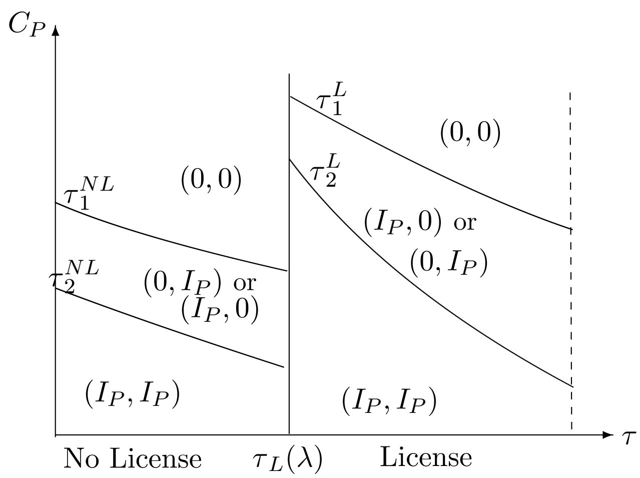


Figure 5: Equilibrium Investments with Green and Non-Green Consumers

In Figure 5, for $\tau < \tau_L(\lambda)$, there is no licensing in equilibrium, whereas for $\tau \geq \tau_L(\lambda)$, licensing occurs in equilibrium. For low values of $C_P$, both firms invest. As $C_P$ increases, firms invest less. For intermediate values of $C_P$ as the emission tax $\tau$ increases, we go from no investment to investment as we switch from no licensing to licensing. For very high levels of $C_P$, as the emission tax $\tau$ increases, we go from no licensing and no investment to licensing and some investment to no investment.

The Subgame Perfect Nash Equilibrium is presented in the following Propositions.

**Proposition 4** *For a given $C_P$, the Subgame Perfect Nash Equilibrium is*

*For $\tau < \tau_L(\lambda)$, there is no license, and* *(i) if $\tau > \tau_1^{NL}(\lambda, C_P)$, firms do not invest $(I_1^*, I_2^*) = (0, 0)$, they both produce the dirty product, and set their prices $p_1^* = p_2^* = c + \tau\gamma_H$;*

*(ii) if $\tau \in [\tau_2^{NL}(\lambda, C_P), \tau_1^{NL}(\lambda, C_P)[$, firm $i$ invests while firm $j$ does not $(I_i^*, I_j^*) = (I_P, 0)$ for $i, j = 1, 2$ and $i \neq j$, firm $i$ produces the cleaner product at $p_{iP}^G = c + \tau\gamma_H - \varepsilon$ and firm $j$ does not produce;*

*(iii) if $\tau < \tau_2^{NL}(\lambda, C_P)$, both firms invest $(I_1^*, I_2^*) = (I_P, I_P)$, but only one of them gets a patent and produces the cleaner product at $p_{iP}^G = c + \tau\gamma_H - \varepsilon$ while the other does not produce.*

**Proposition 5** *For $\tau \geq \tau_L(\lambda)$, licensing occurs, and*

*(i) if $\tau > \tau_1^L(\lambda, C_P)$, firms do not invest $(I_1^*, I_2^*) = (0, 0)$, they both produce the dirty product, and set their prices $p_1^* = p_2^* = c + \tau\gamma_H$;*

*(ii) if $\tau \in [\tau_2^L(\lambda, C_P), \tau_1^L(\lambda, C_P)[$, firm $i$ invests while firm $j$ does not $(I_i^*, I_j^*) = (I_P, 0)$ for $i, j = 1, 2$ and $i \neq j$, firm $i$ licenses its technology to firm $j$ and both firms produce the cleaner product at price $p_{iP} = c + \tau\gamma_P + r^*$;*

*(iii) if $\tau < \tau_2^L(\lambda, C_P)$, both firms invest $(I_1^*, I_2^*) = (I_P, I_P)$, the firm that gets a patent licenses it to the other firm, and both firms produce the cleaner product at $p_{iP} = c + \tau\gamma_P + r^*$.*

For low levels of the emission tax and the patenting cost, both firms invest. For a given level of the emission tax relatively small, as the patenting cost increases, only one firm has an incentive to invest, while the other does not. Eventually, as the patenting cost becomes too large, none of the firms invest. For a given intermediate level of the patenting cost $C_P$, as the emission tax $\tau$ increases, investment at first increases due to a change from no licensing to licensing. However, when the patenting cost is high, initially, none of the firms invest in the area where there is no licensing as the emission tax $\tau$ increases. Then, as the emission tax increases further, both firms invest as we go from no licensing to licensing.

### 4.3 Equilibrium Emission levels

We now calculate the total emission level in equilibrium.

For large values of $C_P$, if $\tau < \tau_L(\lambda)$, for low levels of the emission tax $\tau$, none of the firms invest, and therefore the total emission is $2\gamma_H D(c + \tau\gamma_H, c + \tau\gamma_H; \gamma_H)$ where $D(c + \tau\gamma_H, c + \tau\gamma_H; \gamma_H)$ is defined by (5). If $\tau \geq \tau_L(\lambda)$, initially, both firms invest or only one of them invests, which leads to the same total emission level as defined in (37). For even larger levels of $\tau$ such that $\tau > \tau_1^L(\lambda, C_P)$, none of the firms invest, which leads to the emission level $2\gamma_H D(c + \tau\gamma_H, c + \tau\gamma_H; \gamma_H)$. This leads to the following proposition.

**Proposition 6** *For sufficiently large values of $C_P$, in equilibrium, the total emission level is given by:*

$$e_G^* = \begin{cases} \lambda(v - c - \gamma_H(\tau + \underline{G} - \frac{1-\lambda}{\lambda})) & \textit{if } \tau < \tau_L(\lambda) \\ \lambda\frac{1}{2}(v - c - \gamma_P(\tau + \underline{G} - \frac{1-\lambda}{\lambda})) & \textit{if } \tau_L(\lambda) \leq \tau < \tau_1^L(\lambda, C_P) \\ \lambda(v - c - \gamma_H(\tau + \underline{G} - \frac{1-\lambda}{\lambda})) & \tau \geq \tau_1^L(\lambda, C_P) \end{cases} \quad (36)$$

We represent the emission levels as a function of $\tau$ in Figure 6a for a high level of $C_P$ and an intermediate value of $\lambda$.

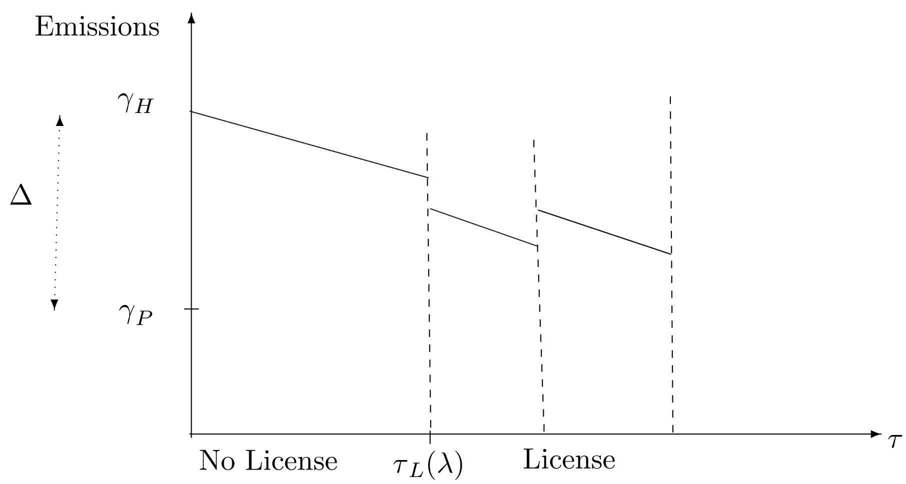


Figure 6b: Equilibrium Emission with Green and Non-Green Consumers for large $C_P$

Initially, the emission level decreases as the emission tax increases; the emission level is relatively high (as firms do not invest), then there is a discrete jump downward as firms invest and license. Thus, our model, by endogenizing the investment and licensing decisions of firms, is able to capture a novel effect of increasing emission taxes. However, as the emission tax increases further, there is a jump upward as firms do not invest anymore. There is a paradox: as the emission tax increases, the tax bill is too large for firms, and thus, they do not invest. This implies that policy makers are most effective at reducing emissions by choosing an intermediate level of emission tax which is large enough to induce licensing and at the same time, small enough to avoid the paradox.

We note that making the patentability requirement stricter, that is, lowering $\gamma_P$, decreases $\tau_L(\lambda)$, and increases $\tau_1^L(\lambda, C_P)$, which leads to lower emission levels for greater range of $\tau$. Reducing $C_P$ does not affect $\tau_L(\lambda)$ but is shown to increase $\tau_1^L(\lambda, C_P)$.

If $C_P$ is sufficiently low, both firms invest no matter whether there is a license or not. If $\tau < \tau_L(\lambda)$, there is no license in equilibrium, and if firm $i$ produces the cleaner product and firm $j$ the dirty product, the total emission is $\gamma_P D_i(p_{iP}^G, p_j^G; \gamma_P) + \gamma_H D_j(p_{iP}^G, p_j^G; \gamma_H)$ where $D_i(p_{iP}^G, p_j^G; \gamma_P)$ and $D_j(p_{iP}^G, p_j^G; \gamma_H)$ are defined by (21) and (22), respectively.

If $\tau \geq \tau_L(\lambda)$, there is a license in equilibrium, and each firm produces the cleaner product and gets the demand $D^L$ as defined by (34). The emission is thus $2\gamma_P D^L$.

**Proposition 7** *For sufficiently small values of $C_P$, in equilibrium, the total emission level is given by:*

$$e_G^* = \begin{cases} \lambda \frac{\gamma_H}{4\gamma_H - \gamma_P}(3(v-c) - (2+\gamma_P)(\tau + \underline{G} - \frac{1-\lambda}{\lambda})) & \textit{if } \tau < \tau_L(\lambda) \\ \lambda \frac{1}{2}(v - c - \gamma_P(\tau + \underline{G} - \frac{1-\lambda}{\lambda})) & \textit{if } \tau \geq \tau_L(\lambda) \end{cases} \tag{37}$$

We represent the emission levels as a function of $\tau$ in Figure 6b for a low level of $C_P$ and an intermediate value of $\lambda$, let's say $\lambda_1$.

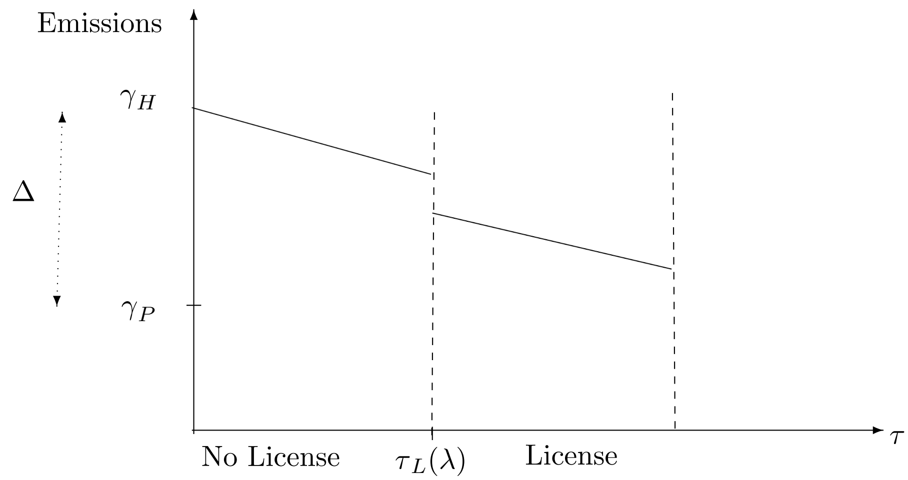


Figure 6a: Equilibrium Emission with Green and Non-Green Consumers for low $C_P$

As the emission tax increases, the total emission is reduced, which fulfil the goal of having an emission tax. However, as we go from no licensing to licensing, there is a discrete jump downward in the emission level. Notice that the cutoff value $\tau_L(\lambda)$ is increasing with $\lambda$. Thus, as there are more green-conscious consumers, it is more likely that there will be no licensing. On the other hand, as there are fewer green-conscious consumers, there will be more licensing. In fact, if $\lambda < \lambda_0$, where $\lambda_0$ is defined by (16), there will be only licensing in equilibrium as the cutoff value $\tau_L(\lambda)$ becomes negative.

For intermediate values of $C_P$, if $\tau < \tau_L(\lambda)$, only one firm invests, but the total emission level is identical to the previous case.

Propositions (6) and (7) together imply that a decrease in the patenting cost restores firms' incentive to invest, thereby mitigating or avoiding the paradoxical effect of increasing emission taxes.

# 5 Conclusion

In this paper, we analyzed how patent policy instruments work in conjunction with emission taxes to impact firms' investment and licensing decisions and emission levels. Our framework allowed for strategic behaviour of firms within a duopolistic setting, and for heterogeneity across consumers in terms of the degree to which they care about the environment, key factors that were shown to play a significant role in determining policy implications.

Given the market of a product, the production of which causes pollution, we modeled the implementation of a cleaner technology as a reduction of the emission per unit of output ratio. We assumed that the product is vertically differentiated in terms of its emission-output ratio with green conscious consumers preferring (to different degrees) products with lower emission-output ratios. Within this setting, we endogenized firms' investment in and patenting and licensing of green innovations. Subsequent to these decisions, firms were assumed to engage in price competition in the product market.

A key finding of the paper is that the greater the proportion of consumers that are green conscious, the less likely that firms are to engage in licensing the green innovation. While in the absence of green conscious consumers, there is always licensing and implementation of the green innovation by all firms, for high proportions of green conscious consumers, firms would rather differentiate their products by using production technologies with different emission-output ratios. For this reason, higher proportions of green conscious consumers is associated with higher levels of emissions such that policy-makers may consider implementing technology standards in order to effectively force licensing when this proportion is sufficiently high. An alternative means of inducing licensing in equilibrium is to increase the emission tax beyond a certain threshold. This has the desirable effect of causing emissions to fall discretely, as we move from an equilibrium without licensing to one with licensing.

We find that there exists a second threshold level of the tax beyond which increasing the emission tax leads to increasing the emission level. This paradox can be mitigated by decreasing patenting costs. Making the patentability requirement stricter also achieves this, and has the added benefit of yielding a lower level of emissions as long as the tax is below the threshold, as compared to reducing patenting costs.

## Appendix

### Demands when the market is covered

If both firms sell the same product, $\gamma_1 = \gamma_2 = \gamma$, and if $\underline{G} < \overline{G} < \widetilde{G}_3$, the demand for each firm $i$, for $i, j = 1, 2$ and $i \neq j$ is

$$D_i(p_i, p_j; \gamma) = \begin{cases} 0 & \text{if } p_i > p_j \\ \frac{1}{2}\lambda + \frac{1}{2}(1-\lambda) & \text{if } p_i = p_j \leq v \\ \lambda + (1-\lambda) & \text{if } p_i < p_j \leq v \end{cases} \tag{38}$$

as all the green consumers buy the product.

On the other hand, if $\widetilde{G}_3 < \underline{G} < \overline{G}$, the demand for each firm $i$, for $i, j = 1, 2$ and $i \neq j$ is

$$D_i(p_i, p_j; \gamma) = \begin{cases} 0 & \text{if } p_i > p_j \\ \frac{1}{2}(1-\lambda) & \text{if } p_i = p_j \leq v \\ (1-\lambda) & \text{if } p_i < p_j \leq v \end{cases} \tag{39}$$

as there is no demand from the green consumers.

If $\gamma_i = \gamma_P < \gamma_j = \gamma_H$, such that firm $i$ has the cleaner product, if $\underline{G} < \widetilde{G}_1 < \overline{G} < \widetilde{G}_2$, the demands are

$$D_i(p_i, p_j; \gamma_P) = \begin{cases} \lambda(\overline{G} - \frac{p_i - p_j}{\Delta}) & \text{if } p_i > p_j \\ \lambda + \frac{1}{2}(1-\lambda) & \text{if } p_i = p_j \leq v \\ \lambda + (1-\lambda) & \text{if } p_i < p_j \leq v \end{cases} \tag{40}$$

and

$$D_j(p_i, p_j; \gamma_H) = \begin{cases} \lambda(\frac{p_i - p_j}{\Delta} - \underline{G}) + (1-\lambda) & \text{if } p_i > p_j \\ \frac{1}{2}(1-\lambda) & \text{if } p_i = p_j \leq v \\ 0 & \text{if } p_i < p_j \leq v \end{cases} \tag{41}$$

If $\widetilde{G}_1 < \underline{G} < \widetilde{G}_2 < \overline{G}$, the demands are

$$D_i(p_i, p_j; \gamma_P) = \begin{cases} \lambda(\frac{v - p_i}{\gamma_P} - \underline{G}) & \text{if } p_i > p_j \\ \lambda(\frac{v - p_i}{\gamma_P} - \underline{G}) + \frac{1}{2}(1-\lambda) & \text{if } p_i = p_j \leq v \\ \lambda(\frac{v - p_i}{\gamma_P} - \underline{G}) + (1-\lambda) & \text{if } p_i < p_j \leq v \end{cases} \tag{42}$$

and

$$D_j(p_i, p_j; \gamma_H) = \begin{cases} (1-\lambda) & \text{if } p_i > p_j \\ \frac{1}{2}(1-\lambda) & \text{if } p_i = p_j \leq v \\ 0 & \text{if } p_i < p_j \leq v \end{cases} \tag{43}$$

If $\widetilde{G}_1 < \underline{G} < \overline{G} < \widetilde{G}_2$, the demands are

$$D_i(p_i, p_j; \gamma_P) = \begin{cases} \lambda & \text{if } p_i > p_j \\ \lambda + \frac{1}{2}(1-\lambda) & \text{if } p_i = p_j \leq v \\ \lambda + (1-\lambda) & \text{if } p_i < p_j \leq v \end{cases} \tag{44}$$

and

$$D_j(p_i, p_j; \gamma_H) = \begin{cases} (1-\lambda) & \text{if } p_i > p_j \\ \frac{1}{2}(1-\lambda) & \text{if } p_i = p_j \leq v \\ 0 & \text{if } p_i < p_j \leq v \end{cases} \tag{45}$$

**Non-green consumers and license**

Consider the case where firm $i$ does not invest and firm $j$ invests $I_P$, applies for a patent, and offers a license to firm $i$. If firm $i$ accepts it, both firms are producing the cleaner good $\gamma_P$. Firm $j$ offers a license $(r, F)$ where $r$ is the per-unit royalty rate, and $F$ a fixed fee. For any license $(r, F)$ that is accepted by firm $i$, firm $j$ obtains

$$D_j(p_i, p_{jP})(p_{jP} - c - \tau\gamma_P) + rD_i(p_i, p_{jP}) + F,$$

and firm $i$ obtains

$$D_i(p_i, p_{jP})(p_i - c - \tau\gamma_P - r) - F,$$

where

$$D_i(p_i, p_{jP}) = \begin{cases} 1 & \text{if} \quad p_i < p_{jP} \text{ and } p_i \leq v \\ \frac{1}{2} & \text{if} \quad p_i = p_{jP} \leq v \\ 0 & \text{if} \quad p_i > \min\{p_{jP}, v\} \end{cases}$$

If $p_{jP} > p_i = v$, firm $j$ does not get any demand, and thus obtains the payoff $r + F$, while firm $i$ gets $(v - c - \tau\gamma_P - r) - F$, which cannot be an equilibrium as firm $j$ will reduce its price $p_{jP}$. If $p_{jP} = p_i = v$, firm $j$ obtains $\frac{1}{2}(v - c - \tau\gamma_P + r) + F$ and firm $i$ gets $\frac{1}{2}(v - c - \tau\gamma_P - r) - F$, which is not an equilibrium, as each firm could reduce its price and capture all the market. Indeed, if firm $j$ reduces its price to $p_{jP} < p_i$, it will obtain $(p_{jP} - c - \tau\gamma_P + r) + F$. Firm $i$ will also reduce its price. Therefore, the unique equilibrium is for each firm to choose $p^* = c + \tau\gamma_P + r$, so that both firms produce the cleaner good, with player $j$ getting $r + F$ and firm $i$ gets $-F$.

Firm $j$ chooses $(r, F)$ that maximize $r + F$ such that $-F \geq 0$ and $p^* = c + \tau\gamma_P + r \leq v$. Therefore, $r^* = v - c - \tau\gamma_P$ and $F^* = 0$, so that in equilibrium firm $j$ obtains $v - c - \tau\gamma_P$ while firm $i$ gets 0 even of both firms produce the cleaner good.

**Green and non-green consumers and no license**

Demanded quantities must be positive at the equilibrium prices. Therefore, if $\underline{G} < \widetilde{G}_1 < \widetilde{G}_2 < \overline{G}$, the equilibrium prices are $(p^G_{iP}, p^G_j)$ where

$$p^G_{iP} = \frac{1}{4\gamma_H - \gamma_P}[c(2\gamma_H + \gamma_P) + 2v\Delta + 3\tau\gamma_H\gamma_P - \gamma_P\Delta\underline{G} + \gamma_P\Delta\frac{1-\lambda}{\lambda}],$$

and

$$p^G_j = \frac{1}{4\gamma_H - \gamma_P}[3\gamma_H c + v\Delta + \gamma_H\tau(2\gamma_H + \gamma_P) - 2\Delta\gamma_H\underline{G} + 2\Delta\gamma_H\frac{1-\lambda}{\lambda}].$$

At these equilibrium prices, the demand for firm $i$ is

$$D_i(p^G_{iP}, p^G_j; \gamma_P) = \lambda(\widetilde{G}_2 - \widetilde{G}_1) = \frac{\lambda}{4\gamma_H - \gamma_P}\frac{\gamma_H}{\gamma_P}(2(v-c) - \gamma_P\tau - \gamma_P\underline{G} + \gamma_P\frac{1-\lambda}{\lambda}),$$

and the demand for firm $j$ is

$$D_j(p^G_{iP}, p^G_j; \gamma_H) = \lambda(\widetilde{G}_1 - \underline{G}) + (1-\lambda) = \frac{\lambda}{4\gamma_H - \gamma_P}(v - c - 2\tau\gamma_H - 2\underline{G}\gamma_H + 2\gamma_H\frac{1-\lambda}{\lambda}),$$

where

$$\widetilde{G}_1 = \frac{p^G_{iP} - p^G_j}{\Delta},$$

and

$$\widetilde{G}_2 = \frac{v - p^G_{iP}}{\gamma_P}.$$

Demand $D_i(p^G_{iP}, p^G_j; \gamma_P) \geq 0$ if

$$\tau < 2\frac{(v-c)}{\gamma_P} - \underline{G} + \frac{1-\lambda}{\lambda},$$

which is always satisfied, as

$$\frac{(v-c)}{2\gamma_H} - \underline{G} < 2\frac{(v-c)}{\gamma_P} - \underline{G} + \frac{1-\lambda}{\lambda}.$$

Demand $D_j(p_{iP}^G, p_j^G; \gamma_H) \geq 0$ if

$$\tau < \frac{v-c}{2\gamma_H} - \underline{G} + \frac{1-\lambda}{\lambda},$$

which always satisfied, as

$$\frac{(v-c)}{2\gamma_H} - \underline{G} \leq \frac{v-c}{2\gamma_H} - \underline{G} + \frac{1-\lambda}{\lambda}.$$

Furthermore, $p_{iP}^G > p_j^G$ if $\tau < \tau_E(\lambda)$ where

$$\tau_E(\lambda) \equiv \frac{v-c}{2\gamma_H} + \frac{2\gamma_H - \gamma_P}{2\gamma_H}(\underline{G} - \frac{1-\lambda}{\lambda}).$$

We show that $\tau_E(\lambda)$ is increasing and concave as

$$\frac{\partial \tau_E(\lambda)}{\partial \lambda} = \frac{1}{2\lambda^2 \gamma_H}(2\gamma_H - \gamma_P) > 0,$$

and

$$\frac{\partial^2 \tau_E(\lambda)}{\partial \lambda^2} = -\frac{1}{\lambda^3 \gamma_H}(2\gamma_H - \gamma_P) < 0.$$

Lastly, we need to verify that at prices $(p_{iP}^G, p_j^G)$, we have $\underline{G} < \widetilde{G}_1 < \widetilde{G}_2 < \overline{G}$. We check that $\underline{G} < \widetilde{G}_1$ if $\tau < \tau_L(\lambda)$ with

$$\tau_L(\lambda) \equiv \frac{v-c}{2\gamma_H} - \underline{G} - \frac{2\gamma_H - \gamma_P}{2\gamma_H}\frac{1-\lambda}{\lambda} < \tau_E(\lambda), \tag{46}$$

and $\widetilde{G}_2 < \overline{G}$ if $\tau > \tau_C(\lambda)$ where

$$\tau_C(\lambda) \equiv \frac{2\gamma_H + \gamma_P}{\gamma_H \gamma_P}\frac{v-c}{3} - \underline{G} - \frac{\gamma_H - \gamma_P}{3\gamma_H}\frac{1-\lambda}{\lambda} - \frac{4\gamma_H - \gamma_P}{3\gamma_H}. \tag{47}$$

We show that $\tau_L(\lambda)$ and $\tau_C(\lambda)$ are increasing and concave as

$$\frac{\partial \tau_L(\lambda)}{\partial \lambda} = \frac{2\gamma_H - \gamma_P}{2\gamma_H}\frac{1}{\lambda^2} > 0, \text{ and } \frac{\partial \tau_C(\lambda)}{\partial \lambda} = \frac{\gamma_H - \gamma_P}{3\gamma_H}\frac{1}{\lambda^2} > 0,$$

and

$$\frac{\partial^2 \tau_E(\lambda)}{\partial \lambda^2} = -\frac{2\gamma_H - \gamma_P}{2\gamma_H}\frac{1}{\lambda^3} < 0, \text{ and } \frac{\partial^2 \tau_E(\lambda)}{\partial \lambda^2} = -\frac{\gamma_H - \gamma_P}{3\gamma_H}\frac{1}{\lambda^3} < 0.$$

We further show that $\tau_L(\lambda) < \tau_E(\lambda)$ as $\frac{4\gamma_H - \gamma_P}{2\gamma_H}\underline{G} > 0$. Depending on the parameter values, we can have that $\tau_L(\lambda) > \tau_C(\lambda)$ or $\tau_L(\lambda) < \tau_C(\lambda)$. However, as long as there are some values of $\lambda$ for which $\tau_L(\lambda) > \tau_C(\lambda)$, our findings hold.