Environmental Policy Design Under Environmentally Concerned and Future Oriented Consumers\textsuperscript{i}

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Abstract

We examine how future considerations and environmental concerns affect environmental policy design when consumers face a green product and a brown alternative with long-lasting polluting impacts. Our results show that a subsidy for the green firm yields a strictly higher social welfare than a context without regulation. We also show that social welfare improves when the regulator takes consumers’ future considerations into account compared to the case in which future considerations are ignored. In addition, our findings suggest that improving perceived environmental quality of the green good can increase social welfare if a high perceived quality is already established, the abatement cost is low, and the environmental damage is high. Finally, we discuss how change in consumers’ patience impacts the optimal level of subsidy and social welfare.

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1. Introduction

Human activities have had some major irreparable impacts on the environment; e.g. plastic waste accumulation in the marine environment (Li et al. 2016; Moore 2008), Amazon rainforest deforestation (Foley et al. 2007), ocean ecosystem alteration due to land-based and ocean-based activities (Halpern et al. 2008). As a result of such ongoing challenges to the environment and consequently hazards for humans, consumers have become more conscious about the immediate impacts of these actions as well as the potential longer-term impacts on the environment.

Besides consumers’ environmental concerns, the extent to which consumers consider the future consequences of their behavior also impacts their present-day decisions (Milfont and Demarque 2015; Milfont et al. 2012), including decisions impacting the environment. Strathman et al. (1994) empirically show that future considerations are strongly and positively correlated with pro-environmental behaviors. Their work provides strong support for significant predictive ability of future considerations above other personality measures, showing there is significant variance in humans’ environmental behavior best explained by future considerations.

Subsequent research provides further support about future orientations being significant predictors of pro-environmental behaviors (Corral-Verdugo and Pinheiro 2006; Bruderer Enzler 2015). This

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1 Ocean ecosystem alteration results in sea population at the head of the food chain carrying long-lasting and large amounts of dangerous chemicals making them unsuitable for humans to eat (Elmgren 2001).

2 Compared with recent years, more Americans are worried about protecting the environment rather than economic growth and energy production, with a peak in the past two years. For more details see http://news.gallup.com/opinion/polling-matters/207608/public-opinion-context-trump-environmental-actions.aspx?g_source=CATEGORY_ENVIRONMENT_AND_ENERGY&g_medium=topic&g_campaign= tiles and http://news.gallup.com/poll/206159/americans-tilt-toward-protecting-environment-alternative-fuels.aspx?g_source=CATEGORY_ENVIRONMENT_AND_ENERGY&g_medium=topic&g_campaign= tiles.

3 Furthermore, according to Stolarski et al. (2015), individuals emphasize the present and the future the most but are least interested in the past.

4 In addition, Stolarski et al. (2015) demonstrate similarities between time perspective and traits in the Big Five model further supporting temporal orientation is a dispositional human trait.
leads us to propose that the design of environmental policy not only needs to consider consumers’ environmental concerns, but also consumers’ interpersonal differences in future orientations. As a consequence, consumers in our model face a two-dimensional preference space, based on their environmental concerns and future considerations. Environmental regulation is represented by an abatement subsidy. Several papers study how a tax or subsidy policy may impact consumers and producers’ behavior when there are negative externalities (see Spraggon and Oxoby 2009; Ulph 1996; Stavins 1996; Xepapadeas 1991; Oates et al. 1989). However, accounting for the existence of environmentally friendly consumers is a relatively modern trend.

We are interested in answering the following question: does a two-dimensional preference space affect the setting of an environmental policy? We study a duopoly market producing a good with different environmental impacts. That is, each firm produces a good that is only different in its environmental quality. Perceived environmental quality is defined as the environmental quality of the green firm’s product recognized by consumers. The long-lasting environmental impacts take place during either the consumption or production process, and the two products are the same in any other attribute.5

Bansal and Gangopadhyay (2003) use a vertical differentiation model to investigate welfare implications of discriminatory and uniform tax-subsidy policies with environmentally aware consumers. Their results show that, while both a uniform and discriminatory subsidy policy improve environmental quality, a uniform or discriminatory tax policy on the polluting firm may do the opposite. Espinola-Arredondo and Zhao (2012) examine a horizontal differentiation setting in which they analyze the impact of a tax and subsidy on welfare. Their model considers two types

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5 Examples include foam disposable containers vs biodegradable disposable containers, recyclable packaging materials vs non-recyclable packaging materials, organic vs chemical pest control products, and organic vs nonorganic laundry detergents.
of consumers: green and neutral consumers. They also show that both a tax and a subsidy policy improve social welfare. However, in these papers consumers do not have any time considerations which is proven by empirical work to play an important role in environmental decision making.

Joireman et al. (2001) integrate social value orientation and future orientation in an empirical framework. They study the pro-environmental behavior when it has short-run negative consequences for the self (spending resources to protect the environment) and long-run positive consequences for the self and others (clean environment). Their results show that individuals with higher future considerations expressed stronger pro-environmental intentions and behaviors. This emphasizes the importance of incorporating temporal orientation in the study of environmental policies. Milfont and Gouveia (2006) study the combined empirical relations of time perspective and values to environmental attitudes. Their work shows that future orientation is a predictor of the environmental preservation. These results further support the distinction between the social and temporal orientation in environmental decision making. Our model accounts for both to achieve more accuracy in the design of an environmental policy.

In spite of such empirical evidences, there is no theoretical work, to the best of our knowledge, integrating environmental and future orientations. In this paper, we provide a theoretical framework to analyze welfare implications of an environmental policy in a complete information setting. We allow for heterogeneous time preferences and a continuum of environmental concerns among consumers.

We show that in a context in which consumers exhibit environmental and future considerations, a subsidy to the green firm generates a higher social welfare than when the regulation is absent. Moreover, including future considerations when setting an environmental policy always yields a higher welfare compared to the situation in which future considerations are ignored. In addition, a
more severe environmental damage and a higher efficiency of abatement technologies call for a higher subsidy.

Our results also suggest that impacts of a change in patience depends on the initial and final location of the consumer becoming more patient. There is no change in the optimal subsidy and social welfare when consumers become marginally more patient (i.e., they are still impatient). However, when already patient consumers become more patient, the optimal subsidy and social welfare improve if the perceived environmental quality differential is high, the cleaning cost is small, and the brown product imposes a high marginal damage on the environment. If the relatively impatient consumers become patient enough, the social planner offers higher subsidy and the optimal social welfare improves for all parameter values. A similar result is obtained when there is a change in the perceived environmental quality of the green good. When the perceived environmental quality improves, social welfare also improves if the perceived environmental quality is already high, abatement costs are low, and environmental damage is high.

The structure of the paper is as follows. Section 2 describes the model and its equilibrium outcomes, section 3 analyzes social planner’s regulation and its welfare implications, the comparative statics with optimal environmental policy are studied in section 4, and section 5 contains some concluding remarks.

2. Model

Consider a continuum of consumers, where \( \theta \) represents a consumer’s environmental concern and it follows a standard uniform distribution, \( \theta \sim U \ [0,1] \). If \( \theta = 0 \), the consumer is unconcerned about environmental attributes, and if \( \theta = 1 \), she considers environmental attributes extremely

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6 In this setting, a positive change in the perceived environmental quality can be produced by an improvement in technology used by the green firm.
important. Similarly, consumer’s consideration of future consequences $\delta_i$ follows an arbitrary discrete distribution represented by probability (density) of $y_i(\delta_i)$, for all $i \in \{1, \ldots, n\}$, where $\delta_n$ represents the most patient consumers and $\sum_{i=1}^{n} y_i(\delta_i) = 1$.\(^7\) For compactness, we will use $y_i(\delta_i) = y_i$ in subsequent sections of the paper. Without loss of generality, we order values of patience such that $\delta_{i+1} > \delta_i$. At the extreme left-side end of this continuum, individuals assign a small weight to future consequences of their decision, while at the extreme right-side end, individuals highly care about future impacts.\(^8\)

The area below the square in Figure 1 depicts the location of consumers in this model. We can see different combinations of consumers on horizontal and vertical axes.

![Figure 1: consumers with two attributes](image)

For instance, point A represents a consumer with a low consideration of future consequences and highly concerned about environmental attributes. This context could represent an older individual who still purchases green products to protect the planet.\(^9\) On the other hand, point B represents a

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\(^7\) Individuals tend to discount future payoffs ($\delta_i \leq 1$) and they vary in their discount rates (e.g. Chabris et al. 2010; Hardisty and Weber 2009; Green and Myerson 2004; Kirby 1997). In addition, a meta-analysis by Milfont et al. (2012) shows that past-present time perspective has a trivial impact on environmental behaviors. Therefore, we only focus on future time perspective to model consumers’ behavior.

\(^8\) It is important to note that there is no direct indication of environmental engagement when consumers’ time preferences are determined (e.g., the Zimbardo Time Perspective Inventory by Zimbardo and Boyd 1999; Consideration of Future Consequences by Strathman et al. 1994).

\(^9\) Albert and Duffy (2012) and Read and Read (2004) show that older people discount future more.
consumer with the opposite preferences. It depicts the case of an individual who works and saves for her future but does not care about green purchasing.

We consider two firms producing the same good, however, the green firm (Firm 1) does not generate pollution while the brown firm (Firm 2) produces a certain amount of pollution.\(^{10}\)

Consumer \(i\)'s utility is:

\[
    u_i = \begin{cases} 
    v - p_1 + \theta e(1 + \delta_i), & \text{if consumer buys from Firm 1} \\
    v - p_2, & \text{if consumer buys from Firm 2}, 
    \end{cases}
\]

where \(v\) is the utility from consuming the good,\(^{11}\) prices \(p_1\) and \(p_2\) are charged by the green and brown firm, and \(e\) denotes the perceived environmental quality of the green product. We assume this parameter is exogenous and strictly positive since, for a given price, all consumers prefer the clean product.\(^{12}\) Thus, \(\theta e(1 + \delta_i)\) is the extra utility derived from purchasing from the green firm.\(^{13}\) As concerns towards environmental issues increases (higher \(\theta\)), consumers obtain a higher utility from green consumption and are willing to pay higher prices for green products (e.g. Roe et al. 2001; Lin and Huang 2012).

As mentioned earlier, individual differences play a crucial role in this component of extra utility from consuming the environmentally clean product. If the value of \(\theta\) and \(\delta_i\) are relatively low (i.e., the consumer cares about neither the environment, nor future payoffs), or the value of \(\delta_i\) is high, but the value of \(\theta\) is close to zero (i.e., the consumer does not care about the environment but cares

\(^{10}\) We use “green product” and environmentally friendly product interchangeably which both stand against “brown product” which is not environmentally friendly and produces pollution either during production process or consumption process. According to Durif et al. (2010), a green product’s design uses recycling resources or has lower environmental toxic damage throughout its entire life cycle.

\(^{11}\) This is the same for both firms since we assumed the only difference between the goods is their environmental quality. We assume \(v\) is large enough so that all consumers buy from either firm.

\(^{12}\) That is, environmental quality is a vertical attribute (Bansal and Gangopadhyay 2003).

\(^{13}\) Consumers derive a positive utility from buying green and doing good for themselves and for the society as a whole (e.g. see Espinola-Arredondo and Zhao 2012; Lin and Huang 2012; Bansal and Gangopadhyay 2003).
about future payoffs) the third component of the utility of a green buyer is small, meaning that if a consumer does not care about the environment, they do not receive any extra utility from purchasing from Firm 1; as expected. However, for a consumer with high $\theta$ and high $\delta_i$ (i.e., consumer puts a high importance on both the environment and future payoff), this extra utility will be at its highest.

Since the only stipulated difference between the product of Firm 1 and Firm 2 is their environmental impact, we can normalize Firm 2’s production costs to zero and $c \equiv c_1 - c_2$ represents Firm 1’s cost differential, capturing its additional cost from producing the green good.\textsuperscript{14}

Therefore, the profit function for the green firm is:

$$\pi_1 = (p_1 - c) \times D_1,$$

and the profit function for the brown firm is:

$$\pi_2 = p_2 \times D_2,$$

where $D_1 (D_2)$ is the demand for Firm 1 (Firm 2, respectively).

The structure of the game is as follows: (1) the regulator provides an environmental subsidy per unit to Firm 1 (only in the regulated market);\textsuperscript{15} (2) the firms simultaneously and independently choose profit-maximizing prices; and (3) consumers decide which firm to purchase from to maximize their utility given prices and environmental quality. We next examine the demand for Firm 1 and Firm 2 in the first stage of the game.

\textsuperscript{14} This additional cost can be due to using a more expensive cleaning technology, cleaner inputs used in manufacturing process, or abatement efforts (Brécard et al., 2009; Bonini and Oppenheim, 2008).

\textsuperscript{15} In the case of taxing, the government implements an environmental tax per unit of good produced by Firm 2.
2.1. Third stage- market share

The demand for Firm 1’s good is generated by consumers who receive a higher utility from consuming the product of Firm 1 than that of Firm 2. The locus of consumers who are indifferent between buying from Firm 1 and Firm 2 satisfies:

\[ v - p_1 + \theta e(1 + \delta_i) = v - p_2, \]

or equivalently,

\[ \theta = \frac{p_1 - p_2}{e} \times \frac{1}{1 + \delta_i}. \]

We should note that the consumer price of a green (environmentally friendly) product is higher than its brown (polluting) alternative, thus, \( p_1 - p_2 > 0 \). Finally, if the environmental quality of the green product improves, then the demand for Firm 1 increases.

Figure 2 represents the demand for each firm by drawing the line \( \theta = \frac{p_1 - p_2}{e} \times \frac{1}{1 + \delta_i} \) in the \((\theta, \delta_i)\) quadrant. The area above the demand line represents the demand for Firm 1 \((D_1)\), while the area below the line shows the demand for Firm 2 \((D_2)\). At this point, two cases may happen: Case (i) where \( p_1 - p_2 > e \), or \( \frac{p_1 - p_2}{e} > 1 \) (solid line in Figure 2) and Case (ii) where \( p_1 - p_2 \leq e \), or \( \frac{p_1 - p_2}{e} \leq 1 \) (dashed lines in Figure 2). Consider that brown preferences in Case (i) are higher than those in Case (ii) and, as a consequence, green preferences in Case (ii) are higher than green preferences in Case (i).
Let $y \equiv \sum_{\delta_i > \hat{\delta}} y_i$ denote the portion of the population whose patience is greater than $\hat{\delta}$ (they are relatively patient), where cutoff $\hat{\delta}$ is defined as the horizontal intersection of the demand curve for Firm 1 and $\theta = 1$, as depicted in Figure 3. Therefore, $\hat{\delta} = \frac{p_1 - p_2}{e} - 1$. Note that if $y = 1$, all individuals’ patience is greater than $\hat{\delta}$, and Case (ii) becomes a special case of Case (i).\(^{16}\)

### 2.2. Second stage- prices

The following lemma describes the demand for each firm.

\(^{16}\) In addition, the cumulative distribution function of $\delta$ is $CDF(\delta) = 1 - y$. 

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Lemma 1. For a given pair of prices \((p_1, p_2)\) and perceived environmental quality \(e\), Firm 1 and 2’s demands are,

\[
(D_1, D_2) = \left( y - \frac{p_1 - p_2}{e}\sigma, 1 - y + \frac{p_1 - p_2}{e}\sigma \right),
\]

where \(\sigma \equiv \sum_{i: \delta_i \geq \delta_1} \frac{y_i}{1 + \delta_i}\).

When \(e\) increases, the demand for the green firm increases and the demand for the brown firm decreases. That is, improving perceived environmental quality promotes the consumption of the green good. As expected, the demand for each firm is conversely related to its own price and positively related to the rival’s price. The next proposition describes the market equilibrium.

Proposition 1. The equilibrium prices and profit functions for each firm are,

\[
p_1 = \frac{1}{3} \left( 2c + \frac{e(1 + y)}{\sigma} \right), \quad p_2 = \frac{1}{3} \left( c + \frac{e(2 - y)}{\sigma} \right),
\]

\[
\pi_1 = \frac{(e(1 + y) - c\sigma)^2}{9e\sigma}, \quad \pi_2 = \frac{(e(2 - y) + c\sigma)^2}{9e\sigma}.
\]

Inserting equilibrium prices \(p_1\) and \(p_2\) in the condition for Case (i), \(p_1 - p_2 > e\), we obtain \(\frac{c}{e} > 3 - \frac{2y - 1}{\sigma}\). It means that Case (i) arises when Firm 1’s cost disadvantage is relatively high, and the environmental quality is relatively low. That is, a larger portion of consumers would prefer to buy the brown good relative to Case (ii), given the high cleaning cost of the good and low environmental quality of the green product.

Moreover, when \(y = 1\), the condition \(p_1 - p_2 \leq e\) evaluated at the prices obtained in proposition 1 simplifies to \(\frac{c}{e} \leq 3 - \frac{1}{\sigma}\). It means that in order to stay in Case (ii), where a larger portion of consumers prefer to buy green over brown compared to Case (i), the abatement cost needs to be relatively small.
Also, \( \hat{\delta} \) evaluated at equilibrium prices simplifies to \( \frac{1}{3} \left( \frac{c}{e} + \frac{2y-1}{\sigma} \right) - 1 \), and \( y = \sum_{\delta_i > \hat{\delta}} \frac{1}{3} \left( \frac{c}{e} + \frac{2y-1}{\sigma} \right) - 1 \) \( y_i \)

where \( \sigma = \sum_{\delta_i > \hat{\delta}} \frac{y_i}{\sigma + b_i} \). Even though \( y \) depends on \( \hat{\delta} \) and \( \hat{\delta} \) depends on \( y \), in the appendix (see proof of Proposition 1), we show that we can determine the value of \( y \) endogenously.\(^{17}\)

As a benchmark, we first identify the social welfare under no regulation, and describe it in Lemma 2. Social welfare is defined as \( SW_{NR} \equiv CS + PS - Env \), where \( CS \) is the consumer surplus (including consumers purchasing from Firm 1 and 2), \( PS \) is producer surplus (including profits for both firms), and \( Env \) represents environmental damage produced by Firm 2, where \( Env = d \times D_2 \), and \( d > 0 \) is the marginal environmental damage; i.e., the extent of marginal damage that an additional unit of Firm 2's production imposes on the environment.\(^{18}\)

**Lemma 2.** Under no regulation, social welfare is

\[
SW_{NR} = \frac{1}{18e\sigma} \left( 2e\sigma (9v - 3d(2 - y) - c(2 + 5y)) + c\sigma^2 (5c - 6d) \right.
\]

\[
+ e^2 \left( 4(1 - y)y + 9\sigma \left( y + \sum_{\delta_i > \hat{\delta}} y_i \delta_i \right) - 1 \right) \right).
\]

We next examine the case of regulation and provide some comparisons.

\(^{17}\) Moreover, if the distribution for consumers’ patience follows a continuous distribution, we can still find the equilibrium values of \( y \) and \( \sigma \) by finding their limits as \( n \) (as the number of consumers) goes to infinity. In the appendix (see proof of Proposition 1), we demonstrate a few examples of continuous distributions for consumers’ patience.

\(^{18}\) Since only Firm 2 generates negative environmental impact, the environmental damage only depends on the production of Firm 2.
3. First stage- optimal subsidy policy

We now analyze the environmental subsidy policy.\textsuperscript{19} Social welfare with regulation also includes government spending for the subsidy provided to Firm 1 for each unit they produce. Therefore, $SW_R$ becomes, $SW_R = CS + PS - Env - G$, where $G$ is government expenditure when subsidy ($s$) is received by Firm 1.\textsuperscript{20} We operate by backward induction and identify the optimal subsidy.

**Proposition 2.** The optimal level of subsidy $s^*$ for the green firm is,

$$s^* = \frac{e(2y - 1)}{\sigma} + 3d - 2c.$$

As marginal environmental damage increases, $s^*$ increases by moving both the first and second components of the optimal subsidy upward; i.e., it is optimal for the regulator to provide a higher subsidy to the green firm to reduce the increasing negative impacts of pollution. However, as cleaning cost increases, the value of the optimal subsidy decreases by moving both the first and third components of the optimal subsidy downward. This means that when the abatement of pollution for the green good is too costly, it may be unsuitable to promote it from a social welfare point of view, consequently shrinking the subsidy.\textsuperscript{21} In addition, in Case (i), $s^* > 3(e + d - c)$ and in Case (ii) where $y = 1$, $s^* < 3(e + d - c)$. Intuitively, the optimal subsidy is greater when the demand for the green product is relatively low (i.e. in Case (i)) in order to promote more green consumption.

\textsuperscript{19} Our results show that the value of optimal tax on the brown firm is equal to the value of optimal subsidy for the green firm. Also, optimal tax and subsidy are equally welfare-improving. Therefore, we only focus on the case of subsidizing the green firm, (or Firm 1). The proof is available upon request.

\textsuperscript{20} Following Dixit and Kyle (1985), in our model the subsidy for Firm 1 is financed by non-distorting taxes, otherwise, the deadweight loss associated with the costs of raising funds for the subsidy should be subtracted from the social welfare function. In addition, if the government taxes the brown firm, $G$ becomes government revenue and takes a positive value in $SW_R$ function.

\textsuperscript{21} The optimal subsidy without future considerations in Espinola-Arredondo and Zhao (2012) moves in similar directions with changes in $d$ and $c$. 


As a special case, consider that \( e = 0 \), implying that goods produced by Firm 1 and 2 are regarded as homogenous. In this setting, consumers buy the lowest priced good. To maximize social welfare, the regulator provides a subsidy higher than or equal to the cleaning costs when \( c \leq d \) (every unit produced and cleaned by Firm 1 has a cost smaller than the environmental damage caused by every unit produced by Firm 2) and Firm 1 captures all the market. Otherwise, the social planner either does not regulate the market or sets a subsidy less than the cleaning costs and Firm 2 gets all the market share.

4. **Comparative statics under regulation, evaluated at \( s^* \)**

We analyze the comparative statics when the regulator provides the optimal subsidy to each unit produced by Firm 1.

**Corollary 1.** After implementing the optimal subsidy, prices evaluated at \( s^* \), the difference between them, and cutoff \( \hat{\delta} \) simplify to,

\[
\begin{align*}
p_1^* &= 2(c - d) + \frac{e(1 - y)}{\sigma}, \\
p_2^* &= (c - d) + \frac{e(1 - y)}{\sigma}, \\
p_1^* - p_2^* &= c - d, \\
\hat{\delta} &= \frac{c - d}{e} - 1.
\end{align*}
\]

For the case in which \( c \leq d \), Firm 2’s price exceeds that of Firm 1 and since Firm 1 is offering a cleaner product, the demand for Firm 2 becomes negative. Given that the optimal subsidy increases in \( d \) but decreases in \( c \), a large enough marginal environmental damage and small enough abatement costs entail a high subsidy. In this case, the optimal price charged by the green firm becomes smaller than that of the brown firm, and Firm 1 captures all the market. If \( 0 < \frac{c-d}{e} \leq 1 \), Case (ii) is observed in which \( y = 1 \) and \( \hat{\delta} \leq 0 \), and when \( 1 < \frac{c-d}{e} \leq 2 \), Case (i) is observed in
which \( 0 < \delta \leq 1 \). Eventually, when \( \frac{c-d}{e} > 2 \), according to Figure 2, \( \delta > 1 \) and Firm 2 captures all the market demand. In the remaining of the paper, the optimal subsidy focuses on the case in which \( c > d \) so that Firm 2 has positive demand.

With the optimal subsidy in place, if the environmental damage caused by the brown good becomes more intense, prices charged by both firms drop. Due to receiving a higher subsidy, the price of the green product declines faster to encourage more green consumption and \( D_1^* \) increases while \( D_2^* \) decreases. In the equilibrium without regulation, however, prices and demands do not react to changes in the environmental damage. Also, if Firm 1’s cost disadvantage expands, both \( p_1^* \) and \( p_2^* \) increase, but due to receiving a lower subsidy, the increase in \( p_1^* \) is higher than that of \( p_2^* \). Therefore, green consumption drops with an increase in abatement costs and brown consumption increases. In addition, as the environmental quality of the green good improves, green consumption becomes more pervasive, while brown consumption becomes less common.

### 4.1. Welfare comparisons

We next show that a subsidy can improve social welfare and reduce environmental damage.

**Corollary 2.** *The social welfare when the optimal subsidy is provided to the green firm is strictly larger than the social welfare without any regulation.*

Hence, providing a subsidy is beneficial for society in addition to helping to preserve a clean environment. The corollary below shows that taking future considerations into account when designing an environmental policy always yields a higher social welfare compared to the case in which the social planner ignores future considerations but considers environmental concerns.
Corollary 3. Under regulation, the optimal social welfare with future considerations is strictly greater than that arising when the regulator ignores future considerations.

We consider that firms can better observe consumers’ characteristics than the regulator and take consumers’ future considerations into account when setting their prices. In this context, there is a welfare loss when the social planner does not take future considerations into account when designing an environmental policy.

4.2. Impacts of change in perceived environmental quality $e$

Corollary 4. If the perceived environmental quality of the green good increases, both optimal subsidy and social welfare increase when the difference between $c$ and $d$ is relatively small, or $e$ is relatively high.

Incentivizing more green consumption when there is an improvement in the environmental quality is worth it only if $\frac{c-d}{e} < CDF^{-1} \left( \frac{1}{2} \right) + 1$, which means $c - d$ needs to be relatively small, and $e$ needs to be relatively large. In other words, the necessary conditions to offer a higher subsidy for the green firm are that the cost disadvantage of the green good is low, the marginal damage caused by the brown good severe, and the environmental quality already high. It is also worth noting that condition $\frac{c-d}{e} < CDF^{-1} \left( \frac{1}{2} \right) + 1$ is equivalent of $y > \frac{1}{2}$, meaning if at least half of the population is patient enough, optimal subsidy increases with increase in $e$.

Similarly, to achieve a higher social welfare through improving environmental quality, it is important to have an already high quality established. Also, the abatement cost needs to be small and the extent of marginal environmental damage large to make the improved environmental quality worthwhile. On the other hand, when the environmental quality is insignificant with high
cleaning costs and low marginal environmental damage, the improved environmental quality is not worth it.

4.3. Impacts of change in the distribution of patience

To examine the impacts of change in the distribution of patience on the optimal subsidy, we assume consumers with initial patience $b_i$ move to $b_j$, where $j > i$; i.e., $y_i$ portion of consumers become more patient. We consider 3 cases depending on the values of $\delta_i$ and $\delta_j$ relative to $\hat{\delta}$. Note that when patience changes, cutoff $\hat{\delta} \equiv \frac{c-d}{e} - 1$ does not change. Corollary 5 shows how the optimal subsidy and social welfare change when consumers become more patient.

**Corollary 5.** When some of the consumers become more patient, three cases may happen:

**Case 1.** $\delta_i, \delta_j < \hat{\delta}$: $s^*$ and $SW_G(s^*)$ do not change.

**Case 2.** $\delta_i, \delta_j > \hat{\delta}$: $s^*$ increases if and only if $\frac{c-d}{e} < CDF^{-1}\left(\frac{1}{2}\right) + 1$. $SW_G(s^*)$ improves if and only if $\frac{c-d}{e} < \sqrt{\frac{w}{x}}$, where $w \equiv y_i \hat{\delta}_j - y_i \delta_i > 0$, and $x \equiv \frac{y_i}{1+\delta_i} - \frac{y_i}{1+\delta_j} > 0$.

**Case 3.** $\delta_i < \hat{\delta}$, $\delta_j > \hat{\delta}$: $s^*$ and $SW_G(s^*)$ increase for all parameter values.

We next discuss Corollary 5 for each particular case.

**Case 1.** $\delta_i, \delta_j < \hat{\delta}$. In this case, $s^*$ and $SW_G(s^*)$ stay the same. When the impatient consumers become more patient (but they do not exceed $\hat{\delta}$), there is no change in the demand for Firm 1 and 2. Therefore, the equilibrium prices and profits remain the same and the social planner does not alter the optimal subsidy. Consequently, the optimal social welfare does not change.
**Case 2.** $\delta_i, \delta_j > \delta$. According to figure 2, as $y_i$ portion of consumers move to $\delta_j$, $D_1$ increases. But the increase is not large (Compared to Case 3); so, deciding whether or not to encourage more demand for Firm 1 depends on the relative value of $e$ and $c - d$. If the environmental quality of the green good is high, the social planner tends to encourage more green consumption by offering a higher subsidy to the green firm. In addition, if the cleaning cost of the green good is small, or the marginal environmental damage caused by the brown good is high, the optimal subsidy increases to encourage more green consumption. The $\frac{e-d}{e} < CDF^{-1}(1) + 1$ condition simplifies to $y > \frac{1}{2}$; i.e., if at least half of the population is patient enough, optimal subsidy increases in Case 2. The social welfare follows a similar pattern. When already patient consumers of a society become more patient, $SW_G(s^*)$ improves if and only if the green firm produces at high environmental quality, is efficient in its abatement process, and the damage caused by the brown good is relatively severe.

**Case 3.** $\delta_i < \delta, \delta_j > \delta$. In this case, $s^*$ and $SW_G(s^*)$ always increase for all parameter values. As part of the impatient consumers become more patient (and they pass $\delta$), the demand for Firm 1 increases and the amount of increase is substantial (compared to Case 2). In other words, in Case 3 there is a considerable increase in the consumers’ utility from purchasing green and the social planner supports lower prices with higher subsidy, and the optimal social welfare improves for all parameter values.

5. **Conclusions**

The existing literature has analyzed how consumers’ environmental concerns affect the design of environmental policies. In addition, there are several empirical studies showing that future considerations are significant predictors of pro-environmental behaviors. However, this issue has
not been considered in environmental policy design studies. In this paper, we investigate how consumers’ future considerations and environmental concerns impact regulation and social welfare. We analyze a duopoly market producing a homogeneous good with different environmental impacts in a complete information setting.

Our results suggest that when the green firm receives a subsidy, social welfare is higher than that arising in the absence of regulation. In addition, compared to the case in which future considerations are ignored, the regulator improves social welfare when she takes into account consumers’ future considerations in designing a subsidy policy. This has important policy implications, since ignoring future considerations costs the society a welfare loss when designing and implementing environmental policies. Additionally, as the environmental damage caused by the brown firm’s production becomes more severe, the social planner tends to encourage more green consumption and optimal subsidy increases. However, with an increase in abatement costs, green consumption becomes costlier and to discourage green consumption, the social planner reduces the optimal subsidy.

We also find that under a context of high environmental quality with low abatement cost and high environmental damage, improving environmental quality increases social welfare, regardless of individuals’ patience to help preserve the environment and improve social welfare. In addition, when environmental quality is improved, the optimal subsidy provided to the green firm also needs to improve if abatement costs are low, the marginal damage caused by the brown good is severe, and the environmental quality is already high. Intuitively, the regulator finds more support for a higher subsidy when environmental quality is high.

Change in consumers’ patience impacts the optimal subsidy and social welfare differently depending on the initial and final location of the consumer whose patience has changed. A
marginal increase in a consumer’s patience while she still remains impatient does not impact the optimal subsidy or social welfare. But when an already patient consumer becomes more patient, the social planner wants to offer a higher subsidy to encourage more green consumption if the abatement costs are small, environmental damage caused by the brown good is high, or there is a high environmental quality established. Eventually, when the relatively impatient consumers become relatively patient, the social planner offers a higher subsidy to support lower prices, and social welfare improves for all parameter values. Therefore, when designing new policies for different consumer groups, it is important for policy makers to account for differences in individuals’ relative value of future considerations.

The model, results, and implications of this work can be applied in other areas with similar structure; i.e. when short-term individual interests are at odds with long-term collective interests, applications include recycling or dumping household’s waste (Lindsay and Strathman 1997), individuals’ preferences for commuting by cars vs public transportation (Joireman et al. 2004), contributing to common pool resources vs free riding (Kortenkamp and Moore 2006), etc.

A topic for further research in the design of the optimal environmental policy is incorporating uncertainty about consumers’ future considerations and environmental concerns in the regulator’s problem. Since the social planner does not have complete information about the consumers’ characteristics, introducing uncertainty improves the accuracy of the model in developing a welfare-maximizing environmental policy. Also, studying an arbitrary distribution of consumers’ environmental concerns can yield interesting results. In addition, status quo bias, i.e., individuals’ preference for the current situation may hinder them from switching to the socially-optimum alternative when a subsidy is provided (Venkatachalam 2008). Therefore, conducting field studies that integrate future considerations and environmental concerns and investigate the role of
subsidies in the consumers’ decision-making process is imperative. A lot of empirical research (Farrow et al. 2017; Culiberg and Elgaaied-Gambier 2016) cite environmental social norms as an important predictor of environmental behavior in a society. Adding an endogenous norm to the model to study the impact of social norm intervention programs (see e.g., Abrahamse et al. 2013; Schultz et al. 2007) on the optimal subsidy and welfare would be an interesting extension.

References


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6. **Appendix:**

**Proof of Lemma 1**
By taking the integral of the demand function and finding the area below the graph, we obtain the demand for Firm 1. For a given pair of prices \((p_1, p_2)\), Firm 1’s demand is:

\[
D_1 = \sum_{\delta_l > \delta} y_l \left(1 - \frac{p_1 - p_2}{e(1+\delta_0)}\right) = y - \frac{p_1 - p_2}{e} \sum_{\delta_l > \delta} \frac{y_l}{1+\delta_l} = y - \frac{p_1 - p_2}{e} \sigma, \text{ where } \sigma = \sum_{\delta > \delta} \frac{y_l}{1+\delta_l}.
\]

Since the area under the square is 1 and we assume a large enough \(v\) so that all consumers will buy from either of firms, the demand for Firm 2 is, \(D_2 = 1 - D_1 = 1 - y + \frac{p_1 - p_2}{e} \sigma\).

**Proof of Proposition 1**

Case (i): By plugging \(D_1\) and \(D_2\) back into the profit functions, we have,

\[
\pi_1 = (p_1 - c) \times \left(y - \frac{p_1 - p_2}{e} \sigma\right), \quad \pi_2 = p_2 \times \left(1 - y + \frac{p_1 - p_2}{e} \sigma\right).
\]

Taking the first order conditions with respect to \(p_1\) and \(p_2\), and simultaneously solving them, we find that,

\[
p_1 = \frac{1}{3} \left(2c + \frac{e(1+y)}{\sigma}\right), \quad p_2 = \frac{1}{3} \left(c + \frac{e(2-y)}{\sigma}\right). \quad (a)
\]

Plugging these prices back into the demand functions, we obtain,

\[
D_1 = \frac{1}{3} \left(1 + y - \frac{c}{e} \sigma\right), \quad D_2 = \frac{1}{3} \left(2 - y + \frac{c}{e} \sigma\right). \quad (b)
\]

Finally, using \((a)\) and \((b)\) and substituting them into the profit functions, we find,

\[
\pi_1 = \frac{(e(1+y) - c\sigma)^2}{9e\sigma}, \quad \pi_2 = \frac{(e(2-y) + c\sigma)^2}{9e\sigma}.
\]

We use proof by contradiction to show that the value of from \(y + CDF \left(\frac{1}{3} \left(\frac{c}{e} + \frac{2y-1}{\sigma}\right) - 1\right) - 1 = 0\), where \(\hat{\delta} = \frac{p_1 - p_2}{e} - 1\) and substituting \((a)\), we obtain \(\hat{\delta} = \frac{1}{3} \left(\frac{c}{e} + \frac{2y-1}{\sigma}\right) - 1\). Assume both \(y^{(1)}\)
and \( y^{(2)} \) solve this equation, where \( y^{(2)} > y^{(1)} \). Hence, it must be that \( y^{(2)} + CDF\left(\frac{1}{3}\left(\frac{c + 2y^{(2)} - 1}{\sigma^{(2)}}\right) - 1\right) = y^{(1)} + CDF\left(\frac{1}{3}\left(\frac{c + 2y^{(1)} - 1}{\sigma^{(1)}}\right) - 1\right) = 0 \). Since \( \sigma = \sum_{i=\hat{\delta}}^{\delta} \frac{y_i}{1 + \delta_i} \), for each \( y_i \) that is added to \( y \), \( \sigma \) increases by \( \frac{y_i}{1 + \delta_i} \) and since \( 0 \leq \delta_i \leq 1 \), the increase in \( \sigma \) is less than that of \( y \).

Therefore, \( 0 < \Delta \sigma < \Delta y < 1 \) and we have,

\[
y^{(2)} = y^{(1)} + kx, \quad \sigma^{(2)} = \sigma^{(1)} + x, \quad \text{s.t.: } x > 0, k > 1.
\]

First, we demonstrate that \( \sigma^{(1)} = \frac{y^{(1)}}{1 + \delta} \), where \( 0 \leq \delta \leq 1 \). We use mathematical induction to prove this. Assume \( \sigma(n) = \sum_{i=m}^{n} \frac{y_i}{1 + \delta_i} \), and \( y(n) = \sum_{i=m}^{n} y_i \), where \( m \) is the smallest \( i \) such that \( \delta_i \geq \delta \).

We want to show \( \sigma(n) = \frac{y(n)}{1 + \delta(n)} \), \( \forall n \in \mathbb{N}, n \geq m \) where \( 0 \leq \delta(n) \leq 1 \).

Base case: we show the statement holds for \( n = m \).

\[
\sigma(m) = \frac{y_m}{1 + \delta_m}, \text{ thus, } y(m) = y_m, \delta(m) = \delta_m \in [0,1].
\]

Inductive step: we show that if \( \sigma(k) \) holds for \( k \geq m \), then \( \sigma(k+1) \) holds, too; i.e.

\[
\exists 0 \leq b(k+1) \leq 1 : \sigma(k+1) = \sum_{i=m}^{k+1} \frac{y_i}{1 + \delta_i} = \frac{y(k+1)}{1 + \delta(k+1)}.
\]

Using the induction hypothesis that \( \sigma(k) \) holds,

\[
\sum_{i=m}^{k+1} \frac{y_i}{1 + \delta_i} = \sum_{i=m}^{k} \frac{y_i}{1 + \delta_i} + \frac{y_{k+1}}{1 + \delta_{k+1}} = \frac{y(k)}{1 + \delta(k)} + \frac{y_{k+1}}{1 + \delta_{k+1}}.
\]

We need to sow that \( \delta(k+1) \) that solves the equation below belongs to \([0,1]\).

\[
\frac{y(k)}{1 + \delta(k)} + \frac{y_{k+1}}{1 + \delta_{k+1}} = \frac{y(k+1)}{1 + \delta(k+1)} = \frac{y(k) + y_{k+1}}{1 + \delta(k+1)}.
\]
\[ \delta(k + 1) = \frac{y(k)\delta(k) (1 + \delta_{k+1}) + y_{k+1}\delta_{k+1} (1 + \delta(k))}{y(k) (1 + \delta_{k+1}) + y_{k+1} (1 + \delta(k))}. \]

Since \( 0 \leq y(k), \delta(k), \delta_{k+1}, \delta_{k+1} \leq 1 \), hence, \( \delta(k + 1) \geq 0 \). Also,

\[-y(k)(1 - \delta(k))(1 + \delta_{k+1}) - y_{k+1}(1 - \delta_{k+1})(1 + \delta(k)) \leq 0, \]

which implies,

\[ y(k)\delta(k)(1 + \delta_{k+1}) + y_{k+1}\delta_{k+1}(1 + \delta(k)) - \left( y(k)(1 + \delta_{k+1}) + y_{k+1}(1 + \delta(k)) \right) \leq 0, \]

hence \( \frac{y(k)\delta(k)(1 + \delta_{k+1}) + y_{k+1}\delta_{k+1}(1 + \delta(k))}{y(k)(1 + \delta_{k+1}) + y_{k+1}(1 + \delta(k))} < 1 \), and, \( \delta(k + 1) \leq 1 \). Therefore, we can write \( \sigma(1) = \frac{y(1)}{1 + \delta} \) where \( 0 \leq \delta \leq 1 \). In addition, since \( 0 \leq \delta \leq 1 \) and \( -1 \leq \frac{1}{1 + \delta} - 1 \leq 0 \), we conclude \( 0 \leq 1 - y(1) \leq 2y(1) \left( \frac{1}{1 + \delta} - 1 \right) + 1 \leq 1. \)

Since \( \frac{x(2k\sigma(1) - 2y(1) + 1)}{\sigma(1)(\sigma(1) + x)} \) is an increasing function of \( k \), and \( k > 1 \) we have,

\[ \frac{2y(2) - 1}{\sigma(2)} - \frac{2y(1) - 1}{\sigma(1)} = \frac{x(2k\sigma(1) - 2y(1) + 1)}{\sigma(1)(\sigma(1) + x)} \geq \frac{x}{\sigma(1)(\sigma(1) + x)} \left( 2 \cdot \frac{y(1)}{1 + \delta} - 2y(1) + 1 \right) \geq 0. \]

Since \( \frac{2y(2) - 1}{\sigma(2)} > \frac{2y(1) - 1}{\sigma(1)} \), for given values of \( c \) and \( e \), we have,

\[ \frac{1}{3} \left( \frac{c}{e} + \frac{2y(2) - 1}{\sigma(2)} \right) - 1 > \frac{1}{3} \left( \frac{c}{e} + \frac{2y(1) - 1}{\sigma(1)} \right) - 1, \]

meaning, \( CDF \left( \frac{1}{3} \left( \frac{c}{e} + \frac{2y(2) - 1}{\sigma(2)} \right) - 1 \right) > CDF \left( \frac{1}{3} \left( \frac{c}{e} + \frac{2y(1) - 1}{\sigma(1)} \right) - 1 \right) \), and consequently,

\[ y(2) + CDF \left( \frac{1}{3} \left( \frac{c}{e} + \frac{2y(2) - 1}{\sigma(2)} \right) - 1 \right) > y(1) + CDF \left( \frac{1}{3} \left( \frac{c}{e} + \frac{2y(1) - 1}{\sigma(1)} \right) - 1 \right), \]

which is in contradiction with the assumption that both \( y(1) \) and \( y(2) \) solve \( y + CDF \left( \frac{1}{3} \left( \frac{c}{e} + \frac{2y - 1}{\sigma} \right) - 1 \right) - 1 = 0. \)
Below, we demonstrate some examples of continuous distributions of consumers’ patience and derive the values of $\sigma$ and $y$.

I. Consumers’ consideration of future consequences follows standard uniform distribution, i.e.

$$\delta_i = \frac{i}{n}, \quad y_i = \frac{1}{n}, \quad i \in \{1, \ldots, n\}, \quad n \to \infty.$$  

Figure 4 shows the distribution of consumers over the patience line for different values of $n$.

From Riemann sum,

$$\lim_{n' \to \infty} \sum_{i=1}^{n'} f \left( a + \frac{b - a}{n'} i \right) \frac{b - a}{n'} = \int_a^b f(x) \, dx.$$  

If we set $a = 0, b = 1, n' = n, f(x) = \frac{1}{1+x}$, we have that $\lim_{n \to \infty} \sum_{i=1}^{n} \frac{1}{1+\frac{i}{n}} = \int_0^1 \frac{1}{1+x} \, dx$. Also, if we set

$$a = 0, b = 1-y, n' = n(1-y), f(x) = \frac{1}{1+x},$$  

we have that $\lim_{n \to \infty} \sum_{i=1}^{n(1-y)} \frac{1}{1+\frac{i}{n}} = \int_0^{1-y} \frac{1}{1+x} \, dx$.

Therefore,
\[
\sigma = \lim_{n \to \infty} \sum_{i=n(1-y)}^{n} \frac{1}{1 + \frac{i}{n}} = \lim_{n \to \infty} \sum_{i=1}^{n} \frac{1}{1 + \frac{i}{n}} - \lim_{n \to \infty} \sum_{i=1}^{n(1-y)} \frac{1}{1 + \frac{i}{n}} = \int_{0}^{1} \frac{1}{1 + x} \, dx - \int_{0}^{1-y} \frac{1}{1 + x} \, dx
\]

\[
= Ln2 - Ln(2 - y) = Ln \frac{2}{2 - y}
\]

In addition, with the continuous uniform distribution for consumers’ patience, according to Figure 3,

\[
1 - y = CDF\left(\frac{p_1 - p_2}{e} - 1\right) = p_1 - p_2 - 1 = \frac{1}{3} \left(\frac{c}{e} + \frac{2y - 1}{\sigma}\right) - 1 = \frac{1}{3} \left(\frac{c}{e} + \frac{2y - 1}{Ln \frac{2}{2 - y}}\right) - 1,
\]

which implies that, \(\frac{1}{3} \left(\frac{c}{e} + \frac{2y - 1}{Ln \frac{2}{2 - y}}\right) + y - 2 = 0\).

II. The density of consumers’ consideration of future consequences increases as patience increases, such that \(\delta_i = \frac{i}{n}, \gamma_i = \frac{i(1+i)}{n^2} = \frac{6i(i+n)}{n(n+1)(5n+1)}, i \in \{1, ..., n\}, \) and \(n \to \infty\). We find \(\sigma\) and \(y\) for this distribution with a slightly different method. Note that this is a probability mass function since \(\sum_{i=1}^{n} \gamma_i = 1\). Figure 5 shows the distribution of consumers over the patience line for different values of \(n\).
\[ \sigma = \lim_{n \to \infty} \sum_{i=n(1-y)}^{n} \frac{y_i}{1+\delta_i} = \lim_{n \to \infty} \sum_{i=n(1-y)}^{n} \frac{6i}{(n+1)(5n+1)} \]

\[ = \lim_{n \to \infty} \frac{6}{(n+1)(5n+1)} \left( \sum_{i=1}^{n} \frac{n(1-y)}{2} - \sum_{i=1}^{n} i \right) \]

\[ = \lim_{n \to \infty} \frac{6}{(n+1)(5n+1)} \left( \frac{n(n+1)}{2} - \frac{n(1-y)(n(1-y)+1)}{2} \right) \]

\[ = \frac{3 \left( n^2 - (n(1-y))^2 \right)}{n \times 5n} = \frac{3}{5} (1 - (1-y)^2). \]

\[ y = \sum_{\delta^\gamma(\frac{1}{3}(c e + \frac{2y}{\sigma}) - 1)}^{1} y_i = \int_{\frac{1}{3}(c e + \frac{2y}{\sigma}) - 1}^{1} PDF(x) \, dx = 1 - CDF \left( \frac{1}{3}(c e + \frac{2y}{\sigma}) - 1 \right). \]

Therefore, \( y + CDF \left( \frac{1}{3}(c e + \frac{2y}{\sigma}) - 1 \right) - 1 = 0. \)

Also, \( CDF(x) = \sum_{i=1}^{x} y_i = \sum_{i=1}^{x} \frac{6i(i+n)}{n(n+1)(5n+1)} = \frac{6}{n(n+1)(5n+1)} \sum_{i=1}^{x} (i^2 + i \times n) = \]

\[ = \frac{6}{n(n+1)(5n+1)} \left( \frac{x(x+1)(2x+1)}{6} + n \times \frac{x(x+1)}{2} \right) = \frac{x(1+x)(1+3n+2x)}{n(1+n)(1+5n)}, \text{ where } x = a \times n \text{ and } 0 \leq a \leq 1. \]

As the number of consumers goes to infinity,

\[ CDF(x) = \lim_{n \to \infty} \frac{x(1+x)(1+3n+2x)}{n(1+n)(1+5n)} = \lim_{n \to \infty} \frac{x(x)(3n+2x)}{n(n)(5n)} = \lim_{n \to \infty} \frac{x^2(2x + 3n)}{5n^3} \]

\[ = \lim_{n \to \infty} \frac{a^2 \times n^2(2an + 3n)}{5n^3} = \frac{2a^3 + 3a^2}{5}. \]

If we substitute \( a = \delta \) and \( \sigma \) in the above equation, we can find \( y \) as a function of \( \frac{e}{c} \) using root-finding algorithms, such as Newton’s method.
III. Below, we study a bell-shaped distribution for consumers’ patience, specifically \( y_i = \frac{1}{4} \left( \frac{1}{n} - \frac{1}{n^2} \right)^2 \), \( i \in \{1, \ldots, n\} \), and \( n \to \infty \). Figure 6 shows the distribution of consumers over the patience line for different values of \( n \).

\[
\sigma = \lim_{n \to \infty} \sum_{i=n(1-y)}^{n} \frac{y_i}{1 + \delta_i} = \lim_{n \to \infty} \sum_{i=n(1-y)}^{n} \frac{6i(n-i)}{(n+i)(n^2-1)}
\]

\[
= \lim_{n \to \infty} \frac{3n \left( (y+2)(ny+1) + 4n \left( H_{2n-ny-1} - H_{2n} \right) \right)}{n^2 - 1}
\]

\[
= \lim_{n \to \infty} \frac{3n \left( ny(y+2) + 4n \left( H_{2n-ny} - H_{2n} \right) \right)}{n^2} = 3y(y+2) + 12\ln \left( 1 - \frac{y}{2} \right).
\]

Similarly, as the number of consumers goes to infinity,

\[
CDF(x) = \lim_{n \to \infty} \sum_{i=1}^{x} y_i = \lim_{n \to \infty} \frac{x(1+x)(3n-2x+1)}{n(n^2-1)} = \lim_{n \to \infty} \frac{x(x)(3n-2x)}{n(n^2)}
\]

\[
= \lim_{n \to \infty} \frac{a^2 \times n^2(3n-2an)}{n^3} = 3a^2 - 2a^3.
\]

If we substitute \( a = \hat{\delta} \) and \( \sigma \) in the above equation, we can find \( y \) as a function of \( \epsilon \) using root-finding algorithms, such as Newton’s method.
IV. The density of consumers’ consideration of future consequences decreases as patience increases, such that \( \delta_i = \frac{i}{n}, y_i = \frac{1-\frac{i-1}{n}}{\sum_{i=1}^{n} 1-\frac{i-1}{n}} = \frac{2(n-k+1)}{n^2+n}, \) \( i \in \{1, \ldots, n\}, \) and \( n \to \infty. \) Note that this is a probability mass function since \( \sum_{i=1}^{n} y_i = 1. \) Figure 7 shows the distribution of consumers over the patience line for different values of \( n. \)

To find \( \sigma \) and \( y \) for this distribution,

\[
\sigma = \lim_{n \to \infty} \sum_{i=n(1-y)}^{n} \frac{y_i}{1+\delta_i} = \lim_{n \to \infty} \sum_{i=n(1-y)}^{n} \frac{2(n-i+1)}{(n+i)(n+1)} = 4\ln \left( \frac{2}{2-y} \right) - 2y.
\]

As the number of consumers goes to infinity,

\[
CDF(x) = \lim_{n \to \infty} \sum_{i=1}^{x} y_i = \lim_{n \to \infty} \frac{2nx + x - x^2}{n(1+n)} = \lim_{n \to \infty} \frac{2\alpha n^2 + \alpha n - \alpha^2 n^2}{n(n)} = 2\alpha - \alpha^2.
\]

If we substitute \( \alpha = \hat{\delta} \) and \( \sigma \) in the equation \( y + CDF \left( \frac{1}{3} \left( \frac{c}{e} + \frac{2\gamma-1}{\sigma} \right) - 1 \right) - 1 = 0, \) we can find \( y \)

as a function of \( \frac{c}{e} \) using root-finding algorithms, such as Newton’s method.

**Proof of Lemma 2**

The social welfare without regulation is, \( SW_{NR} = CS + PS - Env. \)
\[
CS = \sum_{\delta_i > \delta} y_i \int_0^1 \frac{p_1-p_2}{e(1+\delta_i)} (v - p_1 + \theta e(1 + \delta_i)) d\theta + (1 - y) (v - p_2) + \sum_{\delta_i > \delta} y_i \int_0^1 (v - p_2) d\theta \\
= \sum_{\delta_i > \delta} y_i \left( (v - p_1) \left( 1 - \frac{p_1-p_2}{e(1+\delta_i)} \right) + \frac{e(1+\delta_i)}{2} \left( 1 - \frac{(p_1-p_2)^2}{e(1+\delta_i)} \right) \right) \\
+ \frac{p_1-p_2}{e(1+\delta_i)} (v - p_2) + (1 - y)(v - p_2).
\]

After plugging \( p_1 \) and \( p_2 \) into \( CS \), \( PS \), and \( Env \), we obtain,

\[
CS = v + \frac{(c\sigma + e(2y - 1))^2}{18e\sigma} + \frac{ey}{2} + \frac{e}{2} \sum_{\delta_i > \delta} y_i b_i - \frac{2e(y^2 - y + 1)}{3\sigma} - \frac{c(1+y)}{3},
\]

\[
PS = \pi_1 + \pi_2 = \frac{(e(1+y) - c\sigma)^2}{9e\sigma} + \frac{(e(2y + c\sigma))^2}{9e\sigma}, Env = d \times D_2 = d \times \frac{1}{3} (2 - y + \frac{c}{e\sigma}).
\]

Plugging back all the values into \( SW_{NR} \) and simplifying the equation, we have,

\[
SW_{NR} = \frac{1}{18e\sigma} \left( 2e\sigma(9v - 3d(2-y) - c(2+5y)) + c\sigma^2(5c - 6d) \right. \\
\left. + e^2 \left( 4(1-y)y + 9\sigma \left( y + \sum_{\delta_i > \delta} y_i \delta_i \right) - 1 \right) \right)
\]

**Proof of Proposition 2**

The subsidy is provided to Firm 1 for each unit they produce. So, Firm 1’s profit function under regulation is given by, \( \pi_1 = (p_1 - c)D_1 + s \times D_1 = (p_1 - (c - s)) \times \left( y - \frac{p_1-p_2}{e} \sigma \right) \).
By taking the first order conditions with respect to $p_1$ and $p_2$ and simultaneously solving the equations, we find that,

\[
p_1 = \frac{1}{3} \left( 2(c - s) + \frac{e(1 + y)}{\sigma} \right), \quad p_2 = \frac{1}{3} \left( (c - s) + \frac{e(2 - y)}{\sigma} \right),
\]

\[
D_1 = \frac{1}{3} \left( 1 + y - \frac{(c - s)}{e} \right), \quad D_2 = \frac{1}{3} \left( 2 - y + \frac{(c - s)}{e} \right),
\]

\[
\pi_1 = \frac{(e(1 + y) - \sigma(c - s))^2}{9e\sigma}, \quad \pi_2 = \frac{(e(2 - y) + \sigma(c - s))^2}{9e\sigma}.
\]

In the first stage of the game, the regulator chooses the subsidy that maximizes,

\[
\max_s SW_R = CS + PS - Env - G,
\]

where,

\[
CS = v + \left( \frac{(c - s)\sigma + e(2y - 1)}{18e\sigma} \right)^2 + \frac{ey}{2} + \frac{e}{3} \sum_{\delta_i \geq \delta} y_i \delta_i - \frac{2e(y^2 - y + 1)}{3\sigma} - \frac{(c - s)(1 + y)}{3},
\]

\[
PS = \frac{(e(1 + y) - \sigma(c - s))^2}{9e\sigma} + \frac{(e(2 - y) + \sigma(c - s))^2}{9e\sigma},
\]

\[
Env = d \times \frac{1}{3} \left( 2 - y + \frac{(c - s)}{e} \right), \quad G = s \times D_1 = s \times \frac{1}{3} \left( 1 + y - \frac{(c - s)}{e} \right).
\]

Plugging back all the values into the $SW_R$ and simplifying the equation, we find,

\[
SW_R = \frac{1}{18} \left( 18v - 6d + e \left( 9 + 9 \sum_{\delta_i \geq \delta} y_i \delta_i - \frac{1}{\sigma} \right) + c \left( \frac{(5c - 6d)\sigma}{e} - 14 \right) \right)
\]

\[
- s \times \frac{1}{3} \left( 1 + y - \frac{(c - s)}{e} \right).
\]

Taking the first order condition with respect to $s$, we have,
\[
\frac{dSW_R}{ds} = 0 \rightarrow s^* = \frac{e(2y - 1)}{\sigma} + 3d - 2c.
\]

Checking for second order condition to make sure we have a concave function with a maximum,

\[
\frac{d^2SW_R}{ds^2} = -\frac{\sigma}{9e} < 0.
\]

Below we show in Case (i), \(s^* > 3(e + d - c)\) and in Case (ii), \(s^* < 3(e + d - c)\).

Case (i): \(\frac{c}{e} > 3 - \frac{2y - 1}{\sigma}\), which means, \(c > 3e - \frac{e(2y - 1)}{\sigma}\), and, \(\frac{e(2y - 1)}{\sigma} - 2c > 3e - 3c\). As a result, \(s^* = \frac{e(2y - 1)}{\sigma} + 3d - 2c > 3e + 3d - 3c\).

Case (ii): \(\frac{c}{e} \leq \left(3 - \frac{1}{\sigma}\right)\), which means, \(c < 3e - \frac{e}{\sigma}\), and, \(\frac{e}{\sigma} - 2c < 3e - 3c\). As a result, \(s^* = \frac{e}{\sigma} + 3d - 2c > 3e + 3d - 3c\).

By plugging \(s^*\) back into the demands, prices, and social welfare with regulation we obtain,

\[
p_1^* = 2(c - d) + \frac{e(1 - y)}{\sigma}, \quad p_2^* = (c - d) + \frac{e(1 - y)}{\sigma}, \quad p_1^* - p_2^* = c - d,
\]

\[
D_1^* = y - \frac{(c - d)\sigma}{e}, \quad D_2^* = 1 - y + \frac{(c - d)\sigma}{e}, \quad \hat{\delta}^* = \frac{c - d}{e} - 1,
\]

\[
SW_R(s^*) = v - d - y(c - d) + \frac{e}{2}\left(y + \sum_{\delta_l > \delta} y_l\delta_l\right) + \frac{(c - d)^2\sigma}{2e}.
\]

If \(c \leq d\), \(p_1^* - p_2^* = c - d \leq 0\). According to Figure 3, this means \(y = 1\) and \(D_2^* = \frac{(c-d)\sigma}{e} \leq 0\).

Also, the equation \(y + CDF(\hat{\delta}) - 1 = 0\) simplifies to \(y + CDF\left(\frac{c-d}{e} - 1\right) - 1 = 0\). As \(c\) decreases, \(d\) increases, or \(e\) increases, \(\hat{b}\) falls and as a result \(CDF(\hat{\delta})\) falls. For the equation to hold
\( y \) must increase and since \( 0 < \frac{\Delta \sigma}{\Delta y} < 1 \) (see proof for Proposition 1), \( \sigma \) increases, too. Since consumers’ distribution of patience is discrete, both \( y \) and \( \sigma \) increase discretely. In conclusion, we have,

\[
\frac{\Delta y}{\Delta d} \geq 0, \quad \frac{\Delta y}{\Delta c} \leq 0, \quad \frac{\Delta y}{\Delta e} \leq 0, \quad \frac{\Delta \sigma}{\Delta y} > 0.
\]

We introduce,

\[
y^{(2)} = y^{(1)} + kx, \quad \sigma^{(2)} = \sigma^{(1)} + x, \quad s.t.: x \geq 0, k \geq 1,
\]

where \( y^{(1)}/y^{(2)} \) and \( \sigma^{(1)}/\sigma^{(2)} \) stand for initial/final values of \( y \) and \( \sigma \), respectively. In Proposition 1 we showed that as \( y \) increases from \( y^{(1)} \) to \( y^{(2)} \) and \( \sigma \) increases from \( \sigma^{(1)} \) to \( \sigma^{(2)} \), \( \frac{2y - 1}{\sigma} \) increases,

\[
\frac{\Delta \left( \frac{2y - 1}{\sigma} \right)}{\Delta y} = \frac{1}{kx} \left( \frac{2y^{(2)} - 1}{\sigma^{(2)}} - \frac{2y^{(1)} - 1}{\sigma^{(1)}} \right) \geq 0.
\]

Also, as \( \hat{b} \) falls and \( y \) increases, \( \sigma \) increase, \( 1 - y \) decreases, and consequently \( \frac{1 - y}{\sigma} \) decrease,

\[
\frac{\Delta \left( \frac{1 - y}{\sigma} \right)}{\Delta y} < 0.
\]

We now analyze how the optimal subsidy, prices, and demands change when the parameters of the model change,

\[
\frac{\Delta s^*}{\Delta d} = e \frac{\Delta \left( \frac{2y - 1}{\sigma} \right)}{\Delta y} \times \frac{\Delta y}{\Delta d} + 3\Delta d > 0, \quad \frac{\Delta s^*}{\Delta c} = e \frac{\Delta \left( \frac{2y - 1}{\sigma} \right)}{\Delta y} \times \frac{\Delta y}{\Delta c} - 2\Delta c < 0,
\]

\[
\frac{\Delta p_{1}^*}{\Delta d} = e \frac{\Delta \left( \frac{1 - y}{\sigma} \right)}{\Delta y} \times \frac{\Delta y}{\Delta d} - 2\Delta d < 0, \quad \frac{\Delta p_{2}^*}{\Delta d} = e \frac{\Delta \left( \frac{1 - y}{\sigma} \right)}{\Delta y} \times \frac{\Delta y}{\Delta d} - \Delta d < 0,
\]

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\[
\frac{\Delta p_1^*}{\Delta c} = e^{\frac{(1-y)}{\sigma}} \times \frac{\Delta y}{\Delta c} + 2\Delta c > 0, \quad \frac{\Delta p_2^*}{\Delta c} = e^{\frac{(1-y)}{\sigma}} \times \frac{\Delta y}{\Delta c} + \Delta c > 0.
\]

According to Figure 3, as \(d\) increases, \(c\) decreases, or \(e\) increases, \(\hat{\delta}\) decreases and \(D_1/D_2\) increases/decreases, respectively; i.e.

\[
\frac{\Delta D_1^*}{\Delta d} > 0, \quad \frac{\Delta D_2^*}{\Delta d} < 0, \quad \frac{\Delta D_1^*}{\Delta c} < 0, \quad \frac{\Delta D_2^*}{\Delta c} > 0, \quad \frac{\Delta D_1^*}{\Delta e} > 0, \quad \frac{\Delta D_2^*}{\Delta e} < 0.
\]

**Optimal subsidy if \(e = 0\):**

In this case, we have Bertrand competition with asymmetric costs and consumers purchase from the firm with the lower price. If the regulator sets \(s < c\), the brown firm’s production cost is lower than that of the green firm. Firm 2 charges a price slightly below Firm 1’s cleaning cost assuming the monopoly price is higher than \(c\); otherwise, Firm 2 charges the monopoly price and makes monopoly profit. In this case \(D_1 = 0, D_2 = 1\), and we have,

\[
CS = v - p_2, \quad PS = \pi_1 + \pi_2 = 0 + p_2 = p_2, \quad Env = d, \quad G = 0, \quad SW_R = v - d.
\]

If the regulator sets \(s = c\), firms have identical costs, charge the marginal cost (which is zero), each receive half of the market, and make zero profits. We have,

\[
CS = v - 0 = v, \quad PS = \pi_1 + \pi_2 = 0, \quad Env = \frac{d}{2}, \quad G = \frac{c}{2}, \quad SW_R = v - \frac{d}{2} - \frac{c}{2}.
\]

If the regulator sets \(s > c\), Firm 1 is able to charge a lower price than Firm 2 and receives all the market share. In this case \(D_1 = 1, D_2 = 0\), and we have,

\[
CS = v - p_1, \quad PS = \pi_1 + \pi_2 = (p_1 - (c - s)) \times 1 + 0 = p_1 - (c - s),
\]

\[
Env = 0, \quad G = s, \quad SW_R = v - c.
\]
If \( c < d \), setting \( s > c \) yields the highest social welfare. If \( c = d \), all policies yield the same amount of social welfare, and if \( c > d \), setting \( s < c \) yields the highest social welfare.

**Proof of Corollary 2**

Taking the difference between \( SW_R(s^*) \) (social welfare with optimal regulation) and \( SW_{NR} \) (social welfare with no regulation) yields a welfare improvement equal to,

\[
SW_R(s^*) - SW_{NR} = \frac{s^*2\sigma}{18e} > 0
\]

**Proof of Corollary 3**

Without future considerations, all individuals are located on the \( \theta \) axis which is represented in Figure 8. If \( p_1 - p_2 \geq e \), or \( \frac{p_1 - p_2}{e} \geq 1 \), Firm 2 captures all the demand. Therefore, we assume \( p_1 - p_2 < e \), or \( \frac{p_1 - p_2}{e} > 1 \).

![Figure 8; consumers without future considerations](image)

When the regulator ignores future considerations, the utility of consumers purchasing from Firm 1 is considered, \( v - p_1 + \theta e \). Therefore, the locus of indifferent consumers satisfies, \( v - p_1 + \theta e = v - p_2 \), hence, \( \theta = \frac{p_1 - p_2}{e} \).
The area below $\theta = 1$ and above $\frac{p_1-p_2}{e}$ intersection shows the demand for Firm 1 and the area below the $\frac{p_1-p_2}{e}$ intersection shows the demand for Firm 2. Thus, $(D_1, D_2) = \left(1 - \frac{p_1-p_2}{e}, \frac{p_1-p_2}{e}\right)$.

By plugging $D_1$ and $D_2$ back into the profit functions, taking the first order conditions with respect to $p_1$ and $p_2$, and simultaneously solving them, we find that,

$$
\pi_1 = (p_1 - c) \times \left(1 - \frac{p_1-p_2}{e}\right), \quad \pi_2 = p_2 \times \left(\frac{p_1-p_2}{e}\right),
$$

$$
p_1 = \frac{1}{3}\left(2c + \frac{e(1+y)}{\sigma}\right), \quad p_2 = \frac{1}{3}\left(c + \frac{e(2-y)}{\sigma}\right).
$$

$$
CS = \int_{\frac{p_1-p_2}{e}}^{1} (v - p_1 + \theta e)d\theta + (v - p_2) \left(\frac{p_1-p_2}{e}\right) = v + \frac{e}{2} + \frac{(p_1-p_2)^2}{2e} - p_1
$$

$$
= v + \frac{1}{18}\left(\frac{c^2}{e} - 10c - 2e\right),
$$

$$
PS = \pi_1 + \pi_2 = \frac{(c - 2e)^2}{9e} + \frac{(c + e)^2}{9e}, \quad Env = d \times D_2 = d \times \frac{1}{3}\left(1 - \frac{c}{e}\right).
$$

Plugging back all the values into social welfare and simplifying the equation, we have,

$$
SW_{NR}^{NF} = v + \frac{5c^2 - 6cd - 14ce - 6de + 8e^2}{18e},
$$

where $SW_{NR}^{NF}$ is social welfare without regulation and future considerations. Social welfare when the regulator provides the optimal subsidy to the green firm while ignoring future considerations ($SW_{R}^{NF}$) is,

$$
SW_{R}^{NF} = v + \frac{5(c - s)^2 - 6(c - s)d - 14(c - s)e - 6de + 8e^2}{18e} - s \times \frac{1}{3}\left(2 - \frac{(c - s)}{e}\right),
$$
where \( s \times \frac{1}{3} \left( 2 - \frac{(c-s)}{e} \right) \) represents the total subsidy that the regulator pays to the green firm.

Taking the first order condition with respect to \( s \), we find the level of optimal subsidy for a social planner who does not consider future considerations among consumers,

\[
s_{NF}^* = e + 3d - 2c.
\]

Also, \( \frac{d^2 SW_R}{ds^2} = - \frac{1}{9e} < 0 \), meaning social welfare is a concave function with a maximum.

We know individuals have future considerations. So, when \( s_{NF}^* \) is provided to the green firm, social welfare is,

\[
SW_{R}^{NF}(s_{NF}^*) = v - d - y(c - d) + \frac{(c - d)^2 \sigma}{2e} - \frac{e}{18\sigma} \left( (1 - 2y)^2 + \sigma^2 - \sigma \left( 9 \sum_{\delta_i \geq \delta} y_i \delta_i + 13y - 2 \right) \right).
\]

The difference between optimal social welfare with future considerations and that arising when the social planner ignores future considerations reveals a welfare loss,

\[
SW_R(s^*) - SW_{R}^{NF}(s_{NF}^*) = \frac{e(1 - 2y + \sigma)^2}{18\sigma} \geq 0.
\]

Proof of Corollary 4

Since \( \frac{\Delta y}{\Delta e} \leq 0 \), \( \frac{\Delta \sigma}{\Delta y} > 0 \) (see proof for Proposition 2), we introduce,

\[
y^{(2)} = y^{(1)} + kx, \quad \sigma^{(2)} = \sigma^{(1)} + x, \quad s.t.: x \geq 0, k \geq 1,
\]

\[
e^{(2)} > e^{(1)}, \quad s^{*(1)} = \frac{e^{(1)}(2y^{(1)} - 1)}{\sigma^{(1)}} + 3d - 2c, \quad s^{*(2)} = \frac{e^{(2)}(2y^{(2)} - 1)}{\sigma^{(2)}} + 3d - 2c,
\]
where $y^{(1)}/y^{(2)}$, $\sigma^{(1)}/\sigma^{(2)}$, $e^{(1)}/e^{(2)}$, and $s^{*(1)}/s^{*(2)}$ stand for initial/final values of $y$, $\sigma$, $e$, and $s^*$, respectively.

\[
\Delta SW_R(s^*) = SW_R(s^*)^{(2)} - SW_R(s^*)^{(1)}
\]
\[
= (c - d)^2 \left( \frac{\sigma^{(2)}}{e^{(2)}} - \frac{\sigma^{(1)}}{e^{(1)}} \right) - 2(c - d)(y^{(2)} - y^{(1)})
\]
\[
+ e^{(2)} \left( y^{(2)} + \left( \sum_{\delta_i > \delta} y_i \delta_i \right)^{(2)} \right) - e^{(1)} \left( y^{(1)} + \left( \sum_{\delta_i > \delta} y_i \delta_i \right)^{(1)} \right),
\]

where $SW_R(s^*)^{(1)}/SW_R(s^*)^{(2)}$ and $\left( \sum_{\delta_i > \delta} y_i \delta_i \right)^{(1)}/\left( \sum_{\delta_i > \delta} y_i \delta_i \right)^{(2)}$ stand for initial/final values of $SW_R(s^*)$ and $\sum_{\delta_i > \delta} y_i \delta_i$, respectively. Since,

\[
e^{(2)} > e^{(1)}, \quad \sigma^{(2)} > \sigma^{(1)}, \quad y^{(2)} > y^{(1)}, \quad \left( \sum_{\delta_i > \delta} y_i \delta_i \right)^{(2)} > \left( \sum_{\delta_i > \delta} y_i \delta_i \right)^{(1)},
\]

the first component of $\Delta SW_R(s^*)$ i.e. $(c - d)^2 \left( \frac{\sigma^{(2)}}{e^{(2)}} - \frac{\sigma^{(1)}}{e^{(1)}} \right)$ has an indeterminant sign. This component shrinks as $c - d$ decreases and $e$ increases. The second component i.e. $-2(c - d)(y^{(2)} - y^{(1)})$ is negative and its absolute value declines as $c - d$ declines. The third component i.e. $e^{(2)} \left( y^{(2)} + \left( \sum_{\delta_i > \delta} y_i \delta_i \right)^{(2)} \right) - e^{(1)} \left( y^{(1)} + \left( \sum_{\delta_i > \delta} y_i \delta_i \right)^{(1)} \right)$ is positive and is larger with larger values of $e$. In conclusion, when the difference between $c$ and $d$ is relatively small, or $e$ is relatively high, the first (second) component of $\Delta SW_R(s^*)$ with indeterminant (negative) sign, respectively, shrinks and the third component with positive sign grows larger, making $\Delta SW_R(s^*)$ positive and increasing social welfare.
We had shown in Proposition 2 that when $\gamma$ increases from $\gamma(1)$ to $\gamma(2)$ and $\sigma$ increases from $\sigma(1)$ to $\sigma(2)$, $\frac{2\gamma - 1}{\sigma}$ increases. For $s^*$, if $\gamma(2) \geq \frac{1}{2}$,

$$s^{*(2)} - s^{*(1)} = \left( \frac{e^{(2)}(2\gamma(2) - 1)}{\sigma(2)} + 3d - 2c \right) - \left( \frac{e^{(1)}(2\gamma(1) - 1)}{\sigma(1)} + 3d - 2c \right) > 0.$$ 

Also, according to the equation $\gamma + CDF\left(\frac{c-d}{e} - 1\right) - 1 = 0$, $\gamma \geq \frac{1}{2}$ means,

$$CDF\left(\frac{c-d}{e} - 1\right) = 1 - \gamma \leq \frac{1}{2},$$

i.e., $\frac{c-d}{e} - 1 \leq CDF^{-1}\left(\frac{1}{2}\right)$, and, $\frac{c-d}{e} \leq CDF^{-1}\left(\frac{1}{2}\right) + 1$.

If $\gamma(2) < \frac{1}{2}$, $s^*$ may increase or decrease with improvements in the environmental quality.

**Proof of Corollary 5**

**Case 1. $\delta_i, \delta_j < \bar{\delta}$.**

Since the portion and composition of population located on the right of $\bar{\delta}$ does not change, both $\gamma$ and $\sigma$ remain the same. Consequently, $s^*$ and $SW_R(s^*)$ remain the same.

**Case 2. $\delta_i, \delta_j > \bar{\delta}$.**

Since the portion of population located on the right of $\bar{\delta}$ does not change, $\gamma$ stays the same. Since $j > i$, $\delta_j > \delta_i$, $\sigma$ drops, and $\sum_{b_i:b_j} y_i b_i$ increases,

$$x \equiv \sigma^{(1)} - \sigma^{(2)} = \frac{\gamma_i}{1 + \delta_i} - \frac{\gamma_i}{1 + \delta_j} > 0,$$

$$w \equiv \left( \sum_{\delta_i > \delta} y_i \delta_i \right)^{(2)} - \left( \sum_{\delta_i > \bar{\delta}} y_i \delta_i \right)^{(1)} = y_i \delta_j - y_i \delta_i > 0.$$
In Case 2, the amount of increase/decrease in $D_1/D_2$ is,

$$y_i \left(1 - \frac{c - d}{e(1 + \delta_i)}\right) - y_i \left(1 - \frac{c - d}{e(1 + \delta_j)}\right) = x \frac{c - d}{e} > 0.$$ 

$SW_R(s^*)$ improves if and only if,

$$SW_R(s^*)^{(2)} - SW_R(s^*)^{(1)} = \frac{e w}{2} - \frac{(c-d)^2 x}{2 e}, \text{ or, } \frac{c-d}{e} < \sqrt{x}.$$ 

Also, social welfare under regulation is an increasing function of $e$,

$$\frac{\partial (SW_R(s^*)^{(2)} - SW_R(s^*)^{(1)})}{\partial e} = \frac{1}{2} \left(w + x \left(\frac{c - d}{e}\right)^2\right) > 0.$$ 

For the optimal subsidy, if $y > \frac{1}{2}$, $s^*$ increases. If $y < \frac{1}{2}$, $s^*$ decreases,

$$s^{*(2)} - s^{*(1)} = \frac{e x (2y - 1)}{\sigma^{(1)} \sigma^{(2)}}.$$ 

Case 3. $\delta_i < \delta, \delta_j > \delta$.

$$y^{(2)} = y^{(1)} + y_i, \quad \sigma^{(2)} = \sigma^{(1)} + \frac{y_i}{1 + \delta_j}, \quad \left(\sum_{\delta_i > \delta} y_i \delta_i\right)^{(2)} = \left(\sum_{\delta_i > \delta} y_i \delta_i\right)^{(1)} + y_i \delta_j,$$

$$s^{*(1)} = \frac{e (2y^{(1)} - 1)}{\sigma^{(1)}} + 3d - 2c, \quad s^{*(2)} = \frac{e (2y^{(2)} - 1)}{\sigma^{(2)}} + 3d - 2c.$$ 

In Case 3, the amount of increase (decrease) in $D_1(D_2)$ is, $y_i \left(1 - \frac{c - d}{e(1 + \delta_j)}\right) > 0$, which is bigger than that of Case 2. Also, $SW_R(s^*)$ always improves, $SW_R(s^*)^{(2)} - SW_R(s^*)^{(1)} = \frac{y_i(1 + \delta_j - \frac{c - d}{e})^2}{(1 + \delta_j)} > 0.$
In Proposition 1, we showed that we can write $\sigma^{(1)} = \frac{y^{(1)}}{1+\bar{b}}$, where $0 \leq \bar{b} \leq 1$.

$$s^{*}(2) - s^{*}(1) = \frac{ey_{i}(1 - 2y^{(1)} + 2\sigma^{(1)}(1 + \bar{\delta}))}{\sigma^{(1)}(y_{i} + \sigma^{(1)} + \bar{\delta}\sigma^{(1)})} = \frac{ey_{i}(1 + \bar{\delta})(1 + \bar{\delta} - 2y^{(1)}\bar{\delta} + 2y^{(1)}\delta_{j})}{y^{(1)}(y^{(1)}(1 + \delta_{j}) + y_{i}(1 + \bar{\delta}))}, (c)$$

The sign of $s^{*}(2) - s^{*}(1)$ depends on the sign of $1 + \bar{\delta} - 2y^{(1)}\bar{\delta} + 2y^{(1)}\delta_{j}$. If $(c)$ is an increasing function of $\bar{\delta}$, by substituting $\bar{\delta} = 0$ into $(c)$, we have $(1) = 1 + 2y^{(1)}\delta_{j} > 0$. Thus, $(c) > 0$ for $\bar{\delta} \geq 0$. If $(c)$ is a decreasing function of $\bar{\delta}$, by substituting $\bar{\delta} = 1$ into $(c)$, we have $(1) = 2(1 - y^{(1)}) + 2y^{(1)}\delta_{j} > 0$. Thus, $(c) > 0$ for $\bar{\delta} \leq 1$. Therefore, for Case(3), $s^{*}(2) - s^{*}(1) \geq 0$ for all parameter values.