The importance of foregone options*

Ana Espínola-Arredondo† Félix Muñoz-García†
School of Economic Sciences School of Economic Sciences
Washington State University Washington State University
Pullman, WA 99164 Pullman, WA 99164

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Abstract

Recent experimental evidence supports the influence of a player’s unchosen alternatives in other agent’s actions. This paper examines a tractable theoretical model of reference-dependent preferences in which individuals compare other players’ chosen action with respect to their unchosen alternatives. We analyze the equilibrium prediction in complete information sequential-move games, and compare it with that of standard games where players are not concerned about unchosen alternatives. We show that, without relying on interpersonal payoff comparisons (i.e., with strictly individualistic agents), our model predicts higher cooperation among the players than standard game-theoretic models. We apply our results in three economic contexts: the labor market gift exchange game, the ultimatum bargaining game, and the sequential public good game.

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†205C Hulbert Hall, Washington State University, Pullman, WA 99164-6210. E-mail: anaespinola@wsu.edu.
‡103G Hulbert Hall, Washington State University, Pullman, WA 99164-6210. E-mail: fmunoz@wsu.edu. Phone: (509) 335 8402. Fax (509) 335 1173.
1 Introduction

We often find ourselves comparing what we receive from others with what we could have received. For instance, newlyweds may compare the gifts they receive with those they specifically requested in their wedding list. Similarly, a young economist just hired by a public university might compare his or her actual salary with those of similar economists at the same institution, since this information is readily available at most universities’ websites. Indeed, several examples abound in which individuals compare the actual offer they receive from another individual with respect to the offer they could have received (the offer that the proposer foregoes). In addition, we can expect that such comparison might modify individual behavior, leading agents to positively reciprocate actions they infer to be kind, and negatively reciprocate actions they deem as unkind. For instance, the newlyweds may decide to not send thanks cards to their wedding guests if they consider that gifts did not match those in their wedding registry list. In the case of the young economist, if she observes that her salary is higher than that of similar economists in the same institution, she might react by working extra hours.

Such individual behavior, so common in individual interactions, has been confirmed by multiple experimental studies, whereby an agent’s choices can only be supported by analyzing how she compares other players’ chosen actions with respect to their unchosen alternatives. For example, Brandts and Solà (2001), Falk et al. (2003) and Charness and Rabin (2002) accumulate significant evidence supporting the importance of unchosen alternatives in the ultimatum bargaining game, while Andreoni, Brown and Vesterlund (2002) show the relevance of unchosen alternatives in public good games. In the ultimatum bargaining game for instance, these experimental results show that receivers positively evaluate a given offer that is above the alternative division of the pie, and negatively evaluate the same offer otherwise.

This paper introduces a model that rationalizes this economic conduct in complete information sequential-move games. We assume that every player cares about her material payoff as in standard models. Additionally, we consider that every individual compares other players’ actually chosen actions with respect to a particular action that they could have selected (other players’ foregone actions). This particular action is used by every individual as a reference point to measure the kindness she perceives from other players’ choices. In other words, this paper introduces an alternative definition of kindness based on the concept of foregone options.

We first identify conditions under which players’ equilibrium actions are higher when individuals are concerned about these reference-dependent comparisons than when they are not. This set of conditions allow for a direct prediction about whether we can sustain higher cooperation when players care about foregone options than when they do not. Unlike models with inequity averse individuals where players do care about other individuals’ payoffs (social preferences), this study analyzes conditions under which agents choose higher equilibrium actions without the need to assume that they care about other players’ payoffs, i.e., agents’ preferences can be regarded
as “strictly individualistic.” Second, we show that our model embeds certain existing behavioral approaches to inequity aversion. Finally, we apply our model to two stage complete information games: the labor market gift exchange game, the sequential public good game, and the ultimatum bargaining game. Our equilibrium predictions are not only validated in these applications, but also confirmed by recent experimental data, such as Brandts and Solà (2001) and Falk et al. (2003).

The structure of the paper is as follows. In the next section we discuss the literature on social preferences and intentions-based reciprocity, and their relationship with our paper. In section three, we describe the properties that players’ utility function must satisfy in order to support our results in terms of higher degrees of cooperation. Furthermore, section four analyzes players’ equilibrium strategy in these sequential-move games, and section five applies the model to three economic examples. Finally, the last section discusses some conclusions of the paper as well as further extensions.

2 Related literature

2.1 Theoretical literature on social preferences

The literature on behavioral economics has extensively considered elements other than one’s own payoff in individuals’ utility function. In this respect, some papers on inequity aversion, such as Fehr and Schmidt (1999) and Bolton and Ockenfels (2000) play a prominent role. On one hand, Fehr and Schmidt (1999) consider in their two-player version the following utility function

\[ U_i(x_i, x_j) = x_i - \alpha_i \max\{x_j - x_i, 0\} - \beta_i \max\{x_i - x_j, 0\} \]

where \( x_i \) is player \( i \)'s payoff. Intuitively, \( \alpha_i \) represents the disutility from allocations that are disadvantageously unequal for player \( i \) (i.e., he may feel envy about player \( j \)'s payoffs), while \( \beta_i \) denotes the guilt feeling from being the agent with the highest payoff of the population.\(^1\) On the other hand, Bolton and Ockenfels (2000) also develop a similar model of inequity aversion in which individuals’ utility is assumed to be increasing and concave in their share of total income. These models of social preferences, however, cannot rationalize the experimental evidence presented in the introduction. Indeed, any model which explains such results must necessarily complement the above specification by introducing the importance of unchosen alternatives into player \( i \)'s utility function, as this paper examines.\(^2\)

\(^1\)Blanco et al (2007) present experimental evidence supporting inequity aversion at the aggregate level (across all participants of a particular game) but refuting it at the individual level (for a given participant across games). Their results can be confirmed by our model, whereby participants of a particular game exhibit concerns for unchosen alternatives, but they may use different foregone options across games as a reference point for comparison.

\(^2\)Some axiomatic approaches, such as Segal and Sobel (1999), examine what conditions on players’ preferences must be satisfied in order to obtain utility functions which can be represented as a weighted average of a player’s own material payoff as well as that of others. Our approach differs from theirs, since we not only include players’ actually chosen actions in their utility function (as they do), but also players’ unchosen actions.
2.2 Models on intentions-based reciprocity

This paper is related to that of Charness and Rabin (2002), whereby they analyze the intentions that players express with their actual choices along the game. They assume that agents evaluate multiple characteristics of the equilibrium allocation —including fairness and intentions— by establishing different comparisons between own and social payoffs (i.e., between $x_i$ and $x_j$). Specifically, when only intentions are considered, agent $i$’s utility function in Charness and Rabin’s (2002) model reduces to

$$U_i(x_i, x_j) = \begin{cases} x_i + \theta(x_i - x_j) & \text{if player } j \text{ misbehaved} \\ \frac{x_i}{x_j} & \text{otherwise} \end{cases}$$

where player $j$’s misbehavior can implicitly include player $i$’s concern about player $j$’s foregone options, and where $\theta$ represents the importance of intentions-based reciprocity for player $i$. Note, however, that player $i$’s disutility from player $j$’s misbehavior is scaled up by the difference between player $i$ and $j$’s payoffs, $x_i - x_j$. Certainly, this confounds the elements triggering such perception of misbehavior (which implicitly includes unchosen alternatives), and how this misbehavior is then measured (which considers inequity aversion).\(^3\)

Similarly, Falk and Fischbacher (2006) analyze kindness by considering the product of two elements: the above interpersonal payoff comparison (what they refer as the “outcome term”), and a measure of other players’ intentions which reflects the set of available choices for these players (the “intentions factor”). They assume that the reference standard with which every player compares his own payoff is that of other players, and then he scales up this payoff distribution according to the degree of freedom in the other players’ available choices. Likewise, Rabin’s (1993) and Dufwenberg and Kirchteiger’s (2004) models of reciprocity, for simultaneous and sequential-move games respectively, introduce inequity considerations by assuming that every player compares his payoff with respect to an equitable payoff. In particular, both of these papers assume that the reference point that every player uses to determine the kindness behind other players’ choices is the average between the highest and lowest material payoff, which is compatible with other players choosing an efficient strategy.

Finally, Cox, Friedman and Sadiraj (2007) construct a model in which a player’s preferences become more altruistic with respect to other players when she infers that these players have behaved generously with her. However, their notion of generosity is not equivalent to our definition of kindness, nor does their notion of altruism coincide with our definition of reciprocity, since they assume that players compare their payoffs with that of others in their group. Unlike these models, we do not introduce other people’s payoffs into player $i$’s evaluation of intentions or kindness. Instead, in our model player $i$ measures the kindness in player $j$’s actions by comparing player $j$’s chosen and unchosen (foregone) actions.

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\(^3\)Likewise, most of the experimental literature testing reciprocating behaviors triggered by kind intentions also considers that agent $i$ measures player $j$’s intentions by comparing $x_i$ and $x_j$; see Cox (2001, 2003).
3 Model

The model considers complete information sequential-move games with two players and two stages, since several experimental research testing for kindness and reciprocity uses this class of games. Specifically, we focus on games in which: (1) players’ actions work as strategic substitutes; and where (2) every player benefits from increases in other players’ actions. In particular, let us consider games $G = \langle S_i, S_j; u_i, u_j \rangle$, in which a female leader (player $j$) selects an action $s_j \in S_j \subset \mathbb{R}_+$, and afterwards a male follower (player $i$) chooses an action $s_i \in S_i \subset \mathbb{R}_+$. The leader’s action may represent, for instance, her wage offer to a worker, or her monetary contribution to a public good. Similarly, the follower’s action may denote, respectively, his effort level in a labor market game, or his monetary donation to a charity in the sequential public good game. (Note that for simplicity we describe our model for continuous action spaces. Nonetheless, all our assumptions can be extended to discrete action spaces as well). Every action profile $s = (s_i, s_j) \in S_i \times S_j$ is then mapped into the set of possible outcomes by function $\text{out} : S_i \times S_j \rightarrow X$. And finally, every player $i$ assigns a utility value to every outcome through his utility function. Since the outcome function maps every action profile into a single outcome, there is a unique action profile leading to every terminal node of the game. Hence, for every outcome $\text{out}(s) \in X$ we can identify the unique action profile $s = (s_i, s_j)$ which induces that outcome. Thus, players’ utility function can be represented over action profiles in the form $U_i : S_i \times S_j \rightarrow \mathbb{R}$, i.e., $U_i(s_i, s_j) \in \mathbb{R}$. Let us thereafter denote by single (double) subscripts in the utility function its first (and second) order derivatives, and let us use superscript $\text{NC}$ to refer to the case in which player $i$ is “not concerned” about player $j$’s unchosen alternatives, as opposed to superscript $\text{C}$, which we use when players are “concerned” about each others’ unchosen actions.

**Assumption A1.** Positive but decreasing marginal benefit from other players’ actions, $s_j$. That is, $U^\text{NC}_{s_j}(s_i, s_j) \geq 0 \geq U^\text{NC}_{s_js_j}(s_i, s_j)$ for all $s_i$ and $s_j$.

Thus, every player benefits from increases in other players’ actions, but at a decreasing rate. Note that we are deliberately vague about how $U^\text{NC}_i(s_i, s_j)$ increases (or decreases) in her own action $s_i$. In this way, we can capture models where players’ marginal utility from increasing her action is positive (e.g., contributions in public good games) as well as negative (e.g., effort in labor market games). Next, we assume that player $i$’s utility function is strictly concave in his own actions, $s_i$.

**Assumption A2.** Concavity. $U^\text{NC}_{s_is_i}(s_i, s_j) < 0$ for all $s_i$ and $s_j$.

This assumption guarantees the existence of a unique equilibrium when players’ action space is continuous, which will facilitate the comparison of the equilibrium prediction when players are concerned about unchosen alternatives with respect to the case in which players are not.\footnote{In the case of discrete and binary action spaces, as in certain ultimatum bargaining games, concavity is not necessary. Instead, in order to facilitate the comparison of our results, we only need the subgame perfect equilibrium to be unique.}
Assumption A3. Strategic Substitutability. Player j’s (first mover) utility function satisfies $U^{NC}_{s_i s_j}(s_i, s_j) < 0$ for all $s_i$ and $s_j$.

Thus, the first mover’s marginal benefit from increasing her own action, $s_j$, decreases when the second mover raises his action, $s_i$. That is, the leader considers the follower’s actions as strategic substitutes of her own. This assumption is sensible for a large class of games, where players try to free-ride each others’ actions, e.g., the first mover’s incentives to free-ride the second mover’s donations to the public good or his effort decision. Therefore, A3 eliminates payoff structures such as those in the impunity game, whereby (in a variation of the ultimatum bargaining game) the first mover obtains exactly the same payoff regardless of the second mover’s actions, i.e., unconditional on his acceptance or rejection of the first mover’s offer. In contrast, A3 maintains the first mover’s incentives to free-ride the second mover’s action, since she considers players’ actions as strategic substitutes.

3.1 How kindness enters into players’ preferences

Let us first describe how kindness enters into player $i$’s utility function, and afterwards analyze how players measure the kindness they infer from their opponent’s actions.

Assumption A4. Kindness. For any actions $s_i \in S_i$ and $s_j \in S_j$, $U^C_i(s_i, s_j)$ increases in the kindness that player $i$ infers from player $j$’s choices. In contrast, $U^{NC}_i(s_i, s_j)$ does not vary in the kindness that player $i$ infers from player $j$’s actions. Furthermore, $U^C_i(s_i, s_j) = U^{NC}_i(s_i, s_j)$ when player $i$ infers neither kindness nor unkindness from player $j$’s choices.

Therefore, when player $i$ cares about foregone options and interprets a greater kindness from player $j$’s actions, his utility level increases. If he does not assign any value to foregone options, then he infers neither kindness or unkindness from player $j$’s choices, and his utility level is unchanged. Note that if player $i$ infers kindness, his utility level is higher when he is concerned about foregone options than when he is not. Otherwise (when he infers unkindness), his utility level is lower. Let us next describe how this kindness affects player $i$’s marginal utility.

Assumption A5. Reciprocity. For any actions $s_i \in S_i$ and $s_j \in S_j$, $U^C_{s_i}(s_i, s_j)$ increases in the kindness that player $i$ infers from player $j$’s choices. In contrast, $U^{NC}_{s_i}(s_i, s_j)$ does not vary in the kindness that player $i$ infers from player $j$’s actions.

Hence, A5 specifies that player $i$’s marginal utility from rising his action $s_i$ increases in the kindness he infers from player $j$’s choices. If he does not assign a value to foregone options, then he infers neither kindness or unkindness from player $j$’s actions, keeping his marginal utility unchanged. In particular, this assumption leads player $i$ to increase his action (positive reciprocity) when he infers kindness, and to decrease it (negative reciprocity) when he infers unkindness.
3.2 How players measure kindness

Let us now describe how players evaluate the kindness they infer from other players’ actions. In particular, we assume that player $i$ measures kindness through the following distance function, $D_i(s_i, s_j)$, and that he infers kindness when the outcome of the distance function is positive, and unkindness otherwise.

$$D_i(s_i, s_j) = \alpha_i \left[ s_j - s_{Ri}^j (s_i, s_j) \right]$$

for any $\alpha_i \in \mathbb{R}^+$. Thus, player $i$ evaluates player $j$’s kindness by comparing player $j$’s actually chosen action, $s_j$, and a particular reference action that player $i$ uses for comparison, $s_{Ri}^j (s_i, s_j) \in S_j$, among player $j$’s available choices, as we define below.\(^5\) For simplicity, the distance function was chosen to be linear. Nonetheless, from a more general perspective, player $i$’s distance function could be nonlinear, as long as it increases in player $j$’s actually chosen action, $s_j$, and decreases in the reference action that player $i$ uses for comparison.

The reference-dependent measure proposed here is a natural way for player $i$ to assess player $j$’s actions, as the experimental results mentioned in the introduction. This distance function is similar to that in the literature on reference-dependent preferences, such as Köszegi and Rabin (2006). Their model analyzes individual decision making, unlike this paper where we examine its strategic effects. On the other hand, our distance function differs from that in Rabin (1993) for simultaneous-move games and that in Dufwenberg and Kirchsteiger (2004) for sequential-move games. Indeed, these studies assume that player $i$ compares his actual payoff with respect to the “equitable” payoff (his equitable share in the Pareto-efficient payoffs). In contrast, we allow player $i$ to compare player $j$’s actually chosen action with respect to any feasible action, $s_{Ri}^j (s_i, s_j) \in S_j$, leading to equitable or non-equitable payoffs. Let us next define the concept of reference action, $s_{Ri}^j (s_i, s_j)$, which player $i$ uses as a reference point in order to evaluate the kindness that he perceives from player $j$’s actually chosen action, $s_j$.

Definition 1. Player $i$’s reference point function $s_{Ri}^j : S_i \times S_j \rightarrow S_j$, maps the pair $(s_i, s_j)$ of both players’ actually chosen actions, into a reference action $s_{Ri}^j (s_i, s_j) \in S_j$ from player $j$’s set of available choices. In addition, $s_{Ri}^j (s_i, s_j)$ is weakly increasing in $s_i$ and $s_j$, and twice continuously differentiable in $s_i$ and $s_j$.

Hence, player $i$ can use any of player $j$’s available actions in $S_j$ as a reference point.\(^6\) That is, $s_{Ri}^j (s_i, s_j)$ is allowed to be above/below/equal to player $j$’s actually chosen action, $s_j$, which leads to negative/positive/null distances, respectively. The particular sign of such distance affects

\(^5\)We assume that player $i$ compares player $j$’s actions, instead of the payoffs resulting from these action choices. This assumption does not modify our results, since player $i$’s payoffs are increasing in player $j$’s action choices (assumption A1).

\(^6\)We restrict the range of reference points to player $j$’s available choices, $S_j$. More generally, $s_{Ri}^j (s_i, s_j)$ could take values outside $S_j$. We believe, however, that it is more intuitive to assume that player $i$ compares player $j$’s actions with respect to her foregone options than to actions which were not even available to her.
player $i$’s utility function, $U^C_i(s_i, s_j)$, as described above. Additionally, when both players’ action spaces are identical, $S_i = S_j = S$, player $i$’s reference point function becomes $s^R_{ij}: S^2 \rightarrow S$. In this context, the reference point function can be, for instance, $s^R_{ij}(s_i, s_j) = s_i$ for all $s_j$. In such case, $D_i(s_i, s_j) = \alpha_i [s_j - s_i]$, and player $i$ compares player $j$’s chosen action, $s_j$, with respect to her own, $s_i$, i.e., $s_j > s_i$ is perceived by player $i$ as player $j$’s kindness (e.g., her commitment to contribute high donations to the public good), whereas $s_j < s_i$ is evaluated by player $i$ as a unkindness (e.g., free-riding).

Furthermore, we allow player $i$ to modify the reference action he uses to compare player $j$’s actually chosen action, i.e., $s^R_{ij}(s_i, s_j)$ is not restricted to be constant for all $s_j$. In particular, we assume that, for a given increase in player $j$’s action, $s_j$, the reference point, $s^R_{ij}(s_i, s_j)$, does not increase at a higher rate than player $j$’s action does, i.e., $1 \geq \partial s^R_{ij}(s_i, s_j)/\partial s_j$. Intuitively, this condition makes higher values of player $j$’s action meaningful for player $i$, since higher values of $s_j$ increase the outcome of his distance function, i.e., $\partial D_i(s_i, s_j)/\partial s_j = 1 - \partial s^R_{ij}(s_i, s_j)/\partial s_j$; and as we described above, larger distances raise player $i$’s utility level (kindness). As a remark, note that $D_i(s_i, s_j)$ does not depend on any possible randomness over payoffs. Indeed, player $i$’s utility level does not depend on the difference between payoffs he could have received from the outcomes of a certain lottery, but only on payoffs he could have obtained from alternative choices of the other players. This distinction differentiates our approach from regret theory, as in Loomes and Sugden (1982), since our model focuses on agent $i$’s evaluation of other players’ chosen and unchosen actions as a measure of their kindness. Finally, extending assumption A2 to the context of concerned players, we assume that $U^C_i(s_i, s_j)$ is also strictly concave in all player $i$’s actions, $s_i$.

### 3.3 Best response function

Let $s^C_i(s_j) \in \arg \max_{s_i} U^C_i(s_i, s_j)$ denote player $i$’s best response function when he assigns a positive importance to player $j$’s foregone options, and $s^{NC}_i(s_j) \in \arg \max_{s_i} U^{NC}_i(s_i, s_j)$ his best response function when he does not.

**Proposition 1.** Player $i$’s best response function when he is concerned about foregone options is higher than that when he is not if player $i$ infers kindness from player $j$’s actions; and lower if he infers unkindness. That is,

$$s^C_i(s_j) \geq s^{NC}_i(s_j) \text{ for all } s_j \text{ such that } D_i(s_i, s_j) \geq 0$$

$$s^C_i(s_j) < s^{NC}_i(s_j) \text{ for all } s_j \text{ such that } D_i(s_i, s_j) < 0$$

Furthermore, the difference between player $i$’s best response function when he is concerned about foregone options and that when he is not increases in the kindness that player $i$ infers from player $j$’s choices, i.e., $s^C_i(s_j) - s^{NC}_i(s_j)$ is increasing in $D_i(s_i, s_j)$.
Intuitively, player $i$’s interpretation of kind (or unkind) actions triggers a higher (lower) response when he cares about foregone options than when he does not. For example, the worker in the labor market gift exchange game, when perceiving kind actions from the firm manager, exerts a higher effort when he is concerned about the firm manager’s unchosen alternatives (foregone wage offers) than when he is not. Moreover, the greater the kindness that the worker infers from the manager’s wage offer, the stronger is the increase in his effort level relative to the case in which he is not concerned about foregone alternatives.

4 Equilibrium analysis

Let us now analyze player $j$’s (first mover) equilibrium action in this sequential game.

Lemma 1. The leader’s marginal utility from increasing her own action $s_j$ is higher when the follower is concerned about her unchosen alternatives than when he is not. That is, for any action $s_j \in S_j$ player $j$’s (first mover) utility function satisfies,

$$\frac{\partial U_{jNC}(s_i^C(s_j), s_j)}{\partial s_j} \geq \frac{\partial U_{jNC}(s_i^{NC}(s_j), s_j)}{\partial s_j}$$

From this lemma, the following proposition is immediately derived.

Proposition 2. If assumptions A1-A5 are satisfied, then $s_j^C \geq s_j^{NC}$. That is, the leader’s equilibrium action when dealing with a follower who is concerned about foregone options, $s_j^C$, is weakly higher than her equilibrium action when facing a follower not concerned about foregone options, $s_j^{NC}$.

Hence, in the subgame perfect Nash equilibrium strategy profile of the game with positive concerns for foregone options the leader chooses a higher equilibrium action than that in the game with no concerns for unchosen alternatives. This result is especially relevant for certain games, such as the labor market gift exchange and the sequential public good game, where the introduction of concerns for foregone options leads to higher levels of cooperation among the players. In particular, as we show in section 5 for different economic applications, the fact that the follower is sensitive to the leader’s unchosen alternatives attenuates the leader’s incentives to shift most of the burden to the follower (reducing free-riding) which ultimately triggers higher actions from the leader as well.

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7 As a remark, note that the follower moves his action choice in the opposite direction than the first mover moves her when he regards actions as strategic substitutes (negatively sloped best response function); whereas he moves it in the same direction when actions are strategic complements (positively sloped best response function).

8 These results can be easily generalized to sequential-move games with $N$ players. In such settings, every player measures the kindness he infers from the actually chosen strategies of every player who played before him. The outcome of each of these individual comparisons can then be added up (or scaled in a weighted average), in order to evaluate player $i$’s distance function. Despite the greater generality of such model, nonetheless, its results and intuition are already captured by the two-player setting we consider in this paper.
4.1 Remarks on inequity aversion and reciprocity

In this subsection, we show that (under certain conditions) our model can also support some of the results of the literature on inequity aversion and intentions-based reciprocity as special cases.

**Proposition 3.** Assume $s^R_i(s_i, s_j) = s_i$ for all $s_j$. Then, player $i$’s preferences can be represented as a weighted average of her material payoffs and those of player $j$, as in Segal and Sobel (1999).

$$U^C_i(s_i, s_j) = \gamma_i U^NC_i(s_i, s_j) + \gamma_j U^NC_j(s_j, s_i)$$

where $\gamma_i, \gamma_j \in \mathbb{R}$

The above proposition specifies that when player $i$ compares player $j$’s actually chosen action, $s_j$, with that chosen by himself, $s_i$, her utility function $U^C_i(s_i, s_j)$, considers both his own payoff and that of the other player. Therefore, in such context our model captures players’ concerns for inequity aversion (or altruism) as a special case. In addition, this model also captures the literature on intentions-based reciprocity as a special case. Indeed, the above utility representation embodies Charness and Rabin’s (2002) model for the case that player $i$ infers misbehavior from player $j$’s actions, and for $\gamma_i = 1 + \theta$ and $\gamma_j = -\theta$. That is,

$$U^C_i(s_i, s_j) = (1 + \theta) U^NC_i(s_i, s_j) - \theta U^NC_j(s_j, s_i)$$

Therefore, when players use their own action $s_i$ as a reference point to compare other players’ actually chosen action, $s_j$, our model can also capture players’ concerns about inequity aversion and intentions-based reciprocity as special cases.

5 Applications

5.1 Labor market gift exchange game

Let us first apply our model to a labor market gift exchange game, where the proposer is identified as a firm making a wage offer $w \in [0, 1]$ to a worker, who decides what level of effort to exert. In traditional models without considerations about foregone wage offers, the subgame perfect equilibrium of this game predicts that the firm offers the lowest possible wage and that workers exert zero effort regardless of the wage offer made by the firm. These models have found however limited experimental evidence. Indeed, Fehr and Gachter (2000) summarize a series of experiments on labor markets where they confirm the existence of a positive correlation between the wage offered by the firm and the effort exerted by the worker. We next introduce a utility function for the worker that satisfies the properties considered in section 3, and that can rationalize the above experimental
results\(^9\).

\[ U_i^C (w, e) = w - e^2 + \alpha_i (w - w^F) e \]

where \(w^F\) is the foregone wage offer that the worker uses as a comparison against the actual wage offer, \(w\); and \(e\) denotes the amount of effort exerted by the worker. The above utility function coincides with the standard utility function of a worker who exerts costly effort when the parameter denoting the importance of foregone options, \(\alpha_i\), approaches zero. Let us next find the worker’s equilibrium effort when he is concerned about foregone wage offers.

**Lemma 2.** In the gift exchange game where the worker assigns a value \(\alpha_i\) to the distance between the firm’s actual wage offer and its foregone alternative, the worker’s optimal effort level (in the subgame induced after the wage proposal) is given by

\[ e(w) = \max \left\{ \frac{1}{2} \alpha_i (w - w^F), 0 \right\} \]

This optimal effort level is then positive if and only if the wage offer is above the foregone option, \(w > w^F\), for any positive weight to foregone options, \(\alpha_i\), as the following figures illustrate. In particular, figure 1(a) indicates how the worker effort pivots upward — with center at \(w = w^F\) — when his concerns \(\alpha_i\) about the firm’s unchosen alternatives increase.\(^10\) On the other hand, figure 1(b) represents how worker’s effort shifts upwards when the firm’s unchosen alternative decreases. Indeed, if the worker compares the actual wage he receives, \(w\), with respect to the worst wage offer that the firm manager could pay him (e.g., the legal minimum wage), he is easily pleased by most positive wage offers.

---

\(^9\)Different functional forms for \(U_i^C (w, e)\) satisfy assumptions A1 through A5, leading to the results predicted in the previous section. Nonetheless, a simple expression is used here to emphasize intuition.

\(^10\)Note that our results in the labor market gift exchange game are similar to those in Akerlof (1982) since higher salaries induce higher effort levels. In particular, Akerlof’s (1982) results are a special case of ours, when the foregone wage offer is exactly fixed at the “fair wage” level.
Let us finally find the firm’s equilibrium wage offer. Specifically, we assume that the firm’s utility function is given by \( V(w, e) = (v - w)e \), where \( v \) represents the constant productivity of effort (e.g., how worker’s effort is transformed into final output), and \( v > 1 \), since the productivity of effort is assumed to be higher than any of the wage offers, \( w \in [0, 1] \).

**Proposition 4.** In the gift exchange game where the worker assigns an importance of \( \alpha_i \) to the distance between the wage offer foregone by the firm and its actual offer, the subgame perfect equilibrium strategies are the following

- **Firm offers**
  
  \[
  w^* = \frac{v + w^F(w^*)}{2}
  \]

- **Worker accepts any offer** \( w \) **such that** \( w > 0 \). In addition, the worker exerts an effort level of
  
  \[
  e(w) = \max \left\{ \frac{1}{2} \alpha_i (w - w^F(w)), 0 \right\}
  \]

As the above proposition specifies, the firm’s optimal offer \( w^* \) is higher than the worker’s foregone option, \( w^F(w^*) \), since \( v > 1 \). In addition, \( w^* \) is increasing in the foregone option, \( w^F \), that the worker uses to make the comparison\(^{11} \) with respect to \( w^* \). Clearly, the above equilibrium predictions are closer to the actual experimental results observed in the literature, Fehr and Gachter (2000), which find a positive correlation between the wage offered by the firm and the exerted effort levels from the worker.\(^{12} \)

Interestingly, these results are not only supported by experimental evidence, but also by recent empirical work. In particular, Mas (2006) shows that police arrest rates and average sentence length decline (and crime reports raise) when the wage increase that police unions obtain is lower than their wage demands, relative to when it is higher. Hence, police union wage demands would work as the reference point which they use in their negotiations for higher salaries with government officials. Finally, these results also provide an interesting explanation for the existence of wage differentials across industries. Indeed, as Krueger and Summers (1988) show, industry wage differentials are significant even after controlling for individual characteristics and firm quality; which suggests that these differentials are not just due to unobserved differences in labor quality. Our model then rationalizes this result by predicting that firms’ equilibrium wage offer, after controlling for worker’s productivity, may vary depending on the particular reference point that each worker uses for comparison.

\(^{11}\)Note that, for simplicity, we assume that the worker compares all wage offers with respect to the same foregone option, i.e., \( w^F(w) = 0 \). Similar results are nonetheless applicable for the more general case in which \( w^F(w) \neq 0 \), and they are included in the proof of proposition 4 at the appendix.

\(^{12}\)Many authors have rationalized the above findings by using the efficiency wage theory arguments. Nonetheless, our model can explain cooperative behavior between employers and workers without relying on the worker’s opportunity cost of shirking, or his outside options if he is fired.
5.2 Sequential public good game

The second game where we introduce the importance of the proposer’s foregone options is the sequential public good game (PGG thereafter). Specifically, we consider a sequential solicitation game where a first mover is asked to submit a donation, \( s_j \in [0, 1] \), for the provision of a public good, and observing her donation, a follower decides his own contribution, \( s_i \in [0, 1] \). In particular, leader and follower’s utility functions are, respectively

\[
U_j^{NC} (s_j, s_i) = z_j + [m (s_i + s_j)]^{0.5}
\]

\[
U_i^C (s_i, s_j) = z_i + [m (s_i + s_j) (1 + \alpha_i (s_j - s_j^F))]^{0.5}
\]

Both of these functions are linear in the private good, \( z \), and their nonlinear part takes into account the utility derived from the total public good provision \( G = s_i + s_j \) (relevant for both players) and the distance \( \alpha_i (s_j - s_j^F) \), comparing the first mover’s actual donation and her foregone contribution, which is only relevant for the follower. For simplicity, let us assume in this application that the follower uses the same foregone contribution \( s_j^F \) for all action choices of the leader. Finally, \( m \geq 0 \) denotes the return every player obtains from total contributions to the public good. Note how foregone options are introduced into the follower’s utility function. When the relevance he assigns to the leader’s unchosen alternatives approaches zero, \( \alpha_i = 0 \), the follower only cares about the private and public good consumption. However, when he assigns a positive importance to foregone options, he experiences a higher utility from contributing to the public good when the leader’s donation is higher than her foregone contribution, \( s_j > s_j^F \), or a lower utility otherwise, \( s_j < s_j^F \).

Lemma 3. In the sequential PGG, where the follower assigns weight \( \alpha_i \) to the distance between the leader’s actual contribution, \( s_j \), and the foregone contribution, \( s_j^F \), the follower’s best response function \( s_i^C (s_j) \) is given by

\[
s_i^C (s_j) = \begin{cases} 
\frac{m(1-\alpha_i s_j^F)}{4} + \frac{\alpha_i m - 4}{4} s_j & \text{if } s_j \in \left[0, \frac{\alpha_i s_j^F - m}{\alpha_i m - 4}\right] \\
0 & \text{if } s_j \geq \frac{\alpha_i s_j^F - m}{\alpha_i m - 4}
\end{cases}
\]

Figure 2 compares the second mover’s best response function when he is concerned about foregone options, \( s_i^C (s_j) \), and when he is not, \( s_i^{NC} (s_j) \).
Specifically, note that the introduction of the importance of foregone options into the second mover’s utility function induces a counterclockwise rotation in $s^C_i(s_j)$, with center at $s_j = s^F_j$, making $s^C_i(s_j)$ steeper than $s^{NC}_i(s_j)$, i.e., contributions become more strategically complementary. Hence, the second mover reduces his donation when the first mover’s contribution is below her foregone donation $s_j < s^F_j$, but increases it when such contribution exceeds her foregone donation $s_j > s^F_j$, i.e., the second mover relatively “reciprocates” the first mover’s contributions. Let us now find the first mover’s equilibrium contribution in this game.

**Proposition 5.** In the sequential PGG, where the follower assigns a weight $\alpha_i$ to the leader’s foregone options, the leader’s donation in the subgame perfect Nash equilibrium of the game is

$$s^*_j = \begin{cases} s^F_j + \frac{m^2\alpha_i}{16} - \frac{1}{\alpha_i} & \text{if } \alpha_i \geq \bar{\alpha}_i \\ 0 & \text{if } \alpha_i < \bar{\alpha}_i \end{cases}$$

where $\bar{\alpha}_i = \frac{4\sqrt{4\alpha_i^2 + m^2 - 2s^F_j}}{m^2}$.

Thus, the first donor submits a zero contribution when the second donor’s concerns for foregone options are low enough, $\alpha_i < \bar{\alpha}_i$. Clearly, when $\alpha_i = 0$ the first donor also submits a null donation, which coincides with the equilibrium prediction in standard PGGs. However, when the second donor’s concerns for foregone options increase enough, $\alpha_i > \bar{\alpha}_i$, the first mover is induced to submit positive contributions that can trigger further donations from the second mover (given his reciprocating behavior described in the previous figure). Additionally, note that as expected, the
leader’s contribution is increasing in the follower’s concerns for foregone options, \( \alpha_i \), and in the foregone contribution that he uses as a reference point for comparison, \( s_j^F \).

Particularly, the above results specify that by having a second mover concerned about the first mover’s foregone options, the latter is induced to contribute (weakly) higher amounts than those she would donate in the case of facing a responder with no concerns about her unchosen alternatives. From a more general perspective, by introducing a follower concerned about the leader’s foregone options, we are able to obtain (weakly) higher levels of cooperation in the public good provision.

5.3 Ultimatum bargaining game

Let us finally apply our model to the ultimatum bargaining game where a (female) proposer \( j \) is called to choose how to divide a pie (of size normalized to one) between the (male) responder \( i \) and herself, and the responder either accepts or rejects such division, \( s_i \in \{A, R\} \). In particular, let \((s_j, 1 – s_j)\) represent the actual division offered by player \( j \), where \( s_j \) denotes the share of the pie accruing to the responder, and \( 1 – s_j \) represents the remaining share of the pie that the proposer keeps for herself. Hence, \( s_j \) represents the offer that the proposer makes to the responder, and \( s_j^F \) denotes the foregone offer that the responder uses for comparison. Specifically, the responder’s utility function is given by \( U_i^C(s_i, s_j) = s_j + \alpha_i (s_j - s_j^F) \), where \( \alpha_i \geq 0 \). Note that when the division of the pie that the proposer offers to the responder \( s_j \) is above (below) her foregone offer, \( s_j^F \), the responder perceives kindness (unkindness) from the proposer. From the above utility function, we obtain the responder’s acceptance rule.

**Lemma 4.** In the ultimatum bargaining game with a responder who assigns a weight \( \alpha_i \geq 0 \) to the proposer’s foregone offers, \( s_j^F \), the responder accepts any offer \( s_j \) if and only if \( s_j \geq \bar{s}_j \), where \( \bar{s}_j = \frac{\alpha_i}{1 + \alpha_i} s_j^F \).

Let us emphasize some interesting insights from the above lemma, illustrated in figure 3 below. Clearly, when \( \alpha_i = 0 \) the responder’s acceptance rule collapses to \( \bar{s}_j = 0 \). Indeed, when the responder does not assign any weight to the proposer’s unchosen actions, then any positive division of the pie is accepted by the responder, as in standard ultimatum bargaining games. Furthermore, the responder’s acceptance threshold \( \bar{s}_j \) is increasing in \( \alpha_i \) and in \( s_j^R \), i.e., he becomes more demanding. Finally, if the proposer wants to obtain any positive payoff from the game, she must make an offer which is accepted by the responder, as we show below.
Figure 3. $\bar{s}_j$, for $s_j^F = 0.5$ and $s_j^F = 0.8$.

**Proposition 6.** In the ultimatum bargaining game where the responder assigns an importance of $\alpha_i \geq 0$ to the options that the proposer forwent, the following strategy profile describes the unique subgame perfect equilibrium of the sequential game.

Responder accepts any offer $s_j$ such that $s_j \geq \bar{s}_j$, where $\bar{s}_j = \frac{\alpha_i}{1 + \alpha_i} s_j^F$.

Proposer offers $s_j^* = \frac{\alpha_i}{1 + \alpha_i} s_j^F$, for any parameter values.

Unlike models where the receiver is not concerned about foregone options —where the proposer keeps the entire pie for himself—the distribution of equilibrium payoffs is now less unequal, as the following corollary specifies.

**Corollary 1.** The distribution of equilibrium payoffs $(x_i, x_j)$ in the ultimatum bargaining game where the responder assigns importance $\alpha_i$ to the proposer’s foregone offer, $s_j^F$, is

$$(x_i, x_j) = \left( \frac{\alpha_i}{1 + \alpha_i} s_j^F, 1 - \frac{\alpha_i}{1 + \alpha_i} s_j^F \right)$$

**5.3.1 Experimental evidence**

Let us finally relate our theoretical results with those of the experimental literature. In particular, Falk et al. (2003) and Brandts and Solà (2001) show the existence of a relationship between the receiver’s acceptance threshold and the particular foregone offer that the proposer did not make. Indeed, both of these studies show that, conditional on offer $(0.2, 0.8)$ being made, the acceptance rate increases in the distance between the proposer’s chosen and unchosen alternatives, as the following figures illustrate.
In particular, note that the first column of figure 4(a), where $s_j - s_j^F = 0.2 - 0.5 = -0.3$, represents a negative distance between the proposer’s actual and foregone offer, from which the receiver infers “unkindness.” On the other hand, column 3, where $s_j - s_j^F = 0.2 - 0 = 0.2$ (and the distance is positive) denotes the case in which the receiver interprets “kindness” from the proposer’s offer, since she could have offered him less than she actually did. Finally, column 2 illustrates the case in which the proposer has no degree of freedom in choosing her particular offer to the receiver. i.e., the proposer’s offer is $(0.2,0.8)$ and her alternative is also $(0.2,0.8)$. In this case, the outcome of the distance function is zero, what leads the receiver to neither perceive “kindness” nor “unkindness” from the proposer’s actions. Interestingly, the fact that the acceptance rate in the second column is exactly higher than when he perceives “unkindness” (column 1) but lower than when he infers “kindness” (column 3) supports our results. A similar intuition is also applicable to Brandts and Solà’s (2001) results as figure 4(b) suggests. Hence, our theoretical prediction about the proposer’s offer goes in the same direction as these experimental results. Indeed, proposers are observed to make low offers when kindness can be inferred from such offers (positive distances), and high offers when they are interpreted in terms of unkindness (negative distances).

6 Conclusions

Different experimental papers, such as Brandts and Solà (2001), Falk et al. (2003), and Andreoni et al. (2002), accumulate a significant evidence about the importance of a player’s unchosen alter-
natives on other players’ actions. Foregone options, in particular, may work as standards against which every individual evaluates the kindness of other players in the population. Importantly, these studies suggest that arguments on social preferences alone cannot explain their experimental results without complementing their approach by considering the importance of a players’ unchosen alternatives inside his opponents’ utility function.

This paper examines a tractable theoretical model that introduces these unchosen alternatives into individuals’ preferences via a reference point. We first analyze the equilibrium prediction in complete information sequential-move games, and then compare it with that of standard games where players are not concerned about unchosen alternatives. We show that, without relying on interpersonal payoff comparisons (i.e., with “strictly individualistic” agents), our model predicts higher levels of fairness in the resulting allocation, as well as higher cooperation among the players, than standard game-theoretic models. In addition, we demonstrate that this approach embeds as special cases some results of existing behavioral models: from inequity aversion to intentions-based reciprocity. Furthermore, when applying our model to different sequential games, we obtain interesting results. First, worker’s effort and firm’s proposed wages are higher than in the usual labor market gift exchange model. Second, equilibrium donations in the sequential public good game are higher than the predictions for standard models. Finally, the equilibrium allocation in the ultimatum bargaining game is fairer than that resulting from standard game-theoretic predictions.

There are several natural extensions to the model introduced in this paper. First, it would be interesting to experimentally test under which payoff structures we can rationalize observed behavior using individuals’ preferences over equitable payoffs, and in which environments human conduct is instead mainly explained by the players’ “strictly individualistic preferences” suggested in this paper. One direct test of the dominance of these two behavioral motives is, for example, the following ultimatum bargaining game. The proposer is allowed to make only two divisions of the pie, of size normalized to one. In the first treatment she can offer (0.4, 0.6), giving 0.4 to the responder and keeping 0.6 for herself, or the equitable payoff (0.5, 0.5). In the second treatment, the first division of the pie is fixed in (0.4, 0.6), but the second division is now (0.6, 0.4) instead. Note that, conditional on the first offer, (0.4, 0.6), being made, the distance between the actual offer, 0.4, and the alternative offer is higher in the first treatment, 0.4 – 0.5 = –0.1, than in the second, 0.4 – 0.6 = –0.2. Hence, according to our equilibrium predictions, we should observe more rejections in the second treatment than in the first. However, if we observe higher percentage of rejections in the first than in the second treatment, it must be that responders in the first treatment evaluate the equitable payoffs that the proposer did not select as a more desirable goal than the higher individual payoff he could have received in the second treatment.

Second, in this paper the space of available alternatives was exogenously determined before the beginning of the game. However, it would be interesting to allow players to strategically select their available choices before the game starts, given that the kindness other players perceive from their chosen actions depends on which available strategies are not chosen. That is, by strategically
selecting her set of available alternatives, a player may induce other players to infer a greater kindness from her actions. This strategic selection of available choices is observed in different contexts, where a player uses one of her unchosen alternatives as an excuse to support her actual choices, since the equilibrium payoff associated with that particular unchosen action would have been certainly worse than that from her chosen action. For instance, we frequently encounter references to unchosen alternatives in the way many public policies are announced to the media. Indeed, these presentations are often accompanied with statements like “The government had to choose between policies A and B, and choosing A would have been so bad that we should better select B.” These statements are certainly effective when they induce the listener to positively evaluate the chosen action and relative to the unchosen action A. These extensions can certainly enhance our understanding of the role of players’ foregone options on their opponents’ incentives, and how such incentives can lead to higher degrees of cooperation from a strictly individualistic perspective.

7 Appendix

7.1 Proof of Proposition 1

We first show that player $i$’s best response functions when she is concerned about player $j$’s foregone options and when she is not, respectively, $s^C_i (s_j) \in \arg\max_{s_i \in S_i} U^C_i (s_i, s_j)$, and $s^{NC}_i (s_j) \in \arg\max_{s_i \in S_i} U^{NC}_i (s_i, s_j)$, contain a single point. Then, we show the result stated in proposition 1. Note that player $i$’s utility function when she is concerned about player $j$’s unchosen alternatives, $U^C_i (s_i, s_j)$, is strictly concave in $s_i$ and it is defined over a strictly convex domain. This guarantees that player $i$’s best response function $s^C_i (s_j) \in \arg\max_{s_i \in S_i} U^C_i (s_i, s_j)$ contains a single point. A similar argument is also applicable for player $i$’s utility function when she does not assign any relevance to player $j$’s foregone options, $U^{NC}_i (s_i, s_j)$, since it is also strictly concave in $s_i$ and it is defined over a strictly convex domain. Hence, $s^{NC}_i (s_j) \in \arg\max_{s_i \in S_i} U^{NC}_i (s_i, s_j)$ also contains a single point. From assumption A5 we have that a given increase in $D_i (s_i, s_j)$ induces an increase in the marginal utility $U^C_{s_i} (s_i, s_j)$ but does not modify the marginal utility $U^{NC}_{s_i} (s_i, s_j)$. Hence, the unique maximizer of $U^C_i (s_i, s_j)$, $s^C_i (s_j)$, is higher than that of $U^{NC}_i (s_i, s_j)$.

Furthermore, from assumption A5 we have that $U^C_{s_i} (s_i, s_j) - U^{NC}_{s_i} (s_i, s_j)$ increases in $D_i (s_i, s_j)$, i.e., for a given increase in $D_i (s_i, s_j)$ the marginal utility from increasing $s_i$ experiences a larger increase when player $i$ is concerned about foregone options than when he is not, for all $s_j$. Therefore, for a given increase in $D_i (s_i, s_j)$ the unique maximizer of $U^C (s_i, s_j)$, $s^C_i (s_j)$, also experiences a greater increase than the maximizer of $U^{NC} (s_i, s_j)$, $s^{NC}_i (s_j)$, does. Hence, the difference $s^C_i (s_j) - s^{NC}_i (s_j)$ increases in $D_i (s_i, s_j)$. □

7.2 Proof of Lemma 1

From proposition 1 we know that the difference between player $i$’s best response function when she is con-
cerned and unconcerned about foregone options, $s_i^C(s_j) - s_i^{NC}(s_j)$, is weakly increasing in the distance $D_i(s_i, s_j)$. In addition, by assumption A1 we have that player $j$’s utility function $U_j^{NC}(s_j, s_i)$ is strictly increasing in $s_i$. Hence, $U_j^{NC}(s_j, s_i^C(s_j)) - U_j^{NC}(s_j, s_i^{NC}(s_j))$ is weakly increasing in $D_i(s_i, s_j)$. Therefore, for two actions $s_j, s'_j \in S_j$ such that $s'_j > s_j$, we have that $D_i(s_i, s'_j) > D_i(s_i, s_j)$, what implies that

$$U_j^{NC}(s'_j, s_i^C(s_j)) - U_j^{NC}(s_j, s_i^C(s_j)) \geq U_j^{NC}(s_j, s_i^C(s_j)) - U_j^{NC}(s_j, s_i^{NC}(s_j))$$

and rearranging,

$$U_j^{NC}(s'_j, s_i^C(s_j)) - U_j^{NC}(s_j, s_i^{NC}(s_j)) \geq U_j^{NC}(s'_j, s_i^{NC}(s_j)) - U_j^{NC}(s_j, s_i^{NC}(s_j))$$

### 7.3 Proof of Proposition 2

Let us $s_j^C$ and $s_j^{NC}$ denote the leader’s equilibrium strategies when dealing with a concerned and not concerned follower, respectively. Let us prove $s_j^C > s_j^{NC}$ by contradiction. Hence, assume that $s_j^C < s_j^{NC}$. If this is the case, then the leader’s marginal utility from raising her action must be higher when the follower is unconcerned about foregone options than when he assigns a positive importance to them. But this contradicts lemma 2. In particular, recall that lemma 2 states that the marginal utility of raising the proposer’s action is higher for the first mover when the second mover is concerned about unchosen alternatives than when he is not. Hence $s_j^C < s_j^{NC}$ must be false, and proposition 2 is satisfied.

### 7.4 Proof of Proposition 3

Using Segal and Sobel (1999), we know that player $i$’s preferences over player $j$’s actions can be represented by

$$U_i^C(s_i, s_j) = \gamma_i U_i^{NC}(s_i, s_j) + \gamma_j U_j^{NC}(s_j, s_i) \quad \text{where } \gamma_i, \gamma_j \in \mathbb{R}$$

if preferences satisfy continuity and independence, as well as Segal and Sobel’s (1999) condition ($\star$) which states that if $U_i^{NC}(s_i^t, s_j) = U_i^{NC}(s_i, s_j)$, then $s_i^t \sim_i s_i$, which are all satisfied in our model.

### 7.5 Proof of Lemma 2

The worker’s optimal amount of effort to exert as a function of the wage proposal offered by the firm, $e(w)$, can be obtained from solving the following utility maximization problem

$$\max_{e \in \mathbb{R}^+} w - e^2 + \alpha_i(w - w^F(w)) e.$$  

Differentiating with respect to $e$, and manipulating, we have $e(w) = \begin{cases} \frac{1}{2} \alpha_i (w - w^F(w)) & \text{if } w > w^F(w) \\ 0 & \text{otherwise} \end{cases}$. For sufficiency, just note that the worker will never respond to an offer $w$ by exerting a higher effort level than the one specified in $e(w)$. Indeed, on the one hand, if he exerts higher effort levels, he will have more disutility from such effort than the utility he derives from the third term of the above utility function for $w > w^F(w)$. On the other hand, if he exerts less effort, then the marginal utility from exerting additional effort when $w > w^F(w)$ (third term of the utility function) would be greater than the marginal disutility
from exerting effort (second term). Hence, the worker would be better off by exerting more effort. Hence, the above effort level \( e(w) \) is optimal for the worker when the wage offered is \( w \).

### 7.6 Proof of Proposition 4

As shown in the above lemma 2, the worker’s optimal effort level is given by \( e(x_i) = \max \left\{ \frac{1}{2} \alpha_i \left( w - w^F(w) \right) , 0 \right\} \). Regarding the employer offer, we know that the employer inserts the above best response function into his utility function, in order to find the optimal wage offer. \( \max_{w \in [0,1]} (v - w)e(w) \). Hence,

\[
x_i^* = \frac{v \left( 1 - w^F(w^*) \right) + w^F(w)}{2 - w^F(w^*)} \in \arg \max_{w \in [0,1]} (v - w)e(w)
\]

Note that the employer prefers to offer \( w^* = \frac{v(1 - w^F(w^*)) + w^F(w)}{2 - w^F(w^*)} \), where \( w^* > w^F(w^*) \) since \( v > 1 \) and \( w^F(w^*) < 1 \), and induce a positive effort level from the worker, rather than offering any wage level \( \tilde{w} < w^F(\tilde{w}) \) which induces no effort; see \( e(w) \). Indeed, the employer’s equilibrium utility level from offering \( w^* \) is \( V = (v - w^*) \frac{1}{2} \alpha_i \left( w^* - w^F(w^*) \right) \), which is positive for any parameter values. Instead, if the employer makes any offer \( \hat{w} < w^F(\hat{w}) \), the worker exerts no effort, and \( V = 0 \). Hence, \( w^* \) is indeed the equilibrium wage offer. Finally, in order to check for the worker’s voluntary participation, we need to find what is the minimum offer to be accepted by the worker. That is, we must find a wage offer \( w \) such that \( U^C(w, e) = w - e(w)^2 + \alpha_i(w - w^F(w))e(w) = 0 \).

\[
x_i - \left( \max \left\{ \frac{1}{2} \alpha_i \left( w - w^F(w) \right) , 0 \right\} \right)^2 + \alpha_i(w - w^F) \max \left\{ \frac{1}{2} \alpha_i \left( w - w^F(w) \right) , 0 \right\} = 0
\]

In the case in which the foregone option \( w^F(w) > w \), then the above expression is reduced to \( w = 0 \). That is, any wage offer is accepted. On the other hand, in the case in which \( w^F(w) < w \), then, we can reduce the above expression to \( w = \frac{-2 + \alpha_i^2 w^F(w) + 2\sqrt{1 - \alpha_i^2 w^F(w)}}{\alpha_i^2} \), which is always negative, for any values of \( \alpha_i \) and \( w^F(w) \). Therefore, the minimum offer to be accepted by the worker in both cases \( (w^F(w) > w \) and \( w^F(w) < w \) will be \( \tilde{w} = 0 \), since we are assuming that the firm cannot make any negative offers. Note that in the case that \( w^F(w) = 0 \) then \( w^* \) becomes \( w^* = \frac{v + w^F(w)}{2} \).

### 7.7 Proof of Lemma 3

The responder’s utility maximization problem is just given by

\[
\max_{z_i, G} U^C_i(s_i, s_j) = \max_{z_i, G} z_i + \left[ mG \left[ 1 + \alpha_i (s_j - s^F_j) \right] \right]^{0.5}
\]
subject to $z_i + s_i = w_i$

$s_i + s_j = G$

$s_i, z_i \geq 0$

Differentiating with respect to $s_i$, and manipulating, we find the best response function for the second mover concerned about the first mover’s foregone options.

$$s_i^G(s_j) = \begin{cases} 
\frac{m(1-\alpha_is_j^F)}{4} + \left(\frac{\alpha_m-4}{4}\right)s_j & \text{if } s_j \in \left[0, \frac{\alpha_is_j^F m-m}{\alpha_m-4}\right) \\
0 & \text{if } s_j \geq \frac{\alpha_is_j^F m-m}{\alpha_m-4} 
\end{cases}$$

7.8 Proof of Proposition 5

Regarding the first mover (player $i$), we know that he inserts the above best response function of the follower into his utility function, $U_{NC} (s_j, s_i) = w - s_j + \left[m (s_i + s_j)\right]^{0.5}$ which is maximized at $s_j^* = \frac{16(\alpha_is_j^F - 1) + \alpha^2 m^2}{16\alpha_i}$. However, this expression is positive only for sufficiently high values of $\alpha_i$. In particular, $\frac{16(\alpha_is_j^F - 1) + \alpha^2 m^2}{16\alpha_i} > 0$ if and only if $\alpha_i > \frac{4(\sqrt{4\alpha^2_i + m^2 - 2s_j^F})}{m^2} = \bar{\alpha}_i$.

7.9 Proof of Lemma 4

Let $(s_j, 1-s_j)$ denote the proposed allocation that the proposer offers to the responder. We know that the responder will accept any offer $s_j$ if and only if $s_j + \alpha_i(s_j - s_j^F) \geq 0$, that is, $s_j \geq \frac{\alpha_i}{1+\alpha_i}s_j^F = \bar{s}_j$. Let us now check for sufficiency. Note that the responder does not to accept any offer $s_j < \bar{s}_j$. Instead, accepting any offer $s_j < \bar{s}_j$ would imply negative utility levels, and the responder would be better off by rejecting such an offer, obtaining zero utility. Thus, $s_j < \bar{s}_j$ cannot be an equilibrium strategy. Finally we need to check that the responder does not reject any offer above $\bar{s}_j$. Let us assume that the responder sets an acceptable threshold $\tilde{s}_j > \bar{s}_j$. Then, any offer $s_j$ such that $\bar{s}_j < s_j < \tilde{s}_j$ would be rejected, and the responder would find that accepting it constitutes a profitable deviation. Therefore, the acceptance threshold cannot be strictly above $\bar{s}_j$. Hence, the responder does not accept any offer $s_j \in [0, \bar{s}_j)$, but accepts any offer weakly above this threshold level $\bar{s}_j$.

7.10 Proof of Proposition 6

From lemma 6 we know the responder’s acceptance threshold. Since the proposer wants to maximize the remaining portion of the pie which is not offered to the receiver –and guarantees that the receiver accepts such division– he offers $\frac{\alpha_i}{1+\alpha_i}s_j^F$. This is preferred by the proposer rather than not participating when his remaining share of the pie $1 - \frac{\alpha_i}{1+\alpha_i}s_j^F > 0$. That is, the proposer makes the minimal offer $\frac{\alpha_i}{1+\alpha_i}s_j^F$ if and only if $s_j^F < \frac{1+\alpha_i}{\alpha_i}$. Since $s_j^F \in [0, 1]$ and $1 < \frac{1+\alpha_i}{\alpha_i}$ for any $\alpha_i \geq 0$, then the previous condition $s_j^F < \frac{1+\alpha_i}{\alpha_i}$ is satisfied for any $\alpha_i \geq 0$. Therefore, the proposer makes the minimal offer $\frac{\alpha_i}{1+\alpha_i}s_j^F$ for any parameter values.
References


