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Abstract

This paper characterizes a set of Nash equilibria in a first-price sealed-bid auction with the right of first refusal using two bidders and asymmetric information regarding the bidders' value distributions. The equilibria for multiple bidders and a more general value distribution are also presented.

Keywords: repeated auction; right of first refusal; asymmetric information

JEL classification: C72; D44; D82

1. Introduction

A repeated auction is an infinite series of auctions used to award the same, indivisible object in each period. Some repeated auctions include the right of first refusal, where the incumbent has the opportunity to continue to hold the right to the awarded object by matching the best bid of any entering bidder in each period. This paper characterizes a set of Nash equilibria in a first-price repeated auction with the right of first refusal and asymmetric information.

Repeated auctions have been studied since Demsetz [4] called for the use of franchise bidding to award monopoly contracts. It has been found that the treatment of and assumptions concerning the incumbent affects the outcome of the repeated auction. The importance of the incumbent is seen in Osmundsen [11] when repeated auctions are applied to nonrenewable resource extraction franchises, and in Laffont and Tirole [8] as incumbent's make investment choices in the first period. Here, the specifics of the incumbent position affect the outcomes, and may characterize actual repeated auctions.

Often the bidders in repeated auctions are assumed to be the same in each period making collusion among bidders an important aspect of the repeated auction process. Such collusion is the focus of the work discussed in Phillips, Menkhaus, and Coatney [12], Fabra [5], Aoyagi [1], and Skrzypacz and Hopenhayn [13]. However, the characteristics of the object or length of the periods may make the assumption of a different pool of entrant bidders in each period more realistic. In this work, the entrant bidders are not the same in each period.

The right of first refusal exists in many different types of contracts that can be thought of as repeated auctions. Undeveloped land, residential property, and commercial property contracts all may include the right of first refusal to provide current tenants the option to the property in the next period. Securities sales by owners often allow other owners the right of first refusal in order to control ownership from period to period. Employment contracts, especially those of athletes and entertainers may empower the current employer with the right of first refusal in future periods as encouragement to support those unproven talents early in their careers. Similarly, various types of investment settings provide investors the right of first refusal on developed innovation as an incentive to invest in research. Often, procurement contracts, such as those for municipal garbage collection or cable television service, include the right of first refusal. Also, many of the concession contracts in US National Parks also include the right of first refusal to provide consistent service.

This article is the first to model the equilibria in a first price repeated auction with the right of first refusal. Others have considered the theory of the right of first refusal. Walker [15] examines the legal aspects of the right of first refusal. He provides an excellent discussion of the purpose and uses of the right, including insuring against bargaining breakdown and inhibiting exit from a market, but also warns of the potential to limit competitive bidding. In the first economic analysis of the right of first refusal, Kahan [7] models the value of the right in negotiations. He logically presents that the value of the right will depend on the relative valuations of the good. However, Kahan does not consider the right of first refusal in an auction context. Additional theory and an experimental examination of the right of first refusal is presented in Grosskopf and Roth [6]. They conclude that the specific characteristics of the right can work to advantage or disadvantage the right holder. Chouinard [3] models one-period, first-price auctions for National Park concession contracts with and without the right of first refusal. She concludes the elimination of the right increases the service provided by concessioners. Finally, Bikhchandani, Lippman and Ryan [2] discuss the impacts of the right of first refusal on the seller and potential buyers. They rigorously model the right in a second price auction, and conclude that the right is inefficient (the bidder with the highest value does not necessarily win), and the seller will forego surplus.

Using techniques similar to Bikhchandani, Lippman and Ryan, equilibria of first-price repeated auctions with the right of first refusal are modelled. Here, initially, two bidders with uniform value functions are considered. The model is first extended to investigate multiple bidders and then second, a more general value function. Comparisons with auctions without the right of first refusal are given and the efficiency of the auctions are considered.

2. Models and results

There are 2 risk neutral players participating in infinitely repeated auctions for the same object, such as a monopoly right. At the beginning of each period the seller puts the monopoly right for one period up for bid. In the first auction, typical bidder i's valuation, v_i, is independently drawn from the uniform distribution. The values are normalized to lie on the support [0,1]. The valuation distribution is common knowledge. In subsequent auctions, the entrant's value is independently drawn from the unit uniform distribution, and the incumbent's value is common knowledge. The revelation of the incumbent's value is similar to the work of Vickrey [14], where it is assumed one bidder knows the valuation of the other. Landsberger, Rubinstein, Wolfstetter and Zamir [9] evaluate first price auctions given the ranking of bidder evaluations is known. They suggest this knowledge may come from previous auction experience, access to other's financial resources or other idiosyncratic features of bidders. In the types of auctions considered here, it is reasonable that entrant bidders may be able to observe the incumbent, as one period of the repeated auction may last several years. This may allow entrants to discover the incumbent's value of the right. The entrant may also have access to the financial records of an incumbent with a monopoly right awarded by a government agency.

The largest bid submitted wins the monopoly right. If two bidders submit identical bids, a winner will be selected randomly. The winner transfers her bid amount to the seller, and holds the object in that period. In each auction after the initial auction, the entrant submits a bid first. The incumbent learns the entrant's bid, and then submits a bid. A bidder may decide to not submit a bid, but no bid will be accepted if it is smaller than the previous winning bid. This is a rule in several repeated auctions, especially in procurement auctions or those auctions where the winner provides a service. The incumbent wins the auction if the incumbent's bid is some arbitrary small amount, ϵ, more than the entrant's bid. Otherwise, the entrant wins, given that both bids are greater than or equal to the previous winning bid. If only one acceptable bid is submitted, that bid will win the auction. Once a bidder does not win an auction, that bidder will not bid in subsequent auctions. No collusion exists amount bidders.

2.1 Characterization of bidding strategies

In this repeated first-price auction, the initial auction represents a symmetric, private value auction. There are initially two bidders, bidder 1 and bidder 2, that submit bids, b_1 and b_2. In the initial auction, the bidders are indistinguishable, which results in a symmetric game. Although bidders in this auction take the future auctions into account when determining the initial bid, the uninformed incumbent is not able to learn about the entrants' values through time. Thus, the auction can be viewed as a static game of incomplete information.

To formulate this repeated auction as a static game of incomplete information, action spaces, type spaces, beliefs, and payoff functions must be defined. A bidder's action is to submit a bid, b_i. A bidder can submit any positive bid. Thus, the action space, A_i, is $A_i = [0, \infty]$. A bidder's type, T_i, is determined by the bidder's valuation, v_i, which must lie between zero and one, thus the type space is $T_i = [0, 1]$. The bidder's belief about the other bidder is that the other's value, or type, is uniformly distributed on $[0, 1]$. The payoff function, P_i, of a bidder depends on the bids and values of both bidders. The payoff function is

$$P_i(b_1, b_2; v_1, v_2) = \begin{cases} v_i - b_i + \delta u^*(b_I) & if \;\; b_i > b_j \\ 0 & otherwise. \end{cases} \tag{1}$$

where i $\neq$ j. Here, $u^*(b_I)$ is the expected value of being the incumbent in future auctions, and δ is the discount factor.

The strategy for a bidder is a bid function, $b_i(v_i)$, which specifies the bid when that bidder is any type, v_i. A Bayesian Nash equilibrium strategy for a bidder will maximize the expected payoff of the game for that bidder. Bidder 1's expected payoff of winning the initial auction is the payoff in the first period, $v_1 - b_1$, plus the discounted expected value of being the incumbent in future auctions, $\delta u^*(b_I)$, all multiplied by the probability that bidder 1 wins the initial auction. The equilibrium strategy for bidder 1 will therefore solve the following,

$$\max_{b_1} \quad [v_1 - b_1 + \delta u^*(b_I)]Pr(b_2 < b_1). \tag{2}$$

In order to solve this maximization, the discounted expected value of being the incumbent must first be determined. To begin, the probability of winning any auction as the incumbent is considered. Once a bidder has been awarded the contract, this bidder becomes the incumbent, and submits a bid of b_I in each subsequent auction.

Since the incumbent holds the right of first refusal, the only way for an entrant to win any subsequent auction is to submit a bid, b_E, that the incumbent will not choose to match. Only an entrant with a higher value than the incumbent is willing to submit a winning bid. Thus, the probability of winning any auction as the incumbent is the probability of having a higher value than the entrant, $Pr(v_I < v_E)$. Since v_E is uniformly distributed on the support [0,1], the probability of winning as an incumbent equals v_I.

Now, to solve for the expected value of being the incumbent, consider what the incumbent will receive if she serves as the incumbent for one period and then loses in the next period, and the probability of this outcome. This is then considered for every win-loss scenario. When the total expected value of being the incumbent is viewed from the initial period, prior to the bidder bidding as the incumbent, then each period has to be discounted. This discounted expected value of being the incumbent can be written as

$$\delta u^*(b_I) = (v_I - b_I)(1 - v_I) \sum_{T=1}^{\infty} [v_I^T (\sum_{i=1}^{T} \delta^i)]. \quad (3)$$

Solving for the infinite summations, and substituting the discounted expected value of being the incumbent into the initial auction optimization, bidder 1 solves[1]

$$\max_{b_1} \quad [(v_1 - b_1) + (v_1 - b_1)(\frac{v_1 \delta}{1 - v_1 \delta})] Pr(b_2 < b_1). \quad (4)$$

To solve for bidder 1's initial bid, the probability that bidder 1 wins the initial auction must be found. Assume that bidder 2 bids using the optimal bidding strategy, $b_2^*(v_2)$. Thus, in general, the probability of bidder 1 winning the initial auction by

[1]The bid of bidder 1 is equated to the bid of the incumbent. This is because the incumbent will not change the bid from the initial bid. The rules of the auction prevent a lower bid, and nothing will be gained by increasing the bid.

bidding b_1 when bidder 2 plays the strategy $b_2^*(v_2)$ is

$$\begin{aligned} \rho(b_1) &= Pr(b_2^*(v_2) < b_1) \\ &= Pr(v_2 < \sigma(b_1)) \\ &= F(\sigma(b_1)). \end{aligned} \tag{5}$$

Here, σ is the inverse function of b_2^*. By substituting for the probability of winning in the initial auction, the first order condition associated with bidder 1's maximization is

$$f(\sigma(b_1))\sigma'(b_1)(\frac{v_1 - b_1}{1 - v_1\delta}) - F(\sigma(b_1))\frac{1}{1 - v_1\delta} = 0. \tag{6}$$

Using the information that v_1 is uniformly distributed and equal to $\sigma(b_1)$, the first order condition simplifies to the following differential equation,

$$\sigma'(b_1)(\frac{\sigma(b_1) - b_1}{1 - \sigma(b_1)\delta}) = \sigma(b_1)(\frac{1}{1 - \sigma(b_1)\delta}). \tag{7}$$

Solving the differential equation yields $\sigma(b_1) = 2b_1$ and $\sigma'(b_1) = 2$. Substituting these back into the first order condition, an expression for b_1 results. The optimal bidding strategy for bidder 1, is

$$b_1 = \frac{v_1}{2}. \tag{8}$$

In the initial auction, the two bidders are symmetric so, bidder 2 will bid, $b_2 = \frac{v_2}{2}$. Thus, the bidder with the highest value will win the auction in the first period. The bidding strategies for the first period auction represent a Bayesian Nash equilibrium. Substituting $\frac{v_2}{2}$ for $b^*(v_2)$ in the maximization for bidder 1, yields an optimization of $Max\frac{v_1 - b_1}{1 - v_1 * \delta} * 2b_1$. Solving this gives $b_1^* = \frac{v_1}{2}$. Thus, the bids describe the best response to the other's equilibrium bidding strategy. Therefore, neither bidder would like to change their bid given the action of the other bidder.

Now, consider what an entrant will bid in auctions after the initial auction. As explained above, the entrant will bid only if she knows she has a larger value for the monopoly than the incumbent. Otherwise, because the incumbent may exercise the right of first refusal, the incumbent will match the bid of the entrant and always win the auction. Thus, when the entrant bids, the incumbent must choose to not match the bid. The entrant must bid the value of the monopoly right of the incumbent, v_I. The entrant's dominant strategy will be

$$b_E = \begin{cases} v_I & if \quad v_I \leq v_E \\ 0 & otherwise. \end{cases} \tag{9}$$

The incumbent will not match this bid plus the small amount ϵ, because doing so would lead to a negative payoff for the incumbent. This positive bid is just large enough to win the award, so it maximizes the entrant's expected payoff. This bidding strategy will continue for all future periods. The entrant does not bid more due to the right of first refusal offered to her as the incumbent.

The incumbent must submit a bid at least as high as the previous winning bid, b_{-t}, or else the seller will not accept the bid. Thus, the incumbent will continue to maximize her expected payoff by bidding its previous bid. However, if the entrant submits a positive bid which is less than or equal to ϵ less than v_I, the incumbent will bid ϵ more than the entrant's bid. A rational entrant will never submit such a bid, but it is included here for completeness. The incumbent's complete bidding strategy will be

$$b_I = \begin{cases} b_E + \epsilon & if \;\; 0 < b_E \leq v_I - \epsilon, and \;\; b_E + \epsilon > b_{-t} \\ b_{-t} & if \;\; b_E = 0, or \;\; b_E < b_{-t} \\ 0 & otherwise. \end{cases} \tag{10}$$

The incumbent's bidding strategy continues to represent a Bayesian Nash equilibrium. The bid maximizes the expected payoff of the incumbent given the equilibrium strategy of the entrant. The rules of the auction will not allow the incumbent to lower the bid, and a higher bid would decrease the expected payoff to the incumbent due to the right of first refusal. The entrant's bidding strategy involves no uncertainty. This strategy strictly dominates any other possible strategy. Thus, the entrant would never want to bid differently given what the incumbent bids. Thus, all the optimal bids for this auction represent a Nash equilibrium. This repeated auction is efficient, in that the bidder with the highest value always win. The initial period results do not differ from the one period first price auction as seen in Milgrom [10]. This is a bit surprising as the value of holding the right of first refusal is included. However, the asymmetric information of the values and the right of first refusal lead to this result.

2.2 N Bidders

Permitting any positive number of bidders in each auction makes the application of the model to most auctions more flexible and appropriate. In the following model, the rules of the auction remain unchanged from the previous auction. However, now a positive number of bidders greater than two, N, exist in each period.

To solve for the optimal bid when there are N bidders in each period, bidder 1 will maximize the expected payoff of winning the initial auction, as in the auction with only two bidders each period. The equilibrium bidding strategy for bidder 1 will solve

$$\max_{b_1} \quad [v_1 - b_1 + \delta u^*(b_I)]Pr(b_{-1} < b_1). \tag{11}$$

Here, b_{-1} represents all the bids not submitted by bidder 1. Thus, there are $N-1$ bids in b_{-1}.

Using the logic and definitions from the 2 bidder auction, and substituting for the discounted expected value of being the incumbent with N bidders and the probability of winning in the initial auction, bidder 1's maximization problem becomes

$$\max_{b_1} \quad (v_1 - b_1)(\frac{1 - v_1^{N-1} - \delta v_1^{2N-2}}{1 - v_1^{N-1} - \delta v_1^{N-1}})F(\sigma(b_1))^{N-1}. \tag{12}$$

Using the information that v_1 is uniformly distributed, and equal to $\sigma(b_1)$, the above first order condition simplifies to the following differential equation,

$$(N-1)\sigma'(b_1)(\sigma(b_1) - b_1) = \sigma(b_1). \tag{13}$$

Solving the differential equation yields $\sigma(b_1) = \frac{b_1 N}{N-1}$ and $\sigma'(b_1) = \frac{N}{N-1}$. Substituting these solutions into the first order condition, an expression for b_1 results. The optimal bidding strategy for bidder 1, is

$$b_1 = v_1(\frac{N-1}{N}). \tag{14}$$

In the initial auction, all bidders are symmetric. Each bidder will follow the same strategy. Thus, the bidder with the highest value will win the auction in the first period. These bidding strategies for the first period auction represent a Bayesian Nash equilibrium. The bids describe the best response to the other's equilibrium bidding strategy.

The incumbent will continue to bid the amount from the previous period, b_{-t}, if any positive bid is submitted by the incumbent. The incumbent will examine the largest of the entrants' bids, $max(b_E)$, to determine if a bid should be submitted.

$$b_I = \begin{cases} max(b_E) + \epsilon & if \;\; 0 < max(b_E) \leq v_I - \epsilon, and \;\; max(b_E) + \epsilon > b_{-t} \\ b_{-t} & if \;\; max(b_E) = 0, or \;\; max(b_E) < b_{-t} \\ 0 & otherwise. \end{cases} \tag{15}$$

This bid continues to represent a Bayesian Nash equilibrium for the incumbent. The bid maximizes the expected payoff of the incumbent given the equilibrium strategy of entrants. The rules of the auction will not allow the incumbent to lower the bid, and a higher bid would decrease the expected payoff to the incumbent.

Now, consider what entrants will bid in auctions after the initial auction. Entrants will bid only if they have a higher value than the incumbent. Otherwise, because the incumbent may exercise the right of first refusal, the incumbent will match the best bid of the entrants and always win the auction. Thus, when the entrants bid, the incumbent must choose to not match the best bid. In this auction with N bidders, an entrant must consider not only the incumbent but also the other $N-2$ entrants.

An entrant will maximize the expected payoff of winning the auction,

$$\max_{b_E} \quad [v_E - b_E + \delta u^*(b_{I_E})] Pr(b_{-E} < b_E). \tag{16}$$

This maximization is the payoff in the first period, $v_E - b_E$, plus the discounted expected value of being the incumbent in future auctions, $\delta u^*(b_{I_E})$ all multiplied by the probability that the entrant wins the auction, $Pr(b_{-E} < b_E)$. Here, b_{-E} represents all the bids not submitted by the entrant being considered. Thus, there are $N-2$ bids in b_{-E}. Also, I_E denotes an incumbent who previously bid as an entrant, not the initial incumbent.

The expression for the expected value of being the incumbent is the same for an entrant bidder as it is for an initial bidder. If the entrant wins the auction, she will then bid against $N-1$ entrants in subsequent auctions. The only difference in the maximization of an entrant and an initial bidder enters through the probability of winning in the first auction in which they bid. An initial bidder faces $N-1$ other bidders. However, the entrant will only bid if she knows she has a larger value than the incumbent. Therefore, an entrant that submits a positive bid will only have to have a bid larger than $N-2$ other bidders. Using the previously explained process of assuming all other entrants bid optimally, the probability an entrant wins in the first auction in which it bids equals $F(\sigma(b_E))^{N-2}$.

Thus, an entrant that submits a positive bid will solve

$$\max_{b_E} \quad (v_E - b_E)\left(\frac{1 - v_E^{N-1} - \delta v_E^{2N-2}}{1 - v_E^{N-1} - \delta v_E^{N-1}}\right) F(\sigma(b_E))^{N-2}. \tag{17}$$

Using the information that v_E is uniformly distributed, and equal to $\sigma(b_E)$, the first order condition simplifies to the following differential equation,

$$(N-2)\sigma'(b_E)(\sigma(b_E) - b_E) = \sigma(b_E). \tag{18}$$

Solving the differential equation yields $\sigma(b_E) = \frac{(N-1)b_E}{N-2}$ and $\sigma'(b_E) = \frac{N-1}{N-2}$. Substituting these expressions into the first order condition, an expression for b_E results. The optimal bidding strategy for an entrant is

$$b_E = \begin{cases} \frac{v_E(N-2)}{N-1} & if \;\; \frac{v_E(N-2)}{N-1} \geq v_I, \;\; and \;\; v_E \geq v_I \\ v_I & if \;\; \frac{v_E(N-2)}{N-1} < v_I, \;\; and \;\; v_E \geq v_I \\ 0 & if \;\; v_E < v_I \end{cases} \tag{19}$$

2.3 Generalized Value Distribution

In this subsection, a flexible distribution that may represent the uniform or a skewed distribution of value is investigated. The probability density function of value is given as $f = ax^{a-1}$ and the associated cumulative distribution is $F = x^a$. Here, the parameter a may take on any positive value. When a equals one, then the uniform distribution of value results.

As before, a Bayesian Nash equilibrium strategy for a bidder will maximize the expected payoff of the auction for that bidder. The equilibrium strategy for bidder 1 will solve

$$\max_{b_1} \quad [v_1 - b_1 + \delta u^*(b_I)]Pr(b_2 < b_1). \tag{20}$$

Substituting the discounted expected value of being the incumbent into the initial auction, bidder 1's solves

$$\max_{b_1} \quad [(v_1 - b_1) + (v_1 - b_1)(\frac{v_1^a \delta}{1 - v_1^a \delta})]Pr(b_2 < b_1). \tag{21}$$

Simplifying this equation leads to an optimization problem of

$$\max_{b_1} \quad (\frac{v_1 - b_1}{1 - v_1^a \delta})Pr(b_2 < b_1). \tag{22}$$

By substituting for the probability of winning in the initial auction, bidder 1's maximization problem becomes

$$\max_{b_1} \quad (\frac{v_1 - b_1}{1 - v_1^a \delta})F(\sigma(b_1)). \tag{23}$$

Using the information that $f(\sigma(b_1)) = a(\sigma(b_1))^{a-1}$, and v_1 equals $\sigma(b_1)$, the above first order condition simplifies to the following differential equation,

$$a\sigma'(b_1)\left(\frac{\sigma(b_1)-b_1}{1-(\sigma(b_1))^a\delta}\right)=\sigma(b_1)\left(\frac{1}{1-(\sigma(b_1))^a\delta}\right). \tag{24}$$

Solving the differential equation yields $\sigma(b_1)=b_1+\frac{b_1}{a}$ and $\sigma'(b_1)=1+\frac{1}{a}$. Substituting these solutions into the first order condition, an expression for b_1 results. The optimal bidding strategy for bidder 1, is

$$b_1=\frac{v_1 a}{a+1}. \tag{25}$$

In the initial auction, the two bidders are symmetric. Bidder 2 will follow the same strategy. Thus, the bidder with the highest value will win the auction in the first period. These bidding strategies for the first period auction represent a Bayesian Nash equilibrium. The bids describe the best response to the other's equilibrium bidding strategy. The incumbent will continue to bid the amount of the previous period if a positive bid is submitted,

$$b_I=\begin{cases} b_E+\epsilon & if \;\; 0<b_E\leq v_I-\epsilon, and \;\; b_E+\epsilon>b_{-t} \\ b_{-t} & if \;\; b_E=0, or \;\; b_E<b_{-t} \\ 0 & otherwise. \end{cases} \tag{26}$$

This bid continues to represent a Bayesian Nash equilibrium for the incumbent.

Now, consider what an entrant will bid in auctions after the initial auction. As explained above, the entrant will bid only if she knows she has a higher value for the the monopoly than the incumbent. Otherwise, because the incumbent may exercise the right of first refusal, the incumbent will match the bid of the entrant and always win the auction. Thus, when the entrant bids, the incumbent must choose to not match the bid. The entrant's dominant strategy will be

$$b_E = \begin{cases} v_I & if \;\; v_E \leq v_I \\ 0 & otherwise \end{cases} \tag{27}$$

The incumbent will not match this bid plus the small amount ϵ, because doing so would lead to a negative payoff. Therefore, this positive bid is just large enough to win the award, so it maximizes the entrant's expected payoff. This bidding strategy will continue for all future periods.

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