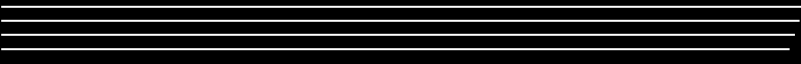




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A Signaling Approach

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Can Mandatory Certification Promote Greenwashing? A Signaling Approach*

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Abstract

This paper examines how positive or negative perceptions about innovation affect firms' strategic behavior when certifying their products. In particular, we consider two types of firm (innovative and non-innovative) which choose between three signals: (1) certified claim, (2) uncertified claim, and (3) no claim. The consumer, either exhibiting positive or negative perceptions, is uninformed about the firm's type and only observes the firm's claim. We find that a separating equilibrium arises in which information about the innovation is revealed to consumers. We also identify a pooling equilibrium in which both types of firm choose the same claim, concealing information from consumers. We show that regulation requiring mandatory certification can hinder information transmission. Our results also indicate that changes in product perceptions do not necessarily facilitate information transmission.

KEYWORDS: Signaling game; innovation; certification; greenwashing.

JEL CLASSIFICATION: D81, D82, L15, Q50.

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1 Introduction

Consumers' perceptions towards innovations are heterogeneous and usually differ to the average scientific opinion; see Messer et al. (2017).¹ Positive or negative perceptions are affected by consumers' beliefs on unobservable safety or health attributes of the product (Bearth and Siegrist, 2016; Siegrist, 2008; Cardello, 2003; Caswell and Mojduszka, 1996), and other environmental, social or ethical factors (Codron et al., 2006).² GMOs illustrate the complexity of how uncertainty about the innovation affects consumer preferences. For instance, Kopicki (2013) reports that 57 percent of survey respondents in the U.S. expressed concerns about GMOs, being health issues and environmental impacts the two major worries. Ganiere et al. (2006) and Yue et al. (2015) find similar results, with more than 30 percent of consumers revealing a negative perception towards GMOs.

Since consumers cannot perfectly observe whether a product uses an innovation (such as GMO ingredients), firms may rely on claims in the product's label to describe the presence or absence of innovation. In such a context, asymmetric information favors the emergence of fraud in the form of greenwashing, that is, a firm making false or misleading claims about the environmental performance of its product; Delmas and Burbano (2011). The practice of greenwashing is neither new nor occasional. Hamilton and Zilberman (2006) report evidence of fraud due to greenwashing in the organic and GMO markets at their early stages. In addition, Terrachoice (2010) finds that 32 percent of a total of 5,296 home and family green products in the U.S. and Canada used false labels, 5 percent more than the previous year. This paper seeks to study how asymmetric information promotes firms' greenwashing behavior and whether the existence of mandatory certification helps to ameliorate such a behavior.

We consider a model with two types of firm (innovative or non-innovative),³ each deciding whether or not to signal to consumers its innovation. Specifically, the firm can either make: (1) a third-party certified claim; (2) a self-reported (uncertified) claim; or (3) no claim. For instance, the firm can claim it uses organic ingredients by acquiring a USDA certification and including it on the product's label; claim that its product uses organic ingredients without a third-party certification (that is, including "organic" in its label without certification); or be silent about whether or not the good is organic. Subsequently, a consumer with either a positive or negative perception about the innovation observes one of the three signals. She updates her beliefs about the firm using the

¹Funk et al. (2015) examine the discrepancies about scientific innovations between U.S. citizens and experts of the American Association for the Advancement of Science (AAAS). For instance, 87 percent of experts from AAAS indicate that climate change is mostly a consequence of human activity, however only half of U.S. adults agree, and only 37 percent sustain it is a grave problem. With regard to genetically modified organisms (GMOs), 88 percent of the experts maintain that are safe, while only 37 percent of Americans believe that GMOs are safe.

²GMOs, nanotechnology, microwave radiation, heat pasteurization and sterilization, among others, are examples of innovations directly influencing perceptions on safety or health attributes; while clean or green technology (i.e., pollution abatement or environmentally friendly production process) influences perceptions on the second group of unobservables.

³According to the United States Department of Commerce (2007), innovation is "the design, invention, development and/or implementation of new or altered products, services, processes, systems, organizational structures, or business models for the purpose of creating new value for customers and financial returns for the firm."

innovation or not, and responds by buying or not buying the product. Our model considers that making no claims is costless for the firm, making an uncertified claim entails some costs (although minor) but can be penalized if consumers sue the company for including false claims in its label, and that acquiring a certification is the most costly signal for the firm.⁴ For generality, we allow for both innovators and non-innovators to acquire certificates, but assume that the latter face a longer certification process and thus experience higher certification costs. When third-party agencies detect the non-innovator’s type during the certification process, acquiring a certification for this type of company is impossible, a setting that our model considers as a special case by making its certification costs infinitely high. We also analyze the effect that a regulation on certified claims produces on the dissemination of information.

We first examine the case in which a consumer has a positive perception about the innovation and, afterwards, extend our setting to the case of negative perceptions. In both contexts we study under which conditions separating and pooling Perfect Bayesian Equilibria (PBEs) are supported. We show that an informative (separating) equilibrium can be sustained when the cost of certification or the penalty from greenwashing are sufficiently high. In particular, under positive (negative) perceptions the innovator (non-innovator) chooses a certified or uncertified claim and the non-innovator (innovator, respectively) does not claim. Regardless of consumers’ perception about the innovation, a lengthy and costly process to obtain the certification, or hard penalties from greenwashing, help to deter a firm that seeks to mimic the claim of its counterpart. We also show the existence of uninformative (pooling) equilibria. We first identify an equilibrium in which both types of firm certify their products, which holds when the certification is easy to obtain or penalties are significantly low. In this context, the non-innovator (innovator) under positive (negative, respectively) perceptions makes false claims.⁵ We also find a pooling equilibrium in which both types choose an uncertified claim or no claim. Both equilibria can be rationalized by a consumer assigning a high probability of facing an innovative firm (e.g., widely used innovation) which induces her to buy the product. As a consequence, the firm can promote purchases by choosing no claims without having to spend on certification.

Our results indicate that, under positive perceptions, when the consumer assigns a higher value to products with the innovation, the uninformative equilibrium is more likely to arise. The non-innovative firm then has incentives to mimic the innovator by choosing a certified or an uncertified claim which conceals its type; at the risk of being caught greenwashing and face future penalties.

⁴Environmental certification costs (monetary fees and time) vary across programs, certifying agencies, and countries. For example, the minimum annual certification fee for the label supported by the Non-GMO Project is approximately US\$1,300, which can be considerably scaled-up as the number of verified products increase and additional services like inspectors are required. On average, the process can take from 3 to 6 months. (For more information, see <https://www.nongmoproject.org/product-verification/technical-administrators/>).

⁵According to Terrachoice (2010), between 2009 and 2010, the consumer electronics industry in North America presented the highest rate of greenwashing compared to other home and family products. Their study reveals that 50 percent of total products surveyed in that category made false claims in regards of toxicity of components, energy efficiency, recycled and biodegradable content, and use of vague environmental jargon. In the food industry, in 2017 the USDA registered 42 complaints related to false organic claims violating the organic regulation. (For more details, see <https://www.ams.usda.gov/services/enforcement/organic/fraudulent-certificates>).

In this context, the consumer receives an uninformative signal, but buys the good since she highly values the innovation. Our findings also indicate that a change in perceptions about innovation does not affect the emergence of pooling equilibria, which still arise when consumers exhibit negative perceptions. We then show that penalties from greenwashing, but not educational programs aimed to change consumers' perceptions, can facilitate information revelation to consumers.

We also consider the effect of a regulation requiring mandatory certification on information transmission. We find that this regulation promotes the informative equilibrium in which the innovative (non-innovative) firm chooses a certified claim while the non-innovative (innovative) chooses no claim when consumers' perceptions are positive (negative, respectively). That is, the cost from failing to comply with the regulation (just a threat, as it is not implemented in equilibrium) reduces the incentives to mimic the other type of firm. In addition, regulation also hinders the existence of the pooling equilibrium in which both types of firm choose uncertified claims, thus helping the dissemination of information. However, regulation can be useless when an informative equilibrium in which only one type of firm chooses an uncertified claim already existed without regulation. In this setting, mandatory certification only adds administrative costs without improving the dissemination of information. Finally, mandatory certification can also be detrimental when both types of firm certify without regulation, since requiring certified claims expands the conditions supporting the uninformative equilibrium, further promoting greenwashing.

Related Literature. Crespi and Marette (2003) examine the role of the label 'contains GMO', finding that it is more effective when the ratio of GMO-averse consumers is high, while the opposite label 'does not contain GMO' is more effective when this ratio is low. Costanigro and Lusk (2014) empirically analyze the signaling effect of labels, showing that labels such as 'contain GMO' vs 'does not contain GMO' may alter beliefs about the likelihood that an unlabeled product is GMO. McCluskey and Winfree (2017) also study firms' incentives to use GMO labels in their products. However, these papers do not examine firms' greenwashing behavior due to asymmetric information.

In the literature analyzing signaling clean technologies, Mahenc (2008) shows that price can act as a signal of high environmental performance to green consumers if a product's marginal cost increases in its environmental performance. Ibanez and Grolleau (2008) propose a duopoly model of vertical product differentiation with endogenous technology and signal choices, allowing for the polluting firm to dishonestly mimic the green firm. They show that a pooling equilibrium in which both green and brown firms use a label arises. While they only consider two possible signals (label or no label), we allow for a richer set of signals (certified claim, uncertified claim, or no claim). From a modeling approach, letting firms choose an uncertified claim is realistic. As shown by Gruere (2013) for the case of environmental claims, different types of labels co-exist; Marconi et al. (2017) provides a similar analysis for the case of fair-trade claims. In addition, we study different consumer's perceptions and the effect of mandatory certification on our equilibrium results.

Atkison and Rosenthal (2014) find that consumers report that both the content in eco-labels and the label source are useful. BreCARD (2014) argues that the profusion of eco-labels (e.g., different third-party agencies, acronyms, and designs) increases consumer confusion, making it difficult for

them to understand which label is better. In the same direction, Harbaugh et al. (2011) show that uncertainty makes labeling and non-labeling equilibria more likely to coexist as the number of labels increases, so consumers face greater strategic uncertainty over how to interpret the presence or absence of a label.

With regards to third-party certification, Hamilton and Zilberman (2006) discuss that a policy increasing certification cost reduces fraud more significantly than policies subsidizing green technologies. However, this paper does not consider the signaling role of certification. In a signaling model, Mason (2011) finds that moderate certification costs reduce greenwashing while sufficiently small certification costs favor the existence of pooling equilibrium. Similarly, Mahenc (2017) studies the misleading certification of eco-labels but from the point of view of the certifier, showing that honest certification requires the third-party certifying company to assign a sufficiently large weight on social welfare. Unlike these papers, we allow firms to make certified, uncertified, or no claims which are used as signals to consumers, we consider consumers exhibiting positive and negative attitudes toward the innovation, and we examine mandatory certification. When our model allows for only two signals (certified claims and no claims), we show that information transmission is hindered, relative to the setting with three signals, and becomes more expensive to implement. However, the uninformative equilibria can arise under larger conditions, thus promoting greenwashing.

The remainder of the paper is organized as follows. The next section presents the time structure of the game and assumptions of the model. Section 3 analyzes the signaling game when the firm, first, faces a consumer with positive preferences towards innovation and, second, considers a consumer with negative preferences. Section 4 examines regulation on certified claims. Finally, section 5 provides a discussion of our results and presents potential extensions for further research.

2 Model

Consider a signaling game between a firm and a consumer. The firm sells a good at a given price, p , where $p > 0$. The firm can be of two types, either innovative (I) if it uses a novel production process, or non-innovative (NI) if it uses a conventional technology. Production costs are type-dependent, $C_i > 0$, where $i \in \{I, NI\}$, and we allow $C_I \geq C_{NI}$ or $C_I < C_{NI}$ for generality.⁶ The label can include: (1) a third-party certification (such as USDA organic); (2) an uncertified claim (such as “organic” without the USDA certification); or (3) no claim (e.g., no information about the organic content of the product).⁷ When consumers exhibit a positive perception towards the innovation, a certified or an uncertified claim implies that the firm acknowledges the innovation, such as organic or Energy Star. However, under negative perceptions, the firm’s claim acknowledges the absence of innovation, such as non-GMOs.

⁶For simplicity, we consider that the innovation cost is sunk, and parameter C_i only captures production cost after the innovation.

⁷If each type could charge a different price, these would become signals to consumers, making any type of claim unnecessary. However, we observe most firms using some form of claim in their product labels, suggesting that claims are common signals in the marketplace. Hence, we focus on the signaling role of claims.

We assume that the cost of each action (per unit sold) is $C_i^{Cert} > C^{UnCert} \geq C^{NoClaim} = 0$. Intuitively, a certification process is more costly than claiming an innovation without the third-party certification, and the latter is at least as costly as not claiming at all in the label (which is, for simplicity, assumed to be costless). Furthermore, $C_I^{Cert} \leq C_{NI}^{Cert}$ implying that the certification process is more expensive for the non-innovative firm than its innovative counterpart.⁸ In contrast, the cost of uncertified claim and no claim are both type-independent. In addition, $p > C_i^{Cert}$ so both types of firm have incentives to certify.

If the product's label includes false information (e.g., the label claims "organic" despite not being so, which is often referred to as "greenwashing"), it faces an expected penalty $K \geq 0$, which is increasing in the probability of being found liable and in the amount of the penalty.

If the firm does not claim, its payoff is $p - C_i$ if the consumer buys, and $-C_i$ if she does not. If the firm chooses a claim without certification, its payoff is $p - C_i - C^{UnCert} - K$ if the consumer buys, and $-C_i - C^{UnCert}$ otherwise. Finally, if the firm certifies, its payoff becomes $p - C_i - C_i^{Cert} - K$ if the consumer buys, and $-C_i - C_i^{Cert}$ otherwise. When the firm makes a claim, both certified and uncertified, the expected penalty is $K > 0$ when the firm greenwashes, but zero when the firm does not lie about its innovation.⁹

The time structure of the game is the following:

1. Nature selects the firm's type (either innovator or non-innovator).
2. The firm privately observes its type, and chooses a certified claim, uncertified claim, or no claim.
3. The consumer does not know the firm's type, but assigns a prior probability q to the firm being innovative. After observing the firm's choice $s = \{Cert, UnCert, NoClaim\}$, she updates her beliefs to $\mu_s \equiv \text{prob}(I|s)$ where $\mu_s \in [0, 1]$. The consumer then responds buying or not buying the product.

Let V_i denote the consumer's valuation of the product, where $i \in \{I, NI\}$ indicates the firm's type. The consumer can exhibit a positive attitude towards innovation if $V_I > p > V_{NI}$, or a negative attitude if $V_I < p < V_{NI}$. Therefore, the expected payoff that the consumer obtains from buying the product is $\mu_s V_I + (1 - \mu_s) V_{NI} - p$, and zero if she does not buy the good regardless of the firm's type. For comparison purposes, the next lemma presents equilibrium behavior in the complete information version of the game where the consumer can perfectly observe the firm's type.

Lemma 1 (Complete information). *Under complete information, both types of firm choose no claim, and the consumer responds buying the product from the innovator (non-innovator) when she exhibits a positive (negative, respectively) perception towards the innovation.*

⁸This condition is true for the case of positive perceptions. However, when we allow for negative perceptions about the innovation (next section), this assumption is reversed. In this case it is more expensive for the innovator to get a certification claiming no innovation than for the non-innovator.

⁹For simplicity, we consider that the firm is subject to penalties only if the consumer buys the product.

Therefore, no type of firm needs to spend resources on certifying its product, or printing an uncertified label, since consumers can perfectly observe whether the firm is innovative or not. As the following section shows, the presence of asymmetric information between firm and consumer leads one or both types of company to spend resources conveying its type to consumers to induce them to buy.

3 Signaling game

We first examine the case of positive perceptions towards the innovation and next discuss the case of negative perceptions. We focus our attention on the separating and pooling equilibrium for each case. All equilibria presented below survive Cho and Kreps' (1987) Intuitive Criterion. Proofs are relegated to the appendix.

3.1 Positive perceptions towards innovation

Let us examine separating equilibria that are supported as a PBE of the game.

Proposition 1. *If the consumer has positive attitudes, the following separating PBEs can be sustained:*

1. *The innovative firm chooses a certified claim and the non-innovative firm does not claim if and only if $p \leq C_{NI}^{Cert} + K$; the consumer after observing a certified claim buys since $\mu_{Cert} = 1$, and she does not buy after observing no claim, $\mu_{NoClaim} = 0$. After observing an uncertified claim (off-the-equilibrium path), she does not buy if and only if beliefs are $\mu_{UnCert} < \bar{\mu} \equiv \frac{p - V_{NI}}{V_I - V_{NI}}$.*
2. *The innovative firm chooses an uncertified claim and the non-innovative firm does not claim if and only if $p \leq C^{UnCert} + K$; the consumer after observing an uncertified claim buys since $\mu_{UnCert} = 1$, and she does not buy after observing no claim, $\mu_{NoClaim} = 0$. After observing a certified claim (off-the-equilibrium path), she buys if and only if $\mu_{Cert} \geq \bar{\mu}$.*

In the first separating equilibrium, the innovative firm has incentives to choose a certified claim. In contrast, the non-innovative firm faces a large greenwashing cost, relative to the market price it would obtain if it were to certify the product. As a result, this firm has no incentives to mimic the innovative firm certifying its claim. In this case, the off-the-equilibrium beliefs induce the consumer to not buy the good when she observes uncertified claims. Hence, the off-the-equilibrium behavior supports the separating equilibrium only when –after observing an uncertified claim– there is no purchase, eliminating any incentive for the firm to deviate. A similar argument applies to the second separating equilibrium, where the innovative firm chooses an uncertified claim while the non-innovative does not claim. This equilibrium can be sustained when the greenwashing cost for the non-innovator (cost from uncertified claim plus potential penalties) is sufficiently large relative

to the market price. In this case, the innovative firm saves the cost of a certified claim which is more expensive than an uncertified one and achieves the purchase of the consumer.

In countries with no third-party certification agency, the first separating PBE cannot arise, leaving only uncertified claims to disseminate information about the firm's innovation; as described in the second separating PBE. Moreover, when greenwashing is heavily penalized, certified and uncertified claims become more informative for consumers, expanding the conditions under which both separating PBEs can be supported. In contrast, when the penalty from greenwashing is nil (or unlikely), the separating PBEs can only be sustained under more restrictive conditions, i.e., when the certification cost is extremely low for the innovator but high for the non-innovator. Our results indicate that the lack of penalties for greenwashing hinders information transmission to consumers about product innovations.

Lemma 2. *Under positive perceptions, the separating strategy profile in which the innovative (non-innovative) firm chooses a certified claim and the non-innovative (innovative, respectively) firm chooses an uncertified claim cannot be sustained as PBE. In addition, the strategy profile in which the innovative firm does not claim and the non-innovative firm makes a certified or an uncertified claim cannot be sustained as PBE.*

Intuitively, the actions from the non-innovator reveal its type in a separating strategy profile, thus inducing consumers to not purchase its product even if it were certified. Therefore, deviating towards no claim saves this type of firm the cost of choosing certified or uncertified claims, providing it with incentives to deviate.

We next analyze the strategy profiles in which both types of firm choose the same strategy.

Proposition 2. *If the consumer has positive attitudes, the following pooling PBEs can be sustained:*

1. *Both types of firm choose a certified claim if and only if $p \geq C_{NI}^{Cert} + K$; the consumer buys only after observing a certified claim if beliefs satisfy $\mu_{Cert} = q \geq \bar{\mu}$ and $\mu_{UnCert}, \mu_{NoClaim} < \bar{\mu}$.*
2. *Both types of firm choose an uncertified claim if and only if $p \geq C^{UnCert} + K$; the consumer buys only after observing uncertified claims if equilibrium beliefs satisfy $\mu_{UnCert} = q \geq \bar{\mu}$, under any off-the-equilibrium beliefs $\mu_{Cert}, \mu_{NoClaim} \in [0, 1]$.*
3. *Both types of firm choose no claim;*
 - i) *the consumer only buys after observing no claim if equilibrium beliefs satisfy $\mu_{NoClaim} = q \geq \bar{\mu}$, under any off-the-equilibrium beliefs $\mu_{Cert}, \mu_{UnCert} \in [0, 1]$;*
 - ii) *the consumer does not buy after observing any claim if $p \geq C^{UnCert} + K$, and if equilibrium beliefs satisfy $\mu_{NoClaim} = q < \bar{\mu}$, and off-the-equilibrium beliefs are $\mu_{Cert}, \mu_{UnCert} < \bar{\mu}$.*

The first pooling PBE, where both innovators and non-innovators certify, can be sustained if the non-innovative firm finds it relatively cheap to certify (i.e., low cost of certification and potential penalties from greenwashing). In this case, the consumer cannot distinguish the type of firm she faces, but buys the product given that innovators are sufficiently likely (high prior q). As a consequence, the emergence of this equilibrium, in which the consumer faces false claims from the non-innovator, could be prevented if certification is significantly costly for the non-innovative firm and/or a high penalty from greenwashing is in place.

In the second pooling PBE both types of firm choose uncertified claims, which can be sustained with a symmetric condition to the first pooling PBE, $p \geq C^{UnCert} + K$, i.e., low cost of uncertified claim and penalties from greenwashing.¹⁰ In the third pooling PBE, both types of firm choose no claim, which is responded with a purchase when innovators are sufficiently likely (see 3i). In this case, the firm has no incentive to make a certified or uncertified claim since it is costly and does not improve its profits. This case can emerge when the cost of greenwashing is sufficiently high and the type of innovation is broadly used, leading consumers to believe that the product contains the innovation despite offering no claims. In case 3ii, in contrast, innovators are unlikely (low priors), inducing consumers to not buy after no claim. Since consumers do not buy the good after observing any other message in this setting, the firm chooses the least expensive option, which is no claim.

Comparative statics. When the positive attitude towards the innovation becomes more significant, as captured by the difference $V_I - V_{NI}$, cutoff $\bar{\mu}$ decreases, thus shrinking the parameter conditions under which the separating equilibrium where the innovative firm certifies its product can be sustained (see Proposition 1). Intuitively, as consumers are more attracted to the innovation, they purchase it under larger conditions. The non-innovative firm then has more incentives to mimic the innovator by choosing a certified or uncertified claim (at the risk of facing future penalties), which conceals its type. In this context, the consumer receives an uninformative signal, but buys the good since its valuation for innovation is high. It implies that an increase in the valuation for green properties¹¹ may promote greenwashing behavior. In contrast, when the certification cost for the non-innovator and its penalties from doing so, $C_{NI}^{Cert} + K$, are sufficiently high, the separating equilibrium in which this firm does not claim can be sustained under larger conditions, and the pooling PBEs in which both firms choose a certified or uncertified claim can be supported under more restrictive settings. If third-party certifying agencies make a thorough investigation of every firm requesting a certification, the non-innovator could never obtain a certification, which is analogous to $C_{NI}^{Cert} \rightarrow \infty$. In that extreme setting, the separating PBEs can be sustained under larger conditions, while the pooling PBEs in Proposition 3 (cases 1 and 2) cannot emerge.

¹⁰However, the second pooling PBE can be supported under two sets of off-the-equilibrium beliefs: those inducing the consumer to not buy the good after any deviation from the uncertified claim, and those that induce no purchase only after no claims. The first case is analogous to that in (1), while in the second case a certification also induces purchase, but entails a larger cost than uncertified claims in equilibrium, ultimately implying that no type of firm has incentives to deviate.

¹¹For instance, the Nielsen Global Survey of Corporate Social Responsibility and Sustainability 2015 indicates that 66 percent of respondents in 60 countries are willing to pay more for sustainable goods, up from 55 percent in 2014, and 50 percent in 2013.

3.2 Negative perceptions towards innovation

In this context, the consumer negatively perceives innovation and, hence, the certification is understood as the absence of such innovation, for instance, non-GMO. In this setting we consider that certification costs are higher for the innovative than the non-innovative firm, that is $C_I^{Cert} \geq C_{NI}^{Cert}$. We first discuss the two separating equilibria that are supported as a PBE.

Proposition 3. *If the consumer has negative attitudes, the following separating PBEs can be sustained:*

1. *The innovative firm does not claim and the non-innovative firm chooses a certified claim if and only if $p \leq C^{UnCert} + K$; the consumer after observing a certified claim buys since $\mu_{Cert} = 0$, and she does not buy after no claim, $\mu_{NoClaim} = 1$. After observing an uncertified claim (off-the-equilibrium path), she does not buy if and only if beliefs satisfy $\mu_{UnCert} > \bar{\mu}$.*
2. *The innovative firm does not claim and the non-innovative firm chooses an uncertified claim if and only if $p \leq C^{UnCert} + K$; the consumer after observing an uncertified claim buys since $\mu_{UnCert} = 0$, and she does not buy after no claim, $\mu_{NoClaim} = 1$. This is supported for any off-the-equilibrium belief $\mu_{Cert} \in [0, 1]$.*

In contrast to the case in which consumers have a positive perception, in this case is the non-innovative firm who chooses a certified (separating 1) or an uncertified (separating 2) claim in order to convey information about the absence of innovation in its product. Negative attitudes towards innovation (i.e., bad opinion about GMOs, artificial sweeteners, etc.) induce the innovator to choose not to claim avoiding the greenwashing penalty.

Lemma 3. *The separating strategy profile in which the innovative (non-innovative) firm chooses a certified claim and the non-innovative (innovative, respectively) firm chooses an uncertified claim cannot be sustained as PBE under negative preferences towards innovation. In addition, the strategy profile in which the innovative firm chooses an uncertified claim and the non-innovative firm chooses a certified claim or no claim cannot be sustained as PBE.*

The dissemination of information about the firm's type induces the innovative firm to deviate from these strategy profiles. In this case a certified or uncertified claim reveals the innovator's type which is responded by a consumer with negative perception with no purchase. Hence, the innovator prefers to save the cost of certification and potential penalties by deviating towards no claim.

Proposition 4. *If the consumer has negative attitudes, the following pooling PBEs can be sustained:*

1. *Both types of firm choose a certified claim if and only if $p \geq C_I^{Cert} + K$; the consumer only buys after observing a certified claim if beliefs satisfy $\mu_{Cert} = q \leq \bar{\mu}$, and $\mu_{UnCert}, \mu_{NoClaim} > \bar{\mu}$.*

2. *Both types of firm choose an uncertified claim if and only if $p \geq C^{UnCert} + K$; the consumer buys only after observing uncertified claims if beliefs satisfy $\mu_{UnCert} = q \leq \bar{\mu}$, and off-the-equilibrium beliefs satisfy: (a) $\mu_{Cert}, \mu_{NoClaim} > \bar{\mu}$; or (b) $\mu_{Cert} \leq \bar{\mu}$ and $\mu_{NoClaim} > \bar{\mu}$.*
3. *Both types of firm choose no claim;*
 - i) *the consumer only buys after observing no claim if equilibrium beliefs satisfy $\mu_{NoClaim} = q \leq \bar{\mu}$, under any off-the-equilibrium beliefs $\mu_{Cert}, \mu_{UnCert} \in [0, 1]$;*
 - ii) *the consumer does not buy after observing any claim if equilibrium beliefs satisfy $\mu_{NoClaim} = q > \bar{\mu}$, and off-the-equilibrium beliefs are $\mu_{Cert}, \mu_{UnCert} > \bar{\mu}$.*

The innovator in the first pooling PBE mimics the non-innovator by making a certified claim, since the potential penalties from greenwashing are relatively low. That is, under negative perceptions, the innovator finds that a sufficiently high price compensates the cost of certification and the cost of lying, achieving the purchase of its good. In addition, the pooling equilibrium in which both types of firm choose an uncertified claim or both types choose no claim coincides with that under positive perceptions. Note that the only difference between the positive and negative case is the consumer’s prior, which for positive (negative) perceptions induces her to buy if the likelihood of facing a innovator is high (low). That is, no matter the consumer’s type (either positive or negative perceptions), a relatively low penalty from greenwashing induces the existence of uninformative equilibria in which the consumer could buy a product with a false claim.

4 Regulation on certified claims

In recent years the debate about whether or not the government should require certified claims for certain types of innovation has heat up. Unlike the European Union, where mandatory GMO labels have been long required, the debate is still ongoing in the US. Proponents of such regulation claim that consumers have “the right to know.” In contrast, opponents argue that a mandatory labeling would impose extra costs to the food industry, and that this “right” can be satisfied by voluntary labels like those from the Non-GMO Project (Bovay and Alston, 2018). We contribute to this debate by showing that mandatory labels may actually be detrimental for information transmission to consumers.

We now consider how this regulation affects the firm’s incentives to certify its product, such as requiring Energy Star or organic certification. Specifically, we assume that firms face an additional cost $\tilde{F} \geq 0$ from failing to comply with the regulation, that is, uncertified claims are now penalized. This cost differs from the greenwashing cost, K , which is a legal cost from being caught lying to consumers. For instance, an innovative firm could choose an uncertified claim, thus not facing greenwashing cost K , but would still bear cost \tilde{F} .

4.1 Regulation with positive attitudes

The following proposition discusses how the separating equilibrium is affected by this type of regulation.

Proposition 5. *If the consumer has positive attitudes and certified claims are required, the following separating PBEs can be sustained:*

1. *The innovative firm chooses a certified claim and the non-innovative firm does not claim if and only if $p \leq C_{NI}^{Cert} + K$; the consumer buys after observing a certified claim since $\mu_{Cert} = 1$, but she does not buy after observing no claim, $\mu_{NoClaim} = 0$. After observing an uncertified claim (off-the-equilibrium path), she buys when $\mu_{UnCert} \geq \bar{\mu}$ but she does not otherwise. No type of firm makes uncertified claims if $\tilde{F} \geq \max \{p - K - C^{UnCert}, C_I^{Cert} - C^{UnCert}\}$.*
2. *The innovative firm chooses an uncertified claim and the non-innovative firm does not claim if and only if $p \leq C^{UnCert} + K$; the consumer buys after observing an uncertified claim since $\mu_{UnCert} = 1$, but she does not buy after observing no claim, $\mu_{NoClaim} = 0$. After observing a certified claim (off-the-equilibrium path), she buys when $\mu_{Cert} \geq \bar{\mu}$ but does not otherwise. No type of firm makes certified claims if and only if $\tilde{F} \leq C_I^{Cert} - C^{UnCert}$ for every belief $\mu_{Cert} \in [0, 1]$.*

Proposition 5.1 differs from Proposition 1.1 since now the separating equilibrium is supported under larger conditions, thus helping the dissemination of information. In particular, the off-the-equilibrium message “uncertified claim” supports the equilibrium when the buyer responds purchasing the product and the penalty from not complying with the regulation is sufficiently high. Therefore, the firm has no incentives to deviate towards uncertified claims. In addition, when consumer’s off-the-equilibrium beliefs after observing an uncertified claim are relatively low, she does not buy, and the innovative firm does not have incentives to make an uncertified claim regardless of the severity of penalty \tilde{F} . In this setting, regulation becomes unnecessary since it does not affect equilibrium behavior. However, if consumer’s off-the-equilibrium beliefs are sufficiently high, only tough non-compliance costs give incentives to the innovative firm to certify its product, thus transmitting more information than when regulation is absent.

In addition, the separating equilibrium in which the innovative firm chooses an uncertified claim while the non-innovative does not claim (Proposition 5.2) is supported under more restrictive conditions than Proposition 1.2. In this case, regulation hinders the existence of this equilibrium, thus limiting information transmission relative to the setting where regulation is absent.

In summary, when the innovative firm certifies while the non-innovative chooses no claim in the absence of regulation, the introduction of compulsory certification is socially desirable. This regulation, however, hinders information dissemination in contexts where firm types were already revealed without the need of certification. In this case, the implementation of regulation yields the same equilibrium behavior but entails unnecessary administrative and monitoring costs.

We next examine the pooling equilibrium under regulation.

Proposition 6. *If the consumer has positive attitudes and certified claims are required, the following pooling PBEs can be sustained:*

1. *Both types of firm choose a certified claim if and only if $p \geq C_{NI}^{Cert} + K$; the consumer buys only after observing a certified claim if beliefs satisfy $\mu_{Cert} = q \geq \bar{\mu}$ and either: (a) $\mu_{UnCert}, \mu_{NoClaim} < \bar{\mu}$; or (b) $\mu_{UnCert} \geq \bar{\mu}$ and $\mu_{NoClaim} < \bar{\mu}$ and $\tilde{F} \geq C_{NI}^{Cert} - C^{UnCert}$.*
2. *Both types of firm choose an uncertified claim if and only if $p \leq C^{UnCert} + K + \tilde{F}$; the consumer buys only after observing uncertified claims if equilibrium beliefs satisfy $\mu_{UnCert} = q \geq \bar{\mu}$, and off-the-equilibrium beliefs satisfy: (a) $\mu_{Cert}, \mu_{NoClaim} < \bar{\mu}$; or (b) $\mu_{Cert} \geq \bar{\mu}$ and $\mu_{NoClaim} < \bar{\mu}$.*
3. *Both types of firm choose no claim;*
 - i) *the consumer only buys after observing no claim if equilibrium beliefs satisfy $\mu_{NoClaim} = q \geq \bar{\mu}$, under any off-the-equilibrium beliefs $\mu_{Cert}, \mu_{UnCert} \in [0, 1]$;*
 - ii) *the consumer does not buy after observing no claim if $p \leq C^{UnCert} + K$, and if equilibrium beliefs satisfy $\mu_{NoClaim} = q < \bar{\mu}$, and either: (a) $\mu_{Cert}, \mu_{UnCert} < \bar{\mu}$; or (b) $\mu_{Cert} < \bar{\mu}$ and $\mu_{UnCert} \geq \bar{\mu}$ and $\tilde{F} \geq K$.*

Relative to Proposition 2.1, regulation expands the conditions under which the pooling equilibrium where both firms make certified claims arises (see Proposition 6.1). Hence, paradoxically, regulation could hinder information transmission, and promote greenwashing by inducing the non-innovative firm to also choose certified claims to avoid the non-compliance cost.

In contrast, regulation shrinks the parameter conditions under which both types of firm choose uncertified claims. Specifically, in the absence of regulation the non-innovative firm chooses uncertified claims as long as prices satisfy $p \geq C^{UnCert} + K$ (including greenwashing costs), while now prices need to be higher to compensate this type of firm for the non-compliance cost. In this case, compulsory certifications facilitate information transmission.

Finally, the pooling equilibrium in which both types of firm make no claim emerges under larger conditions. Intuitively, the non-innovator is now less attracted to deviate towards making uncertified claims since it would face an additional non-compliance cost relative to the context without regulation. Therefore, requiring firms to certify their product expands this pooling equilibrium, ultimately limiting information transmission.

Two signals. When $\tilde{F} \rightarrow +\infty$, uncertified claims become unfeasible, and our results help us predict equilibrium when firms can only choose between two signals (certified claim and no claim). In this context, the informative equilibrium of Proposition 5.2 cannot be sustained. The innovative firm can now only signal its type by acquiring certification, which makes information transmission more expensive. In addition, the uninformative equilibrium of Proposition 6.2 does not arise, which implies that greenwashing occurs under more restrictive conditions.

4.2 Regulation with negative attitudes

We next examine how regulation affects equilibrium behavior when consumers exhibit negative attitudes towards the innovation.

Proposition 7. *If the consumer has negative attitudes and certified claims are required, the following separating PBEs can be sustained:*

1. *The innovative firm does not claim and the non-innovative firm chooses a certified claim if and only if $p \leq C_I^{Cert} + K$; the consumer buys after observing a certified claim since $\mu_{Cert} = 0$, but she does not buy after no claim, $\mu_{NoClaim} = 1$. After observing an uncertified claim (off-the-equilibrium path), she buys when $\mu_{UnCert} \leq \bar{\mu}$ but does not buy otherwise. No type of firm makes uncertified claims if $\tilde{F} \geq \max \{p - K - C^{UnCert}, C_{NI}^{Cert} - C^{UnCert}\}$.*
2. *The innovative firm does not claim and the non-innovative firm chooses an uncertified claim if and only if $C^{UnCert} + \tilde{F} \leq p \leq C^{UnCert} + K + \tilde{F}$; the consumer buys after observing an uncertified claim since $\mu_{UnCert} = 0$, but she does not buy after no claim, $\mu_{NoClaim} = 1$. After observing a certified claim (off-the-equilibrium path), she buys when $\mu_{Cert} \leq \bar{\mu}$ but does not otherwise. No type of firm makes certified claims if $p \leq C_I^{Cert} + K$ and $\tilde{F} \leq C_{NI}^{Cert} - C^{UnCert}$.*

Similarly to the case with positive attitudes, the introduction of regulation expands parameter conditions under which the separating PBE can be sustained, i.e., Proposition 7.1 holds under less demanding conditions than 3.1. Intuitively, the non-innovative firm is less attracted to deviate towards uncertified claims when regulation is present since such a deviation is now punished with non-compliance cost \tilde{F} . Therefore, compulsory certification promotes information transmission helping the consumer avoid the innovation, e.g., GMOs.

In contrast, regulation restricts the range of parameter values for which the non-innovative firm chooses an uncertified claim while the innovative does not claim. Intuitively, the non-innovator needs to face higher prices to compensate the additional (non-compliance) cost of an uncertified claim, whereas the innovator has less incentives to mimic the non-innovator given the punishment from deviating towards an uncertified claim.

We now analyze the pooling equilibrium.

Proposition 8. *If the consumer has negative attitudes and certified claims are required, the following pooling PBEs can be sustained:*

1. *Both types of firm choose a certified claim if and only if $p \geq C_I^{Cert} + K$ and $\tilde{F} \geq C_I^{Cert} - C^{UnCert}$; the consumer only buys after observing a certified claim if beliefs satisfy $\mu_{Cert} = q \leq \bar{\mu}$, and off-the-equilibrium beliefs satisfy: (a) $\mu_{UnCert}, \mu_{NoClaim} > \bar{\mu}$; or (b) $\mu_{UnCert} \leq \bar{\mu}$ and $\mu_{NoClaim} > \bar{\mu}$.*

2. Both types of firm choose an uncertified claim if and only if $p \geq C^{UnCert} + K + \tilde{F}$ and $\tilde{F} \leq C_{NI}^{Cert} - C^{UnCert}$; the consumer buys only after observing uncertified claims if equilibrium beliefs satisfy $\mu_{UnCert} = q \leq \bar{\mu}$, and off-the-equilibrium beliefs satisfy: (a) $\mu_{Cert}, \mu_{NoClaim} > \bar{\mu}$; or (b) $\mu_{Cert} \leq \bar{\mu}$ and $\mu_{NoClaim} > \bar{\mu}$.
3. Both types of firm choose no claim;
 - i) the consumer only buys after observing no claim if equilibrium beliefs satisfy $\mu_{NoClaim} = q \leq \bar{\mu}$, under any off-the-equilibrium beliefs $\mu_{Cert}, \mu_{UnCert} \in [0, 1]$;
 - ii) the consumer does not buy after observing any claim if $\mu_{NoClaim} = q > \bar{\mu}$ and $p \leq C^{UnCert} + K + \tilde{F}$; and if off-the-equilibrium beliefs satisfy: (a) $\mu_{Cert}, \mu_{UnCert} > \bar{\mu}$; or (b) $\mu_{Cert} > \bar{\mu}$ and $\mu_{UnCert} \leq \bar{\mu}$.

In this setting the uninformative equilibrium in which both types of firm decide to certificate is supported under larger conditions. Regulation in this case not only induces the non-innovator to acquire a certificate (for instance, a non-GMO authentication), but also the innovator is now more tempted to mimic the other type by acquiring certification. This is socially inefficient since both types of firm spend resources to acquire certificates under larger conditions, without improving information transmission to consumers. However, regulation shrinks the set of parameter values for which the uninformative PBE in which both types of firm choose uncertified claims can be sustained; and this equilibrium is completely eliminated if the penalty from non-compliance is sufficiently severe, $\tilde{F} > C_{NI}^{Cert} - C^{UnCert}$. In this setting, compulsory certificates successfully reduce greenwashing. Finally, the PBE in which both types of firm decide not to make claims expands with the regulation, which occurs specifically when the non-compliance cost is sufficiently high and prices do not compensate for the costs of certification and greenwashing.

In summary, when a pooling PBE exists in the absence of regulation, the introduction of compulsory certificates only helps shrink the uninformative equilibrium in which both types of firm make uncertified claim, potentially eliminating it; but expands the pooling PBEs in which both firms were already choosing a certified claim or no claims, thus not promoting information transmission.

5 Discussion

Mandatory certification. When consumers exhibit positive (negative) attitudes towards the innovation, our results show that, in the absence of regulation, a separating equilibrium exists in which the innovative (non-innovative) firm certifies its product while the other type of firm does not make a claim. In this setting, certification becomes a clear signal that consumers can use to infer firm types before making their purchases. When regulation is introduced, our findings indicate that this separating equilibrium can be sustained under larger conditions, implying that regulation helps promoting information transmission to consumers. However, we also show that another separating

equilibrium exists where only the innovator (non-innovator) makes an uncertified claim when consumers exhibit positive (negative) attitudes. While firms do not use certified claims in equilibrium, their labeling decisions are successful at disseminating information about the product to consumers. We demonstrate that compulsory certificates in this context actually shrink the parameter conditions for which this separating equilibrium emerges, ultimately limiting information transmission. If penalties from failing to certify are sufficiently severe, our results suggest that this separating PBE cannot be sustained at all, indicating that regulation would completely block information transmission to uninformed customers.

In the case of pooling equilibria where both types of firm certify (or both choose to make no claims), the introduction of regulation expands parameter conditions sustaining these equilibria as, intuitively, it provides stronger incentives for firms certifying their products (making no claims) in the absence of regulation. In contrast, it shrinks the pooling PBE where both types of firm make uncertified claims, since this type of claims are now punished with a fine. Overall, our results suggest that certification laws can promote information transmission, but only in two specific scenarios: (1) when one of the firm types already certifies its products in the absence of regulation (separating equilibrium); and (2) when both types of firm make uncertified claims before regulation (pooling equilibrium). Otherwise, our results show that the introduction of certification laws hinders information transmission. In these contexts, regulation either limits the emergence of separating equilibria where an uncertified product is a sufficiently informative signal to consumers, or expands pooling equilibria where firms can conceal their types from consumers, often leading to greenwashing.

Penalties from greenwashing. In the pooling equilibria of the game, the non-innovator has incentives to make a certified or uncertified claim, thus lying about the characteristics of its product, when the expected penalties from greenwashing are relatively low. For instance, penalties related to false USDA organic claims are usually US\$11,000 per violation and/or the suspension of a product organic certificate.¹² This finding holds both when consumers have a positive perception towards the innovation and when they do not. Therefore, claims such as Energy Star, Green-e, USDA organic, or Rainforest Alliance Certified, are more likely used by all types of firm when penalties are sufficiently low, allowing cheating firms to profit from those claims. Intuitively, the consumer understands that non-innovators may be using the claim, but still buys the product since the probability of facing an innovative company is relatively high. In contrast, when penalties are significant, we should expect the separating equilibrium to emerge under larger conditions, thus allowing certifications to disseminate information about the product. For example, after Volkswagen faced a penalty close to US\$14.7 billion from cheating about its diesel emissions, it is likely that other diesel car manufacturers are now reluctant to falsify their emissions.¹³

Cost of certification. If the cost of certification is considerably higher for non-innovators than

¹²For more information on these penalties see: <https://www.ams.usda.gov/services/enforcement/organic/file-complaint>

¹³As reported by USA Today in April 2017, see <https://www.usatoday.com/story/money/cars/2017/04/14/volkswagen-group-2-liter-diesel-settlement/100464244/>

innovators, our findings indicate that the separating equilibrium arises, where the consumer is perfectly informed about the firm’s type. This setting considers, as an extreme case, industries in which certifying agencies always detect the non-innovating nature of the firm, not providing a certification, which can be modelled by assuming that the non-innovator faces an infinite cost from certifying its product.

In addition, the cost of certifying a product varies significantly across countries and industries, with costs being increasing in the length of the inspection by the third-party agency.¹⁴ Our findings show that countries with higher certification costs help the dissemination of information, as predicted in the separating equilibrium regardless of whether consumers exhibit positive or negative perceptions about the innovation. It remains an empirical question to verify if countries with more expensive certification systems report less cheating from non-organic producers.

Finally, our results help analyze programs that reimburse firms a share of their certification costs, such as the USDA Organic Certification Cost-Share Programs, which reimburse eligible operations up to 75 percent of their certification costs. If this program reduces the certification cost for all types of firm, our findings indicate that the separating (pooling) emerge under more restrictive (larger, respectively) conditions, thus suggesting that the reimbursement program hinders information transmission to consumers.

Innovations that are extensively used. Our results identify a pooling equilibrium in which both types of firm make no claim in its label, which occurs when innovations are extensively used by several companies, such as recycled plastic. In this setting, consumers understand that the innovation is widely used, and respond buying even if it comes with no claim. Anticipating this response, the firm has no need to spend resources in certifying its product. In these cases we should observe no claim about the innovation on the label. For instance, Coca-Cola has committed to double the use of recycled plastic for PET bottles becoming its only input for all packaging by 2025.¹⁵ However, no claim about the use of recycled materials can be found in its current label in the U.S. market.

Change in perceptions. We found that a change in perceptions about innovation –a consumer who had a negative perception about GMOs becomes in favor of it– does not affect the emergence of uninformative (pooling) equilibria. A firm has incentives to make false claims when the greenwashing penalty is low, regardless of consumers’ perceptions. Therefore, consumers becoming more aware about GMOs being safe does not prevent GMO and non-GMO producers from pooling by making the same certified or uncertified claims. As a consequence, expensive educational programs trying to affect consumers’ perceptions about the benefits of GMO products (or other innovations) do not affect information transmission in equilibrium. Instead, only a severe penalty from cheating or a costly certification can facilitate the revelation of information to consumers.

¹⁴For example, the USDA Organic certification and the California Certified Organic Farmers certification may take between 6 and 10 weeks after the site inspection is complete. In addition, Stolze et al. (2012) find that the business costs of organic certification for farmers in 2008 can range from \$1,238 euros in United Kingdom to \$303 euros in Germany or \$462 euros in Italy.

¹⁵For more details see: <https://www.beveragedaily.com/Article/2017/11/07/Coca-Cola-to-double-use-of-recycled-plastic-in-PET-bottles-to-50>

Further Research. Liaukonyte et al. (2015) find that the message “Free of GMOs” on labels may increase the negative perception on GMO goods. An interesting extension of our model could analyze the non-innovator’s incentives to certify its product when such certification lowers the valuation that consumers assign to the innovation, V_I . In addition, our model does not allow the consumer to update her beliefs based on the firm being caught cheating. We could consider a two-period setting in which the consumer observes both the firm’s claim and whether it was caught cheating in the previous period. While the firm being caught cheating fully reveals its type, the fact that it was not caught does not help the consumer perfectly distinguish its type when monitoring is not perfect.

6 Appendix

Proof of Proposition 1

I-type chooses a certified claim and NI-type no claim. Upon observing a certified claim, the consumer updates her beliefs by Bayes’ rule, inferring that the firm must be of I-type, i.e. $\mu_{Cert} = 1$. No claim conveys the opposite information, i.e. $\mu_{NoClaim} = 0$. Off-the-equilibrium beliefs are $\mu_{UnCert} \in [0, 1]$. After observing a certified claim, the consumer buys since $\mu_s V_I + (1 - \mu_s) V_{NI} - p \geq 0$, which yields $V_I \geq p$, where $\mu_s = \mu_{Cert} = 1$; after observing no claim, she does not buy since $\mu_s V_I + (1 - \mu_s) V_{NI} - p \geq 0$, which yields $V_{NI} \geq p$, where $\mu_s = \mu_{NoClaim} = 0$; and she buys after observing the off-the-equilibrium uncertified claim if and only if $\mu_{UnCert} V_I + (1 - \mu_{UnCert}) V_{NI} - p \geq 0$, which implies $\mu_{UnCert} \geq \frac{p - V_{NI}}{V_I - V_{NI}} \equiv \bar{\mu}$. Anticipating consumer’s response, the I-type chooses a certified claim if and only if $p - C_I - C_I^{Cert} \geq -C_I$, implying $p \geq C_I^{Cert}$, which is satisfied by definition, and the firm does not deviate.

- If $\mu_{UnCert} \geq \bar{\mu}$, the I-type firm chooses a certified rather than an uncertified claim if and only if $p - C_I - C_I^{Cert} \geq p - C_I - C^{UnCert}$, which yields $C^{UnCert} \geq C_I^{Cert}$, and does not hold since $C_I^{Cert} > C^{UnCert}$ by definition. Hence, the firm deviates towards an uncertified claim.
- If $\mu_{UnCert} < \bar{\mu}$, the I-type chooses a certified rather than an uncertified claim if and only if $p - C_I - C_I^{Cert} \geq -C_I - C^{UnCert}$, implying $p \geq C_I^{Cert} - C^{UnCert}$, which is satisfied by definition since $p \geq C_I^{Cert}$.

Similarly, the NI-type does not claim if and only if $-C_{NI} \geq p - C_{NI} - C_{NI}^{Cert} - K$, which is satisfied if $p \leq C_{NI}^{Cert} + K$. If $\mu_{UnCert} < \bar{\mu}$, the NI-type does not claim rather than choosing an uncertified claim if and only if $-C_{NI} \geq -C_{NI} - C^{UnCert}$, implying $C^{UnCert} \geq 0$, which holds by definition, and the firm does not deviate. Therefore, this separating equilibrium can be supported as a PBE if and only if $p \leq C_{NI}^{Cert} + K$, and off-the-equilibrium beliefs are $\mu_{UnCert} < \bar{\mu}$.

We next analyze if this PBE survives the *Intuitive Criterion*. The I-type firm has incentives to deviate towards an uncertified claim since $p - C_I - C^{UnCert} \geq p - C_I - C_I^{Cert}$, yielding $C_I^{Cert} \geq C^{UnCert}$, which holds by definition. The NI-type also has incentives to deviate towards an uncertified claim since $p - C_{NI} - C^{UnCert} - K \geq -C_{NI}$, implying $p \geq C^{UnCert} + K$, which

is satisfied by equilibrium conditions. Since both types have incentives to deviate this separating PBE survives the IC.

I-type chooses an uncertified claim and NI-type no claim. Upon observing an uncertified claim, the consumer updates her beliefs to $\mu_{UnCert} = 1$, and if she observes no claim to $\mu_{NoClaim} = 0$. Off-the-equilibrium beliefs are $\mu_{Cert} \in [0, 1]$. After observing an uncertified claim, the consumer buys since $\mu_s V_I + (1 - \mu_s) V_{NI} - p \geq 0$, which yields $V_I \geq p$, where $\mu_s = \mu_{UnCert} = 1$; after observing no claim, she does not buy since $\mu_s V_I + (1 - \mu_s) V_{NI} - p \geq 0$, which implies $V_{NI} \geq p$, where $\mu_s = \mu_{NoClaim} = 0$; and she buys after observing the off-the-equilibrium certified claim if and only if $\mu_{Cert} \geq \bar{\mu}$. Anticipating consumer's response, the I-type firm chooses an uncertified claim if and only if $p - C_I - C^{UnCert} \geq -C_I$, implying $p \geq C^{UnCert}$, which is satisfied by definition, and the firm does not deviate.

- If $\mu_{Cert} \geq \bar{\mu}$, the I-type firm chooses an uncertified claim rather than a certified claim if an only if $p - C_I - C^{UnCert} \geq p - C_I - C_I^{Cert}$, implying $C^{UnCert} \leq C_I^{Cert}$, which holds by definition, and the firm does not deviate.
- If $\mu_{Cert} < \bar{\mu}$, the I-type firm chooses an uncertified claim rather than a certified claim if an only if $p - C_I - C^{UnCert} \geq -C_I - C_I^{Cert}$, which yields $p \geq C^{UnCert} - C_I^{Cert}$, where $C^{UnCert} - C_I^{Cert} < 0$, which is satisfied by definition, and the firm does not deviate.

Similarly, the NI-type does not claim if and only if $-C_{NI} \geq p - C_{NI} - C^{UnCert} - K$, which is satisfied if $p \leq C^{UnCert} + K$.

- If $\mu_{Cert} \geq \bar{\mu}$, the NI-type does not claim rather than choosing a certified claim if an only if $-C_{NI} \geq p - C_{NI} - C_{NI}^{Cert} - K$, which is satisfied if $p \leq C_{NI}^{Cert} + K$.
- If $\mu_{Cert} < \bar{\mu}$, the NI-type firm does not claim rather than choosing a certified claim if an only if $-C_{NI} \geq -C_{NI} - C_{NI}^{Cert}$, yielding $C_{NI}^{Cert} \geq 0$, which holds by definition.

In addition, note that $C_{NI}^{Cert} + K > C^{UnCert} + K$, hence, we require that $p \leq C^{UnCert} + K$. Therefore, this separating equilibrium can be supported as a PBE if and only if $p \leq C^{UnCert} + K$ for any off-the-equilibrium beliefs $\mu_{Cert} \in [0, 1]$.

We next analyze if this PBE survives the *Intuitive Criterion*. The I-type firm does not deviate towards a certified claim since $p - C_I - C^{UnCert} \geq p - C_I - C_I^{Cert}$, yielding $C_I^{Cert} \geq C^{UnCert}$, which holds by definition. The NI-type firm does not deviate towards a certified claim since $p - C_{NI} - C_{NI}^{Cert} - K \geq -C_{NI}$, implying $p \geq C_{NI}^{Cert} + K$, which does not satisfy the equilibrium conditions $p \leq C^{UnCert} + K$, since $C_{NI}^{Cert} > C^{UnCert}$. Therefore, since both types do not have incentives to deviate this separating PBE survives the IC.

I-type chooses a certified claim and NI-type an uncertified claim. Upon observing a certified claim, the consumer updates her beliefs to $\mu_{Cert} = 1$, and if she observes an uncertified claim to $\mu_{UnCert} = 0$. Off-the-equilibrium beliefs are $\mu_{NoClaim} \in [0, 1]$. After observing a certified claim, the consumer buys since $\mu_s V_I + (1 - \mu_s) V_{NI} - p \geq 0$, which yields $V_I \geq p$, where $\mu_s = \mu_{Cert} = 1$; after

observing an uncertified claim, she does not buy since $\mu_s V_I + (1 - \mu_s) V_{NI} - p \geq 0$, which implies $V_{NI} \geq p$, where $\mu_s = \mu_{UnCert} = 0$; and she buys after observing the off-the-equilibrium no claim if and only if $\mu_{NoClaim} V_I + (1 - \mu_{NoClaim}) V_{NI} - p \geq 0$, which implies $\mu_{NoClaim} \geq \frac{p - V_{NI}}{V_I - V_{NI}} \equiv \bar{\mu}$. Anticipating consumer's response, the I-type firm chooses a certified claim if and only if $p - C_I - C_I^{Cert} \geq -C_I - C^{UnCert}$, which is satisfied if $p \geq C_I^{Cert} - C^{UnCert}$.

- If $\mu_{NoClaim} \geq \bar{\mu}$, the I-type firm chooses a certified claim rather than no claim if and only if $p - C_I - C_I^{Cert} \geq p - C_I$, which yields $C_I^{Cert} \leq 0$, and never holds. Hence, the I-type firm has incentives to deviate towards no claim.
- If $\mu_{NoClaim} < \bar{\mu}$, the I-type firm chooses a certified claim rather than no claim if and only if $p - C_I - C_I^{Cert} \geq -C_I$, which implies $p \geq C_I^{Cert}$, which is satisfied by definition, and the firm does not deviate.

Similarly, the NI-type firm chooses an uncertified claim if and only if $-C_{NI} - C^{UnCert} \geq p - C_{NI} - C_{NI}^{Cert} - K$, which is satisfied if $p \leq (C_{NI}^{Cert} - C^{UnCert}) + K$.

- If $\mu_{NoClaim} < \bar{\mu}$, the NI-type firm chooses an uncertified claim rather than no claim if and only if $-C_{NI} - C^{UnCert} \geq -C_{NI}$, implying $C^{UnCert} \leq 0$, which does not hold, and the NI-type firm has incentives to deviate towards no claim.

Therefore, this separating equilibrium cannot be supported as a PBE for any off-the-equilibrium beliefs $\mu_{NoClaim} \in [0, 1]$.

I-type firm chooses an uncertified claim and NI-type a certified claim. Upon observing an uncertified claim, the consumer updates her beliefs to $\mu_{UnCert} = 1$, and if she observes a certified claim to $\mu_{Cert} = 0$. Off-the-equilibrium beliefs are $\mu_{NoClaim} \in [0, 1]$. After observing an uncertified claim, the consumer buys since $\mu_s V_I + (1 - \mu_s) V_{NI} - p \geq 0$, which yields $V_I \geq p$, where $\mu_s = \mu_{UnCert} = 1$; after observing a certified claim, she does not buy since $\mu_s V_I + (1 - \mu_s) V_{NI} - p \geq 0$, which implies $V_{NI} \geq p$, where $\mu_s = \mu_{Cert} = 0$; and she buys after observing the off-the-equilibrium no claim if and only if $\mu_{NoClaim} \geq \bar{\mu}$. Anticipating consumer's response, the I-type chooses an uncertified claim if and only if $p - C_I - C^{UnCert} \geq -C_I - C_I^{Cert}$, which implies $p \geq C^{UnCert} - C_I^{Cert}$, where $C^{UnCert} - C_I^{Cert} < 0$, and the firm does not deviate.

- If $\mu_{NoClaim} \geq \bar{\mu}$, the I-type firm chooses an uncertified claim rather than no claim if and only if $p - C_I - C^{UnCert} \geq p - C_I$, yielding $C^{UnCert} \leq 0$, which does not hold by definition. Hence, the I-type firm has incentives to deviate towards no claim.
- If $\mu_{NoClaim} < \bar{\mu}$, the I-type chooses an uncertified claim rather than no claim if and only if $p - C_I - C^{UnCert} \geq -C_I$, implying $p \geq C^{UnCert}$, which holds by definition, and the firm does not deviate.

Similarly, the NI-type firm chooses a certified claim if and only if $-C_{NI} - C_{NI}^{Cert} \geq p - C_{NI} - C^{UnCert} - K$, which yields $p \leq (C^{UnCert} - C_{NI}^{Cert}) + K$.

- If $\mu_{NoClaim} < \bar{\mu}$, the NI-type chooses a certified claim rather than no claim if and only if $-C_{NI} - C_{NI}^{Cert} \geq -C_{NI}$, implying $C_{NI}^{Cert} \leq 0$, which does not hold. Hence, the NI-type has incentives to deviate. Therefore, this separating equilibrium cannot be supported as a PBE for any off-the-equilibrium beliefs $\mu_{NoClaim} \in [0, 1]$.

I-type firm chooses no claim and NI-type a certified claim. Upon observing no claim, the consumer updates her beliefs to $\mu_{NoClaim} = 1$, and if she observes a certified claim to $\mu_{Cert} = 0$. Off-the-equilibrium beliefs are $\mu_{UnCert} \in [0, 1]$. After observing no claim, the consumer buys since $\mu_s V_I + (1 - \mu_s) V_{NI} - p \geq 0$, which yields $V_I \geq p$, where $\mu_s = \mu_{NoClaim} = 1$; after observing a certified claim, she does not buy since $\mu_s V_I + (1 - \mu_s) V_{NI} - p \geq 0$, which implies $V_{NI} \geq p$, where $\mu_s = \mu_{Cert} = 0$; and she buys after observing the off-the-equilibrium uncertified claim if and only if $\mu_{UnCert} \geq \bar{\mu}$. Anticipating consumer's response, the I-type does not claim if and only if $p - C_I \geq -C_I - C_I^{Cert}$, implying $p \geq -C_I^{Cert}$, and the firm does not deviate.

- If $\mu_{UnCert} \geq \bar{\mu}$, the I-type firm chooses no claim rather than an uncertified claim if and only if $p - C_I \geq p - C_I - C^{UnCert}$, yielding $C^{UnCert} \geq 0$, and the firm does not deviate.
- If $\mu_{UnCert} < \bar{\mu}$, the I-type chooses no claim rather than an uncertified claim if and only if $p - C_I \geq -C_I - C^{UnCert}$, implying $p \geq -C^{UnCert}$, and the firm does not deviate.

Similarly, the NI-type firm chooses a certified claim if and only if $-C_{NI} - C_{NI}^{Cert} \geq p - C_{NI}$, yielding $p \leq -C_{NI}^{Cert}$, which never holds by definition. Hence, the NI-type firm has incentives to deviate towards no claim. Therefore, this separating equilibrium cannot be supported as a PBE for any off-the-equilibrium beliefs $\mu_{NoClaim} \in [0, 1]$.

I-type firm chooses no claim and NI-type an uncertified claim. Upon observing no claim, the consumer updates her beliefs to $\mu_{NoClaim} = 1$, and if she observes an uncertified claim to $\mu_{UnCert} = 0$. Off-the-equilibrium beliefs are $\mu_{Cert} \in [0, 1]$. After observing no claim, the consumer buys since $\mu_s V_I + (1 - \mu_s) V_{NI} - p \geq 0$, which yields $V_I \geq p$, where $\mu_s = \mu_{NoClaim} = 1$; after observing an uncertified claim, she does not buy since $\mu_s V_I + (1 - \mu_s) V_{NI} - p \geq 0$, which implies $V_{NI} \geq p$, where $\mu_s = \mu_{UnCert} = 0$; and she buys after observing the off-the-equilibrium certified claim if and only if $\mu_{Cert} \geq \bar{\mu}$. Anticipating consumer's response, the I-type does not claim if and only if $p - C_I \geq -C_I - C^{UnCert}$, implying $p \geq -C^{UnCert}$, and the firm does not deviate.

- If $\mu_{Cert} \geq \bar{\mu}$, the I-type does not claim rather than choosing a certified claim if and only if $p - C_I \geq p - C_I - C_I^{Cert}$, yielding $C_I^{Cert} \geq 0$, and the firm does not deviate.
- If $\mu_{Cert} < \bar{\mu}$, the I-type does not claim rather than choosing a certified claim if and only if $p - C_I \geq -C_I - C_I^{Cert}$, implying $p \geq -C_I^{Cert}$, and the firm does not deviate.

Similarly, the NI-type firm chooses an uncertified claim if and only if $-C_{NI} - C^{UnCert} \geq p - C_{NI}$, yielding $p \leq -C^{UnCert}$, which cannot be satisfied. Hence, the NI-type has incentives to deviate towards no claim. Therefore, this separating equilibrium cannot be supported as a PBE for any off-the-equilibrium beliefs $\mu_{Cert} \in [0, 1]$.

Proof of Proposition 2

Both types choose a certified claim. Upon observing the equilibrium message, the consumer cannot further update her beliefs about the firm's type, yielding $\mu_{Cert} = q$ in equilibrium, and $\mu_s = \{\mu_{UnCert}, \mu_{NoClaim}\} \in [0, 1]$ off-the-equilibrium. After observing a certified claim, the consumer buys if and only if $qV_I + (1 - q)V_{NI} - p \geq 0$, which yields $q \geq \frac{p - V_{NI}}{V_I - V_{NI}} \equiv \bar{\mu}$; after observing an uncertified claim, she buys if and only if $\mu_{UnCert}V_I + (1 - \mu_{UnCert})V_{NI} - p \geq 0$, which implies $\mu_{UnCert} \geq \frac{p - V_{NI}}{V_I - V_{NI}} \equiv \bar{\mu}$; and after observing no claim, she buys if and only if $\mu_{NoClaim}V_I + (1 - \mu_{NoClaim})V_{NI} - p \geq 0$, which yields $\mu_{NoClaim} \geq \frac{p - V_{NI}}{V_I - V_{NI}} \equiv \bar{\mu}$.

We analyze eight different cases depending on consumers' beliefs:

Case 1: Consumer's beliefs are $q, \mu_{UnCert}, \mu_{NoClaim} \geq \bar{\mu}$, that is, she buys regardless of the message she observes.

Anticipating consumer's response, the I-type chooses a certified rather than an uncertified claim if and only if $p - C_I - C_I^{Cert} \geq p - C_I - C^{UnCert}$, yielding $C^{UnCert} \geq C_I^{Cert}$, which does not hold by definition; and the I-type firm has incentives to deviate towards an uncertified claim. Therefore, this pooling equilibrium in case 1 cannot be supported as a PBE.

Case 2: Consumer's beliefs are $q, \mu_{NoClaim} \geq \bar{\mu}$, and $\mu_{UnCert} < \bar{\mu}$, that is, she buys after observing a certified claim or no claim, and she does not buy otherwise.

Anticipating consumer's response, the I-type firm chooses a certified rather than an uncertified claim if and only if $p - C_I - C_I^{Cert} \geq -C_I - C^{UnCert}$, which is satisfied if $p \geq C_I^{Cert} - C^{UnCert}$.

The I-type chooses a certified claim rather than no claim if and only if $p - C_I - C_I^{Cert} \geq p - C_I$, yielding $C_I^{Cert} \leq 0$, which is not satisfied; and the I-type firm has incentives to deviate towards no claim. Hence, this pooling equilibrium in case 2 cannot be sustained as a PBE.

Case 3: Consumer's beliefs are $q < \bar{\mu}$, and $\mu_{UnCert}, \mu_{NoClaim} \geq \bar{\mu}$, that is, she does not buy after observing a certified claim, and buys otherwise.

Anticipating consumer's response, the I-type chooses a certified rather than an uncertified claim if and only if $-C_I - C_I^{Cert} \geq p - C_I - C^{UnCert}$, implying $p \leq C^{UnCert} - C_I^{Cert}$, which is not satisfied since $C^{UnCert} - C_I^{Cert} < 0$; and the I-type has incentives to deviate towards an uncertified claim. Therefore, this pooling equilibrium in case 3 cannot be supported as a PBE.

Case 4: Consumer's beliefs are $q, \mu_{UnCert} < \bar{\mu}$, and $\mu_{NoClaim} \geq \bar{\mu}$, that is, she does not buy after observing a certified or an uncertified claim, and buys otherwise.

Anticipating consumer's response, the I-type firm chooses a certified rather than an uncertified claim if and only if $-C_I - C_I^{Cert} \geq -C_I - C^{UnCert}$, yielding $C^{UnCert} \geq C_I^{Cert}$, which does not hold by definition; and the I-type has incentives to deviate towards an uncertified claim. Hence, this pooling equilibrium in case 4 cannot be sustained as a PBE.

Case 5: Consumer's beliefs are $q, \mu_{UnCert} \geq \bar{\mu}$, and $\mu_{NoClaim} < \bar{\mu}$, that is, she buys after observing a certified or an uncertified claim, and does not buy otherwise.

Anticipating consumer's response, the I-type chooses a certified rather than an uncertified claim if and only if $p - C_I - C_I^{Cert} \geq p - C_I - C^{UnCert}$, implying $C^{UnCert} \geq C_I^{Cert}$, which is not satisfied by definition; and the I-type firm has incentives to deviate towards an uncertified claim. Therefore,

this pooling equilibrium in case 5 cannot be supported as a PBE.

Case 6: Consumer's beliefs are $q \geq \bar{\mu}$, and $\mu_{UnCert}, \mu_{NoClaim} < \bar{\mu}$, that is, she buys after observing a certified claim, and does not buy otherwise.

Anticipating consumer's response, the I-type firm chooses a certified rather than an uncertified claim if and only if $p - C_I - C_I^{Cert} \geq -C_I - C^{UnCert}$, which is satisfied if $p \geq C_I^{Cert} - C^{UnCert}$.

The I-type chooses a certified claim rather than no claim if and only if $p - C_I - C_I^{Cert} \geq -C_I$, yielding $p \geq C_I^{Cert}$, which holds by definition; and the I-type firm does not deviate.

Similarly, the NI-type firm chooses a certified rather than an uncertified claim if and only if $p - C_{NI} - C_{NI}^{Cert} - K \geq -C_{NI} - C^{UnCert}$, which is satisfied if $p \geq (C_{NI}^{Cert} - C^{UnCert}) + K$.

The NI-type chooses a certified claim rather than no claim if and only if $p - C_{NI} - C_{NI}^{Cert} - K \geq -C_{NI}$, which holds if $p \geq C_{NI}^{Cert} + K$. Hence, since $C_{NI}^{Cert} + K > (C_{NI}^{Cert} - C^{UnCert}) + K > (C_I^{Cert} - C^{UnCert})$, this pooling equilibrium in case 6 can be sustained as a PBE if and only if $p \geq C_{NI}^{Cert} + K$.

Case 7: Consumer's beliefs are $q, \mu_{NoClaim} < \bar{\mu}$, and $\mu_{UnCert} \geq \bar{\mu}$, that is, she does not buy after observing a certified claim or no claim, and buys otherwise.

Anticipating consumer's response, the I-type chooses a certified rather than an uncertified claim if and only if $-C_I - C_I^{Cert} \geq p - C_I - C^{UnCert}$, which implies $p \leq C^{UnCert} - C_I^{Cert}$, where $C^{UnCert} - C_I^{Cert} < 0$; and the I-type has incentives to deviate towards an uncertified claim. Therefore, this pooling equilibrium in case 7 cannot be supported as a PBE.

Case 8: Consumer's beliefs are $q, \mu_{UnCert}, \mu_{NoClaim} < \bar{\mu}$, that is, she does not buy regardless of the message she observes.

Anticipating consumer's response, the I-type chooses a certified rather than an uncertified claim if and only if $-C_I - C_I^{Cert} \geq -C_I - C^{UnCert}$, yielding $C^{UnCert} \geq C_I^{Cert}$, which does not hold by definition; and the I-type firm has incentives to deviate towards an uncertified claim. Hence, this pooling equilibrium in case 8 cannot be sustained as a PBE.

Both types choose an uncertified claim. Upon observing the equilibrium message, the consumer cannot further update her beliefs about the firm's type, yielding $\mu_{UnCert} = q$ in equilibrium, and $\mu_s = \{\mu_{Cert}, \mu_{NoClaim}\} \in [0, 1]$ off-the-equilibrium. After observing a uncertified claim, the consumer buys if and only if $q \geq \bar{\mu}$; after observing a certified claim, she buys if and only if $\mu_{Cert} \geq \bar{\mu}$; and after observing no claim, she buys if and only if $\mu_{NoClaim} \geq \bar{\mu}$.

We analyze eight different cases depending on consumers' beliefs:

Case 1: Consumer's beliefs are $q, \mu_{Cert}, \mu_{NoClaim} \geq \bar{\mu}$, that is, she buys regardless of the message she observes.

Anticipating consumer's response, the I-type chooses an uncertified rather than a certified claim if and only if $p - C_I - C^{UnCert} \geq p - C_I - C_I^{Cert}$, yielding $C_I^{Cert} \geq C^{UnCert}$, which holds by definition; and the I-type does not deviate. The I-type chooses an uncertified claim rather than no claim if and only if $p - C_I - C^{UnCert} \geq p - C_I$, implying $C^{UnCert} \leq 0$, and the I-type firm has incentives to deviate towards no claim. Therefore, this pooling equilibrium in case 1 cannot be supported as a PBE.

Case 2: Consumer's beliefs are $q, \mu_{NoClaim} \geq \bar{\mu}$, and $\mu_{Cert} < \bar{\mu}$, that is, she buys after observing an uncertified claim or no claim, and does not buy otherwise.

Anticipating consumer's response, the I-type chooses an uncertified rather than a certified claim if and only if $p - C_I - C^{UnCert} \geq -C_I - C_I^{Cert}$, yielding $p \geq C^{UnCert} - C_I^{Cert}$, and the I-type firm does not deviate. The I-type firm chooses an uncertified claim rather than no claim if and only if $p - C_I - C^{UnCert} \geq p - C_I$, implying $C^{UnCert} \leq 0$, and I-type firm has incentives to deviate towards no claim. Hence, this pooling equilibrium in case 2 cannot be sustained as a PBE.

Case 3: Consumer's beliefs are $q < \bar{\mu}$, and $\mu_{Cert}, \mu_{NoClaim} \geq \bar{\mu}$, that is, she does not buy after observing an uncertified claim, and buys otherwise.

Anticipating consumer's response, the I-type firm chooses an uncertified rather than a certified claim if and only if $-C_I - C^{UnCert} \geq p - C_I - C_I^{Cert}$, which is satisfied if $p \leq C_I^{Cert} - C^{UnCert}$.

The I-type chooses an uncertified claim rather than no claim if and only if $-C_I - C^{UnCert} \geq p - C_I$, yielding $p \leq -C^{UnCert}$, and the I-type firm has incentives to deviate towards no claim. Therefore, this pooling equilibrium in case 3 cannot be supported as a PBE.

Case 4: Consumer's beliefs are $q, \mu_{Cert} < \bar{\mu}$, and $\mu_{NoClaim} \geq \bar{\mu}$, that is, she does not buy after observing an uncertified claim or a certified claim, and buys otherwise.

Anticipating consumer's response, the I-type chooses an uncertified rather than a certified claim if and only if $-C_I - C^{UnCert} \geq -C_I - C_I^{Cert}$, implying $C_I^{Cert} \geq C^{UnCert}$, which holds by definition; and the I-type does not deviate. The I-type chooses an uncertified claim rather than no claim if and only if $-C_I - C^{UnCert} \geq p - C_I$, yielding $p \leq -C^{UnCert}$, which is not satisfied by definition; and the I-type firm has incentives to deviate towards no claim. Hence, this pooling equilibrium in case 4 cannot be sustained as a PBE.

Case 5: Consumer's beliefs are $q, \mu_{Cert} \geq \bar{\mu}$, and $\mu_{NoClaim} < \bar{\mu}$, that is, she buys after observing an uncertified claim or a certified claim, and does not buy otherwise.

Anticipating consumer's response, the I-type chooses an uncertified rather than a certified claim if and only if $p - C_I - C^{UnCert} \geq p - C_I - C_I^{Cert}$, implying $C_I^{Cert} \geq C^{UnCert}$, and the I-type firm does not deviate. The I-type chooses an uncertified claim rather than no claim if and only if $p - C_I - C^{UnCert} \geq -C_I$, yielding $p \geq C^{UnCert}$, which is satisfied by definition; and the I-type firm does not deviate. Similarly, the NI-type chooses an uncertified rather than a certified claim if and only if $p - C_{NI} - C^{UnCert} - K \geq p - C_{NI} - C_{NI}^{Cert} - K$, implying $C_{NI}^{Cert} \geq C^{UnCert}$, which holds by definition; and NI-type firm does not deviate. The NI-type chooses an uncertified claim rather than no claim if and only if $p - C_{NI} - C^{UnCert} - K \geq -C_{NI}$, which is satisfied if $p \geq C^{UnCert} + K$. Therefore, this pooling equilibrium in case 5 can be supported as a PBE if and only if $p \geq C^{UnCert} + K$.

Case 6: Consumer's beliefs are $q \geq \bar{\mu}$, and $\mu_{Cert}, \mu_{NoClaim} < \bar{\mu}$, that is, she buys after observing an uncertified claim, and does not buy otherwise.

Anticipating consumer's response, the I-type chooses an uncertified rather than a certified claim if and only if $p - C_I - C^{UnCert} \geq -C_I - C_I^{Cert}$, yielding $p \geq C^{UnCert} - C_I^{Cert}$, where $C^{UnCert} - C_I^{Cert} < 0$; and the I-type firm does not deviate. The I-type chooses an uncertified claim rather than no

claim if and only if $p - C_I - C^{UnCert} \geq -C_I$, implying $p \geq C^{UnCert}$, which holds by definition; and the I-type firm does not deviate. Similarly, the NI-type chooses an uncertified rather than a certified claim if and only if $p - C_{NI} - C^{UnCert} - K \geq -C_{NI} - C_{NI}^{Cert}$, which is satisfied if $p \geq (C^{UnCert} - C_{NI}^{Cert}) + K$. The NI-type chooses an uncertified claim rather than no claim if and only if $p - C_{NI} - C^{UnCert} - K \geq -C_{NI}$, which holds if $p \geq C^{UnCert} + K$. Hence, since $C^{UnCert} + K > (C^{UnCert} - C_I^{Cert}) + K$, this pooling equilibrium in case 6 can be sustained as a PBE if and only if $p \geq C^{UnCert} + K$.

Case 7: Consumer's beliefs are $q, \mu_{NoClaim} < \bar{\mu}$, and $\mu_{Cert} \geq \bar{\mu}$, that is, she does not buy after observing an uncertified claim or no claim, and buys otherwise.

Anticipating the consumer's response, the I-type chooses an uncertified rather than a certified claim if and only if $-C_I - C^{UnCert} \geq p - C_I - C_I^{Cert}$, implying $p \leq C_I^{Cert} - C^{UnCert}$, which is not satisfied because $p \geq C_I^{Cert}$ by definition; and the I-type firm has incentives to deviate towards a certified claim. Therefore, this pooling equilibrium in case 7 cannot be supported as a PBE.

Case 8: Consumer's beliefs are $q, \mu_{Cert}, \mu_{NoClaim} < \bar{\mu}$, that is, she does not buy regardless of the message she observes. Anticipating consumer's response, the I-type chooses an uncertified rather than a certified claim if and only if $-C_I - C^{UnCert} \geq -C_I - C_I^{Cert}$, yielding $C_I^{Cert} \geq C^{UnCert}$, which holds by definition; and the I-type firm does not deviate. The I-type chooses an uncertified claim rather than no claim if and only if $-C_I - C^{UnCert} \geq -C_I$, implying $C^{UnCert} \leq 0$, and the I-type firm has incentives to deviate towards no claim. Hence, this pooling equilibrium in case 8 cannot be sustained as a PBE.

Both types do not claim. Upon observing the equilibrium message, the consumer cannot further update her beliefs about the firm's type, yielding $\mu_{NoClaim} = q$ in equilibrium, and $\mu_s = \{\mu_{Cert}, \mu_{UnCert}\} \in [0, 1]$ off-the-equilibrium. After observing no claim, she buys if and only if $q \geq \bar{\mu}$; after observing a certified claim, she buys if and only if $\mu_{Cert} \geq \bar{\mu}$; and after observing an uncertified claim, she buys if and only if $\mu_{UnCert} \geq \bar{\mu}$.

We analyze eight different cases depending on consumers' beliefs:

Case 1: Consumer's beliefs are $q, \mu_{Cert}, \mu_{UnCert} \geq \bar{\mu}$, that is, she buys regardless of the message she observes.

Anticipating consumer's response, the I-type chooses no claim rather than a certified claim if and only if $p - C_I \geq p - C_I - C_I^{Cert}$, yielding $C_I^{Cert} \geq 0$, which is satisfied by definition; and the I-type does not deviate. The I-type chooses no claim rather than an uncertified claim if and only if $p - C_I \geq p - C_I - C^{UnCert}$, implying $C^{UnCert} \geq 0$, and the I-type does not deviate. Similarly, the NI-type chooses no claim rather than a certified claim if and only if $p - C_{NI} \geq p - C_{NI} - C_{NI}^{Cert} - K$, yielding $C_{NI}^{Cert} + K \geq 0$, and the NI-type firm does not deviate. The NI-type chooses no claim rather than an uncertified claim if and only if $p - C_{NI} \geq p - C_{NI} - C^{UnCert} - K$, implying $C^{UnCert} + K \geq 0$, and the NI-type firm does not deviate. Therefore, this pooling equilibrium in case 1 can be supported as a PBE.

Case 2: Consumer's beliefs are $q, \mu_{UnCert} \geq \bar{\mu}$, and $\mu_{Cert} < \bar{\mu}$, that is, she buys after observing no claim or an uncertified claim, and does not buy otherwise.

Anticipating consumer's response, the I-type chooses no claim rather than a certified claim if and only if $p - C_I \geq -C_I - C_I^{Cert}$, implying $p \geq -C_I^{Cert}$, and the I-type firm does not deviate. The I-type chooses no claim rather than an uncertified claim if and only if $p - C_I \geq p - C_I - C^{UnCert}$, yielding $C^{UnCert} \geq 0$, and the I-type firm does not deviate. Similarly, the NI-type chooses no claim rather than a certified claim if and only if $p - C_{NI} \geq -C_{NI} - C_{NI}^{Cert}$, implying $p \geq -C_{NI}^{Cert}$, and the NI-type firm does not deviate. The NI-type chooses no claim rather than an uncertified claim if and only if $p - C_{NI} \geq p - C_{NI} - C^{UnCert} - K$, yielding $C^{UnCert} + K \geq 0$, and the NI-type firm does not deviate. Hence, this pooling equilibrium in case 2 can be sustained as a PBE.

Case 3: Consumer's beliefs are $q < \bar{\mu}$, and $\mu_{Cert}, \mu_{UnCert} \geq \bar{\mu}$, that is, she does not buy after observing no claim, and buys otherwise.

Anticipating consumer's response, the I-type chooses no claim rather than a certified claim if and only if $-C_I \geq p - C_I - C_I^{Cert}$, implying $p \leq C_I^{Cert}$, which is not satisfied; and the I-type firm deviates towards a certified claim. Therefore, this pooling equilibrium in case 3 cannot be supported as a PBE.

Case 4: Consumer's beliefs are $q, \mu_{Cert} < \bar{\mu}$, and $\mu_{UnCert} \geq \bar{\mu}$, that is, she does not buy after observing no claim or a certified claim, and buys otherwise.

Anticipating the consumer's response, the I-type chooses no claim rather than a certified claim if and only if $-C_I \geq -C_I - C_I^{Cert}$, yielding $C_I^{Cert} \geq 0$, and the I-type firm does not deviate. The I-type chooses no claim rather than an uncertified claim if and only if $-C_I \geq p - C_I - C^{UnCert}$, implying $p \leq C^{UnCert}$, and the I-type firm deviates towards an uncertified claim. Hence, this pooling equilibrium in case 4 cannot be supported as a PBE.

Case 5: Consumer's beliefs are $q, \mu_{Cert} \geq \bar{\mu}$, and $\mu_{UnCert} < \bar{\mu}$, that is, she buys after observing no claim or a certified claim, and does not buy otherwise.

Anticipating consumer's response, the I-type chooses no claim rather than a certified claim if and only if $p - C_I \geq p - C_I - C_I^{Cert}$, yielding $C_I^{Cert} \geq 0$, and the I-type firm does not deviate. The I-type chooses no claim rather than an uncertified claim if and only if $p - C_I \geq -C_I - C^{UnCert}$, implying $p \geq -C^{UnCert}$, and the I-type firm does not deviate. Similarly, the NI-type chooses no claim rather than a certified claim if and only if $p - C_{NI} \geq p - C_{NI} - C_{NI}^{Cert} - K$, yielding $C_{NI}^{Cert} + K \geq 0$, and the NI-type firm does not deviate. The NI-type chooses no claim rather than an uncertified claim if and only if $p - C_{NI} \geq -C_{NI} - C^{UnCert}$, implying $p \geq -C^{UnCert}$, and the NI-type firm does not deviate. Therefore, this pooling equilibrium in case 5 can be sustained as a PBE.

Case 6: Consumer's beliefs are $q \geq \bar{\mu}$, and $\mu_{Cert}, \mu_{UnCert} < \bar{\mu}$, that is, she buys after observing no claim, and does not buy otherwise.

Anticipating consumer's response, the I-type chooses no claim rather than a certified claim if and only if $p - C_I \geq -C_I - C_I^{Cert}$, yielding $p \geq -C_I^{Cert}$, and the I-type firm does not deviate. The I-type chooses no claim rather than an uncertified claim if and only if $p - C_I \geq -C_I - C^{UnCert}$, implying $p \geq -C^{UnCert}$, and the I-type firm does not deviate. Similarly, the NI-type firm chooses no claim rather than a certified claim if and only if $p - C_{NI} \geq -C_{NI} - C_{NI}^{Cert}$, yielding $p \geq -C_{NI}^{Cert}$,

and the NI-type does not deviate. The NI-type chooses no claim rather than an uncertified claim if and only if $p - C_{NI} \geq -C_{NI} - C^{UnCert}$, implying $p \geq -C^{UnCert}$, and the NI-type firm does not deviate. Hence, this pooling equilibrium in case 6 can be supported as a PBE.

Case 7: Consumer's beliefs are $q, \mu_{UnCert} < \bar{\mu}$, and $\mu_{Cert} \geq \bar{\mu}$, that is, she does not buy after observing no claim or an uncertified claim, and buys otherwise.

Anticipating consumer's response, the I-type chooses no claim rather than a certified claim if and only if $-C_I \geq p - C_I - C_I^{Cert}$, yielding $p \leq C_I^{Cert}$, which does not hold by definition; and the I-type firm has incentives to deviate towards a certified claim. Therefore, this pooling equilibrium in case 7 cannot be sustained as a PBE.

Case 8: Consumer's beliefs are $q, \mu_{Cert}, \mu_{UnCert} < \bar{\mu}$, that is, she does not buy regardless of the message she observes.

Anticipating consumer's response, the I-type chooses no claim rather than a certified claim if and only if $-C_I \geq -C_I - C_I^{Cert}$, implying $C_I^{Cert} \geq 0$, and the I-type firm does not deviate. The I-type chooses no claim rather than an uncertified claim if and only if $-C_I \geq -C_I - C^{UnCert}$, yielding $C^{UnCert} \geq 0$, and the I-type firm does not deviate. Similarly, the NI-type chooses no claim rather than a certified claim if and only if $-C_{NI} \geq -C_{NI} - C_{NI}^{Cert}$, implying $C_{NI}^{Cert} \geq 0$, and the NI-type firm does not deviate. The NI-type chooses no claim rather than an uncertified claim if and only if $-C_{NI} \geq -C_{NI} - C^{UnCert}$, yielding $C^{UnCert} \geq 0$, and the NI-type firm does not deviate. Hence, this pooling equilibrium in case 8 can be supported as a PBE.

Intuitive Criterion (IC)

Case 1. In this pooling PBE, both firms choose to certify, so the most profitable deviation among all off-the-equilibrium messages is towards no claim. We next analyze which type of firm, if any, has incentives to deviate towards no claim. First, the innovator obtains, at most, a profit of $p - C_I$ from no claim, which exceeds its equilibrium payoff of $p - C_I - C_I^{Cert}$, since $C_I^{Cert} \geq 0$; implying that this type of firm has incentives to deviate. Second, the non-innovator obtains, at most, a profit of $p - C_{NI}$ from no claim, which exceeds its equilibrium payoff of $p - C_{NI} - C_{NI}^{Cert} - K$, since $C_{NI}^{Cert} + K \geq 0$; which implies that this type of firm has incentives to deviate as well. In summary, both types of firm have incentives to deviate towards no claim, entailing that consumers cannot update their off-the-equilibrium beliefs $\mu_{NoClaim}$. Therefore, the pooling PBE in Proposition 2 (Case 1) survives the Cho and Kreps' Intuitive Criterion.

Case 2. In this pooling PBE, both firms choose an uncertified claim. Like in Case 1, the most profitable deviation among all off-the-equilibrium messages is towards no claim. We next study which type of firm, if any, has incentives to deviate towards no claim. First, the innovator obtains, at most, a profit of $p - C_I$ from no claim, which exceeds its equilibrium payoff of $p - C_I - C^{UnCert}$, since $C^{UnCert} \geq 0$; implying that this type of firm has incentives to deviate. Second, the non-innovator obtains, at most, a profit of $p - C_{NI}$ from no claim, which exceeds its equilibrium payoff of $p - C_{NI} - C^{UnCert} - K$, since $C^{UnCert} + K \geq 0$; which implies that this type of firm has incentives to deviate too. Summarizing, both types of firm have incentives to deviate towards no claim, entailing that consumers cannot update their off-the-equilibrium beliefs $\mu_{NoClaim}$. Therefore,

the pooling PBE in Proposition 2 (Case 2) survives the Intuitive Criterion.

Case 3i. In this pooling PBE, both firms make no claim about its product, which is responded with a purchase by consumers. The most profitable deviation among all off-the-equilibrium messages is, then, an uncertified claim. We next analyze which type of firm, if any, has incentives to deviate. First, the innovator obtains, at most, a profit of $p - C_I - C^{UnCert}$ from an uncertified claim, which cannot exceed its equilibrium payoff of $p - C_{NI}$, since $C^{UnCert} \geq 0$; implying that this type of firm has no incentives to deviate. Second, the non-innovator obtains, at most, a profit of $p - C_{NI} - C^{UnCert} - K$ from an uncertified claim, which cannot exceed its equilibrium payoff of $p - C_{NI}$, since $C^{UnCert} + K \geq 0$; which implies that this type of firm has no incentives to deviate either. In summary, no type of firm has incentives to deviate towards an uncertified claim, entailing that consumers cannot update their off-the-equilibrium beliefs μ_{UnCert} . Therefore, the pooling PBE in Proposition 2 (Case 3i) survives the Intuitive Criterion.

Case 3ii. In this pooling PBE, both firms make no claim about the product, which is now responded with no purchase by consumers. Like in Case 3i, the most profitable deviation among all off-the-equilibrium messages is towards an uncertified claim. We next examine which type of firm, if any, has incentives to deviate. First, the innovator obtains, at most, a profit of $p - C_I - C^{UnCert}$ from an uncertified claim, which exceeds its equilibrium payoff of $-C_I$, since $p \geq C^{UnCert} \geq 0$ by definition; implying that this type of firm has incentives to deviate. Second, the non-innovator obtains, at most, a profit of $p - C_{NI} - C^{UnCert} - K$ from an uncertified claim, which exceeds its equilibrium payoff of $-C_{NI}$, if $p \geq C^{UnCert} + K$. Summarizing, when $p \geq C^{UnCert} + K$ holds, both types of firm have incentives to deviate towards an uncertified claim, entailing that consumers cannot update their off-the-equilibrium beliefs μ_{UnCert} , and the pooling PBE in Proposition 2 (Case 3ii) survives the Cho and Kreps' Intuitive Criterion. However, when $p < C^{UnCert} + K$ holds, only the innovator has incentives to deviate, leading consumers to update their off-the-equilibrium beliefs to $\mu_{UnCert} = 1$, thus responding purchasing the good, and the pooling PBE in Proposition 2 (Case 3ii) violates the Intuitive Criterion.

Proof of Proposition 3

I-type firm chooses no claim and NI-type a certified claim. Upon observing no claim, the consumer updates her beliefs to $\mu_{NoClaim} = 1$, and if she observes a certified claim to $\mu_{Cert} = 0$. Off-the-equilibrium beliefs are $\mu_{UnCert} \in [0, 1]$. After observing no claim, the consumer does not buy since $\mu_s V_I + (1 - \mu_s) V_{NI} - p \geq 0$, which yields $V_I \geq p$, where $\mu_s = \mu_{NoClaim} = 1$; after observing a certified claim, she buys since $\mu_s V_I + (1 - \mu_s) V_{NI} - p \geq 0$, which implies $V_{NI} \geq p$, where $\mu_s = \mu_{Cert} = 0$; and she buys after observing the off-the-equilibrium uncertified claim if $\mu_{UnCert} V_I + (1 - \mu_{UnCert}) V_{NI} - p \geq 0$, which yields $\mu_{UnCert} \leq \frac{p - V_{NI}}{V_I - V_{NI}} \equiv \bar{\mu}$. Anticipating consumer's response, the I-type firm does not claim if and only if $-C_I \geq p - C_I - C_I^{Cert} - K$, which is satisfied if $p \leq C_I^{Cert} + K$.

- If $\mu_{UnCert} \leq \bar{\mu}$, the I-type chooses no claim rather than an uncertified claim if and only if $-C_I \geq p - C_I - C^{UnCert} - K$, which holds if $p \leq C^{UnCert} + K$.

- If $\mu_{UnCert} > \bar{\mu}$, the I-type chooses no claim rather than an uncertified claim if and only if $-C_I \geq -C_I - C^{UnCert}$, yielding $C^{UnCert} \geq 0$, and the firm does not deviate.

Similarly, the NI-type chooses a certified claim if and only if $p - C_{NI} - C_{NI}^{Cert} \geq -C_{NI}$, yielding $p \geq C_{NI}^{Cert}$, which is satisfied by definition; and the firm does not deviate.

- If $\mu_{UnCert} \leq \bar{\mu}$, the NI-type chooses a certified rather than an uncertified claim if and only if $p - C_{NI} - C_{NI}^{Cert} \geq p - C_{NI} - C^{UnCert}$, implying $C^{UnCert} \geq C_{NI}^{Cert}$, which does not hold by definition. Hence the NI-type deviates towards an uncertified claim.
- If $\mu_{UnCert} > \bar{\mu}$, the NI-type chooses a certified rather than an uncertified claim if and only if $p - C_I - C_{NI}^{Cert} \geq -C_I - C^{UnCert}$, yielding $p \geq C_{NI}^{Cert} - C^{UnCert}$, which holds by definition.

Therefore, this separating equilibrium can be supported as a PBE if and only if $p \leq C^{UnCert} + K$, and off-the-equilibrium beliefs are $\mu_{UnCert} > \bar{\mu}$.

We next analyze if this PBE survives the *Intuitive Criterion*. In this separating PBE, the only off-the-equilibrium message is uncertified claim. The I-type obtains, at most, a profit of $p - C_I - C^{UnCert} - K$ from deviating towards an uncertified claim, which does not exceed its equilibrium payoff of $-C_I$ from no claim, since $p \geq C^{UnCert} + K$ is not satisfied by equilibrium conditions; implying that this type of firm does not have incentives to deviate. Second, the NI-type obtains, at most, a profit of $-C_{NI} - C^{UnCert}$ from an uncertified claim under $\mu_{UnCert} \geq \bar{\mu}$, which does not exceed its equilibrium payoff of $p - C_{NI} - C_{NI}^{Cert}$ from certifying the product, since $p \geq C_{NI}^{Cert} - C^{UnCert}$ holds by definition; which implies that this type of firm does not deviate either. Therefore, both types of firm do not have incentives to deviate towards an uncertified claim, entailing that consumers cannot update their off-the-equilibrium beliefs μ_{UnCert} . Hence, this separating PBE survives the Cho and Kreps' Intuitive Criterion.

I-type firm chooses no claim and NI-type an uncertified claim. Upon observing no claim, the consumer updates her beliefs to $\mu_{NoClaim} = 1$, and if she observes an uncertified claim to $\mu_{UnCert} = 0$. Off-the-equilibrium beliefs are $\mu_{Cert} \in [0, 1]$. After observing no claim, the consumer does not buy since $\mu_s V_I + (1 - \mu_s) V_{NI} - p \geq 0$, which yields $V_I \geq p$, where $\mu_s = \mu_{NoClaim} = 1$; after observing an uncertified claim, she buys since $\mu_s V_I + (1 - \mu_s) V_{NI} - p \geq 0$, which implies $V_{NI} \geq p$, where $\mu_s = \mu_{UnCert} = 0$; and she buys after observing the off-the-equilibrium certified claim if and only if $\mu_{Cert} \leq \bar{\mu}$. Anticipating consumer's response, the I-type does not claim if and only if $-C_I \geq p - C_I - C^{UnCert} - K$, which is satisfied if $p \leq C^{UnCert} + K$.

- If $\mu_{Cert} \leq \bar{\mu}$, the I-type does not claim rather than choosing a certified claim if and only if $-C_I \geq p - C_I - C_I^{Cert} - K$, which holds if $p \leq C_I^{Cert} + K$.
- If $\mu_{Cert} > \bar{\mu}$, the I-type does not claim rather than choosing a certified claim if and only if $-C_I \geq -C_I - C_I^{Cert}$, yielding $C_I^{Cert} \geq 0$, and the firm does not deviate.

Similarly, the NI-type chooses an uncertified claim if and only if $p - C_{NI} - C^{UnCert} \geq -C_{NI}$, implying $p \geq C^{UnCert}$, which holds by definition, and the firm does not deviate.

- If $\mu_{Cert} \leq \bar{\mu}$, the NI-type firm chooses an uncertified rather than a certified claim if and only if $p - C_{NI} - C^{UnCert} \geq p - C_{NI} - C_{NI}^{Cert}$, which yields $C_{NI}^{Cert} \geq C^{UnCert}$, which is satisfied by definition; and the firm does not deviate.
- If $\mu_{Cert} > \bar{\mu}$, the NI-type firm chooses an uncertified rather than certified claim if and only if $p - C_{NI} - C^{UnCert} \geq -C_{NI} - C_{NI}^{Cert}$, which implies $p \geq C^{UnCert} - C_I^{Cert}$, where $C^{UnCert} - C_I^{Cert} < 0$, and the firm does not deviate.

Therefore, this separating equilibrium can be supported as a PBE if and only if $p \leq C^{UnCert} + K$ for any off-the-equilibrium beliefs $\mu_{Cert} \in [0, 1]$.

We next analyze if this PBE survives the *Intuitive Criterion*. In this separating PBE, the only off-the-equilibrium message is to certify the product. We next evaluate which firm has incentives to deviate towards this message. First, the I-type obtains, at most, a profit of $p - C_I - C_I^{Cert} - K$ from deviating towards a certified claim, which exceeds its equilibrium payoff of $-C_I$ from no claim if $p \geq C_I^{Cert} + K$, which does not hold by equilibrium conditions; implying that this type of firm does not have incentives to deviate. Second, the NI-type obtains, at most, a profit of $p - C_{NI} - C_{NI}^{Cert}$ from a certified claim, which does not exceed its equilibrium payoff of $p - C_{NI} - C^{UnCert}$ from using an uncertified claim, since $C^{UnCert} < C_{NI}^{Cert}$ by assumption; which implies that this type of firm does not deviate either. Therefore, both types of firm do not have incentives to deviate towards a certified claim, entailing that consumers cannot further update their off-the-equilibrium beliefs μ_{Cert} . Hence, this separating PBE survives the Cho and Kreps' Intuitive Criterion.

I-type firm chooses a certified claim and NI-type an uncertified claim. Upon observing a certified claim, the consumer updates her beliefs to $\mu_{Cert} = 1$, and if she observes an uncertified claim to $\mu_{UnCert} = 0$. Off-the-equilibrium beliefs are $\mu_{NoClaim} \in [0, 1]$. After observing a certified claim, the consumer does not buy since $\mu_s V_I + (1 - \mu_s) V_{NI} - p \geq 0$, which yields $V_I \geq p$, where $\mu_s = \mu_{Cert} = 1$; after observing an uncertified claim, she buys since $\mu_s V_I + (1 - \mu_s) V_{NI} - p \geq 0$, which implies $V_{NI} \geq p$, where $\mu_s = \mu_{UnCert} = 0$; and she buys after observing the off-equilibrium no claim if and only if $\mu_{NoClaim} V_I + (1 - \mu_{NoClaim}) V_{NI} - p \geq 0$, which implies $\mu_{NoClaim} \leq \frac{p - V_{NI}}{V_I - V_{NI}} \equiv \bar{\mu}$. Anticipating consumer's response, the I-type chooses a certified claim if and only if $-C_I - C_I^{Cert} \geq p - C_I - C^{UnCert} - K$, which is satisfied if $p \leq (C^{UnCert} - C_I^{Cert}) + K$.

- If $\mu_{NoClaim} \leq \bar{\mu}$, the I-type chooses a certified claim rather than no claim if and only if $-C_I - C_I^{Cert} \geq p - C_I$, yielding $p \leq -C_I^{Cert}$. Hence, the I-type deviates to no claim.
- If $\mu_{NoClaim} > \bar{\mu}$, the I-type chooses a certified claim rather than no claim if and only if $-C_I - C_I^{Cert} \geq -C_I$, implying $C_I^{Cert} \leq 0$. Therefore, the I-type deviates to no claim.

Therefore, this separating equilibrium cannot be supported as a PBE for any off-the-equilibrium beliefs $\mu_{NoClaim} \in [0, 1]$.

I-type firm chooses a certified claim NI-type no claim. Upon observing a certified claim, the consumer updates her beliefs to $\mu_{Cert} = 1$, and if she observes no claim to $\mu_{NoClaim} = 0$. Off-the-equilibrium beliefs are $\mu_{UnCert} \in [0, 1]$. After observing a certified claim, the consumer does not buy since $\mu_s V_I + (1 - \mu_s) V_{NI} - p \geq 0$, which yields $V_I \geq p$, where $\mu_s = \mu_{Cert} = 1$; after observing no claim, she buys since $\mu_s V_I + (1 - \mu_s) V_{NI} - p \geq 0$, which implies $V_{NI} \geq p$, where $\mu_s = \mu_{NoClaim} = 0$; and she buys after observing the off-the-equilibrium uncertified claim if and only if $\mu_{UnCert} \leq \bar{\mu}$. Anticipating consumer's response, the I-type chooses a certified claim if and only if $-C_I - C_I^{Cert} \geq p - C_I$, yielding $p \leq -C_I^{Cert}$. Hence the I-type firm deviates to no claim. Therefore, this separating equilibrium cannot be supported as a PBE.

I-type firm chooses an uncertified claim and NI-type a certified claim. Upon observing an uncertified claim, the consumer updates her beliefs to $\mu_{UnCert} = 1$, and if she observes a certified claim to $\mu_{Cert} = 0$. Off-the-equilibrium beliefs are $\mu_{NoClaim} \in [0, 1]$. After observing a uncertified claim, the consumer does buy not since $\mu_s V_I + (1 - \mu_s) V_{NI} - p \geq 0$, which yields $V_I \geq p$, where $\mu_s = \mu_{UnCert} = 1$; after observing a certified claim, she buys since $\mu_s V_I + (1 - \mu_s) V_{NI} - p \geq 0$, which implies $V_{NI} \geq p$, where $\mu_s = \mu_{Cert} = 0$; and she buys after observing the off-the-equilibrium no claim if and only if $\mu_{NoClaim} \leq \bar{\mu}$. Anticipating consumer's response, the I-type chooses an uncertified claim if and only if $-C_I - C^{UnCert} \geq p - C_I - C_I^{Cert} - K$, which is satisfied if $p \leq (C_I^{Cert} - C^{UnCert}) + K$.

- If $\mu_{NoClaim} \leq \bar{\mu}$, the I-type chooses an uncertified claim rather than no claim if and only if $-C_I - C^{UnCert} \geq p - C_I$, yielding $p \leq -C^{UnCert}$. Hence, the I-type deviates to no claim.
- If $\mu_{NoClaim} > \bar{\mu}$, the I-type chooses an uncertified claim rather than no claim if and only if $-C_I - C^{UnCert} \geq -C_I$, implying $C^{UnCert} \leq 0$. Therefore, the I-type deviates to no claim.

Therefore, this separating equilibrium cannot be supported as a PBE for any off-the-equilibrium beliefs $\mu_{NoClaim} \in [0, 1]$.

I-type firm chooses an uncertified claim and NI-type no claim. Upon observing an uncertified claim, the consumer updates her beliefs to $\mu_{UnCert} = 1$, and if she observes no claim to $\mu_{NoClaim} = 0$. Off-the-equilibrium beliefs are $\mu_{Cert} \in [0, 1]$. After observing a uncertified claim, she does not buy since $\mu_s V_I + (1 - \mu_s) V_{NI} - p \geq 0$, which yields $V_I \geq p$, where $\mu_s = \mu_{UnCert} = 1$; after observing no claim, she buys since $\mu_s V_I + (1 - \mu_s) V_{NI} - p \geq 0$, which implies $V_{NI} \geq p$, where $\mu_s = \mu_{NoClaim} = 0$; and she buys after observing a certified claim if and only if $\mu_{Cert} \leq \bar{\mu}$. Anticipating consumer's response, the I-type chooses an uncertified claim if and only if $-C_I - C^{UnCert} \geq p - C_I$, yielding $p \leq -C^{UnCert}$. Hence, I-type firm deviates towards no claim. Therefore, this separating equilibrium cannot be supported as a PBE.

Proof of Proposition 4

Both types choose a certified claim. In the case of negative perceptions we consider $V_I < p < V_{NI}$. Upon observing the equilibrium message, the consumer cannot further update her beliefs

about the firm's type, yielding $\mu_{Cert} = q$ in equilibrium, and $\mu_s = \{\mu_{UnCert}, \mu_{NoClaim}\} \in [0, 1]$ off-the-equilibrium. After observing a certified claim, the consumer buys if and only if $qV_I + (1-q)V_{NI} - p \geq 0$, which solving for q yields $q \leq \frac{p-V_{NI}}{V_I-V_{NI}} \equiv \bar{\mu}$; after observing an uncertified claim, the consumer buys if and only if $\mu_{UnCert}V_I + (1-\mu_{UnCert})V_{NI} - p \geq 0$, which yields $\mu_{UnCert} \leq \frac{p-V_{NI}}{V_I-V_{NI}} \equiv \bar{\mu}$; and after observing no claim, she buys if and only if $\mu_{NoClaim}V_I + (1-\mu_{NoClaim})V_{NI} - p \geq 0$, which implies $\mu_{NoClaim} \leq \frac{p-V_{NI}}{V_I-V_{NI}} \equiv \bar{\mu}$.

We analyze eight different cases depending on consumers' beliefs:

Case 1: Consumer's beliefs are $q, \mu_{UnCert}, \mu_{NoClaim} \leq \bar{\mu}$, that is, she buys regardless of the message she observes. Anticipating consumer's response, the I-type chooses a certified rather than an uncertified claim if and only if $p - C_I - C_I^{Cert} - K \geq p - C_I - C^{UnCert} - K$, implying $C^{UnCert} \geq C_I^{Cert}$, which does not hold by definition; and the I-type firm deviates towards an uncertified claim. Therefore, this pooling equilibrium in case 1 cannot be supported as a PBE.

Case 2: Consumer's beliefs are $q, \mu_{NoClaim} \leq \bar{\mu}$, and $\mu_{UnCert} > \bar{\mu}$, that is, she buys after observing a certified claim or no claim, and does not buy otherwise. Anticipating consumer's response, the I-type chooses a certified rather than an uncertified claim if and only if $p - C_I - C_I^{Cert} - K \geq -C_I - C^{UnCert}$, which is satisfied if $p \geq (C_I^{Cert} - C^{UnCert}) + K$. The I-type chooses a certified claim rather than no claim if and only if $p - C_I - C_I^{Cert} - K \geq p - C_I$, yielding $C_I^{Cert} + K \leq 0$, which is not satisfied; and the I-type firm deviates towards no claim. Hence, this pooling equilibrium in case 2 cannot be sustained as a PBE.

Case 3: Consumer's beliefs are $q > \bar{\mu}$, and $\mu_{UnCert}, \mu_{NoClaim} \leq \bar{\mu}$, that is, she does not buy after observing a certified claim, and buys otherwise. Anticipating consumer's response, the I-type chooses a certified rather than an uncertified claim if and only if $-C_I - C_I^{Cert} \geq p - C_I - C^{UnCert} - K$, which is satisfied if $p \leq (C^{UnCert} - C_I^{Cert}) + K$. The I-type chooses a certified claim rather than no claim if and only if $-C_I - C_I^{Cert} \geq p - C_I$, implying $p \leq -C_I^{Cert}$, and the I-type firm deviates towards no claim. Therefore, this pooling equilibrium in case 3 cannot be supported as a PBE.

Case 4: Consumer's beliefs are $q, \mu_{UnCert} > \bar{\mu}$, and $\mu_{NoClaim} \leq \bar{\mu}$, that is, she does not buy after observing a certified or an uncertified claim, and buys otherwise. Anticipating consumer's response, the I-type chooses a certified rather than an uncertified claim if and only if $-C_I - C_I^{Cert} \geq -C_I - C^{UnCert}$, yielding $C^{UnCert} \geq C_I^{Cert}$, which is not satisfied by definition; and the I-type firm deviates towards an uncertified claim. Hence, this pooling equilibrium in case 4 cannot be sustained as a PBE.

Case 5: Consumer's beliefs are $q, \mu_{UnCert} \leq \bar{\mu}$, and $\mu_{NoClaim} > \bar{\mu}$, that is, she buys after observing a certified or an uncertified claim, and does not buy otherwise. Anticipating consumer's response, the I-type chooses a certified rather than an uncertified claim if and only if $p - C_I - C_I^{Cert} - K \geq p - C_I - C^{UnCert} - K$, implying $C^{UnCert} \geq C_I^{Cert}$, which does not hold by definition; and the I-type firm deviates towards an uncertified claim. Therefore, this pooling equilibrium in case 5 cannot be supported as a PBE.

Case 6: Consumer's beliefs are $q \leq \bar{\mu}$, and $\mu_{UnCert}, \mu_{NoClaim} > \bar{\mu}$, that is, she buys after observing a certified claim, and does not buy otherwise. Anticipating consumer's response, the

I-type chooses a certified rather than an uncertified claim if and only if $p - C_I - C_I^{Cert} - K \geq -C_I - C^{UnCert}$, which is satisfied if $p \geq (C_I^{Cert} - C^{UnCert}) + K$. The I-type chooses a certified claim rather than no claim if and only if $p - C_I - C_I^{Cert} - K \geq -C_I$, which is satisfied if $p \geq C_I^{Cert} + K$. Similarly, the NI-type chooses a certified rather than an uncertified claim if and only if $p - C_{NI} - C_{NI}^{Cert} \geq -C_{NI} - C^{UnCert}$, implying $p \geq C_{NI}^{Cert} - C^{UnCert}$, which is satisfied by definition $p \geq C_{NI}^{Cert}$; and the NI-type firm does not deviate. The NI-type chooses a certified claim rather than no claim if and only if $p - C_{NI} - C_{NI}^{Cert} \geq -C_{NI}$, yielding $p \geq C_{NI}^{Cert}$, which holds by definition. Hence, since $C_I^{Cert} > C_{NI}^{Cert} > C^{UnCert}$ by definition, then $C_I^{Cert} + K > (C_I^{Cert} - C^{UnCert}) + K$, this pooling equilibrium in case 6 can be supported as a PBE if and only if $p \geq C_I^{Cert} + K$.

Case 7: Consumer's beliefs are $q, \mu_{NoClaim} > \bar{\mu}$, and $\mu_{Uncert} \leq \bar{\mu}$, that is, she does not buy after observing a certified or no claim, and buys otherwise.

Anticipating consumer's response, the I-type chooses a certified rather than an uncertified claim if and only if $-C_I - C_I^{Cert} \geq p - C_I - C^{UnCert} - K$, which is satisfied if $p \leq (C^{UnCert} - C_I^{Cert}) + K$. The I-type chooses a certified claim rather than no claim if and only if $-C_I - C_I^{Cert} \geq -C_I$, implying $C_I^{Cert} \leq 0$, and the I-type firm deviates towards no claim. Therefore, this pooling equilibrium in case 7 cannot be sustained as a PBE.

Case 8: Consumer's beliefs are $q, \mu_{Uncert}, \mu_{NoClaim} > \bar{\mu}$, that is, she does not buy regardless of the message she observes. Anticipating consumer's response, the I-type chooses a certified rather than an uncertified claim if and only if $-C_I - C_I^{Cert} \geq -C_I - C^{UnCert}$, yielding $C^{UnCert} \geq C_I^{Cert}$, which does not hold by definition; and the I-type firm deviates towards an uncertified claim. Hence, this pooling equilibrium in case 8 cannot be supported as a PBE.

Both types choose an uncertified claim. Upon observing the equilibrium message, the consumer cannot further update her beliefs about the firm's type, yielding $\mu_{Uncert} = q$ in equilibrium, and $\mu_s = \{\mu_{Cert}, \mu_{NoClaim}\} \in [0, 1]$ off-the-equilibrium. After observing a uncertified claim, the consumer buys if and only if $q \leq \bar{\mu}$; after observing a certified claim she buys if and only if $\mu_{Cert} \leq \bar{\mu}$; and after observing no claim, she buys if and only if $\mu_{NoClaim} \leq \bar{\mu}$.

We analyze eight different cases depending on consumers' beliefs:

Case 1: Consumer's beliefs are $q, \mu_{Cert}, \mu_{NoClaim} \leq \bar{\mu}$, that is, she buys regardless of the message she observes. Anticipating consumer's response, the I-type chooses an uncertified rather than a certified claim if and only if $p - C_I - C^{UnCert} - K \geq p - C_I - C_I^{Cert} - K$, implying $C_I^{Cert} \geq C^{UnCert}$, which is satisfied by definition; and the I-type firm does not deviate. The I-type chooses an uncertified claim rather than no claim if and only if $p - C_I - C^{UnCert} - K \geq p - C_I$, implying $C^{UnCert} + K \leq 0$, which does not hold; and the I-type firm deviates towards no claim. Therefore, this pooling equilibrium in case 1 cannot be supported as a PBE.

Case 2: Consumer's beliefs are $q, \mu_{NoClaim} \leq \bar{\mu}$, and $\mu_{Cert} > \bar{\mu}$, that is, she buys after observing an uncertified claim or no claim, and does not buy otherwise. Anticipating consumer's response, the I-type chooses an uncertified rather than a certified claim if and only if $p - C_I - C^{UnCert} - K \geq -C_I - C_I^{Cert}$, which is satisfied if $p \geq (C^{UnCert} - C_I^{Cert}) + K$. The I-type chooses an uncertified claim rather than no claim if and only if $p - C_I - C^{UnCert} - K \geq p - C_I$, yielding $C^{UnCert} + K \leq 0$,

which is not satisfied and the I-type firm deviates towards no claim. Hence, this pooling equilibrium in case 2 cannot be sustained as a PBE.

Case 3: Consumer's beliefs are $q > \bar{\mu}$, and $\mu_{Cert}, \mu_{NoClaim} \leq \bar{\mu}$, that is, she does not buy after observing an uncertified claim, and buys otherwise. Anticipating consumer's response, the I-type chooses an uncertified rather than a certified claim if and only if $-C_I - C^{UnCert} \geq p - C_I - C_I^{Cert} - K$, which is satisfied if $p \leq (C_I^{Cert} - C^{UnCert}) + K$. The I-type chooses an uncertified claim rather than no claim if and only if $-C_I - C^{UnCert} \geq p - C_I$, yielding $p \leq -C^{UnCert}$, and the I-type firm deviates towards no claim. Therefore, this pooling equilibrium in case 3 cannot be supported as a PBE.

Case 4: Consumer's beliefs are $q, \mu_{Cert} > \bar{\mu}$, and $\mu_{NoClaim} \leq \bar{\mu}$, that is, she does not buy after observing an uncertified or a certified claim, and buys otherwise. Anticipating consumer's response, the I-type chooses an uncertified rather than a certified claim if and only if $-C_I - C^{UnCert} \geq -C_I - C_I^{Cert}$, implying $C_I^{Cert} \geq C^{UnCert}$, which is satisfied by definition; and the I-type firm does not deviate. The I-type chooses an uncertified claim rather than no claim if and only if $-C_I - C^{UnCert} \geq p - C_I$, yielding $p \leq -C^{UnCert}$, and the I-type firm deviates towards no claim. Hence, this pooling equilibrium in case 4 cannot be sustained as a PBE.

Case 5: Consumer's beliefs are $q, \mu_{Cert} \leq \bar{\mu}$, and $\mu_{NoClaim} > \bar{\mu}$, that is, she buys after observing an uncertified or a certified claim, and does not buy otherwise. Anticipating consumer's response, the I-type chooses an uncertified rather than a certified claim if and only if $p - C_I - C^{UnCert} - K \geq p - C_I - C_I^{Cert} - K$, implying $C_I^{Cert} \geq C^{UnCert}$, which holds by definition; and the I-type firm does not deviate. The I-type chooses an uncertified claim rather than no claim if and only if $p - C_I - C^{UnCert} - K \geq -C_I$, which is satisfied if $p \geq C^{UnCert} + K$.

Similarly, the NI-type chooses an uncertified rather than a certified claim if and only if $p - C_{NI} - C^{UnCert} \geq p - C_{NI} - C_{NI}^{Cert}$, yielding $C_{NI}^{Cert} \geq C^{UnCert}$, which holds by definition; and the NI-type firm does not deviate. The NI-type chooses an uncertified claim rather than no claim if and only if $p - C_{NI} - C^{UnCert} \geq -C_I$, implying $p \geq -C^{UnCert}$, which is satisfied; and the NI-type firm does not deviate. Therefore, this pooling equilibrium in case 5 can be supported as a PBE if and only if $p \geq C^{UnCert} + K$.

Case 6: Consumer's beliefs are $q \leq \bar{\mu}$, and $\mu_{Cert}, \mu_{NoClaim} > \bar{\mu}$, that is, she buys after observing an uncertified claim, and does not buy otherwise. Anticipating consumer's response, the I-type chooses an uncertified rather than a certified claim if and only if $p - C_I - C^{UnCert} - K \geq -C_I - C_I^{Cert}$, which is satisfied if $p \geq (C^{UnCert} - C_I^{Cert}) + K$. The I-type chooses an uncertified claim rather than no claim if and only if $p - C_I - C^{UnCert} - K \geq -C_I$, which is satisfied if $p \geq C^{UnCert} + K$. Similarly, the NI-type chooses an uncertified rather than a certified claim if and only if $p - C_{NI} - C^{UnCert} \geq -C_{NI} - C_{NI}^{Cert}$, yielding $p \geq C^{UnCert} - C_{NI}^{Cert}$, where $C^{UnCert} - C_{NI}^{Cert} < 0$; and the NI-type firm does not deviate. The NI-type chooses an uncertified claim rather than no claim if and only if $p - C_{NI} - C^{UnCert} \geq -C_{NI}$, implying $p \geq C^{UnCert}$, which holds by definition; and the NI-type firm does not deviate. Hence, since $C^{UnCert} + K > (C^{UnCert} - C_I^{Cert}) + K$, this pooling equilibrium in case 6 can be sustained as a PBE if and only if $p \geq C^{UnCert} + K$.

Case 7: Consumer's beliefs are $q, \mu_{NoClaim} > \bar{\mu}$, and $\mu_{Cert} \leq \bar{\mu}$, that is, she only buys after observing a certified claim. Anticipating consumer's response, the I-type chooses an uncertified rather than a certified claim if and only if $-C_I - C^{UnCert} \geq p - C_I - C_I^{Cert} - K$, which is satisfied if $p \leq (C_I^{Cert} - C^{UnCert}) + K$. The I-type chooses an uncertified claim rather than no claim if and only if $-C_I - C^{UnCert} \geq -C_I$, yielding $C_I^{Cert} \leq 0$, and the I-type firm deviates towards no claim. Therefore, this pooling equilibrium in case 7 cannot be supported as a PBE.

Case 8: Consumer's beliefs are $q, \mu_{Cert}, \mu_{NoClaim} > \bar{\mu}$, that is, she does not buy regardless of the message. Anticipating consumer's response, the I-type chooses an uncertified rather than a certified claim if and only if $-C_I - C^{UnCert} \geq -C_I - C_I^{Cert}$, implying $C_I^{Cert} \geq C^{UnCert}$, which holds by definition; and the I-type firm does not deviate. The I-type chooses an uncertified claim rather than no claim if and only if $-C_I - C^{UnCert} \geq -C_I$, yielding $C^{UnCert} \leq 0$, and the I-type firm deviates towards no claim. Hence, this pooling equilibrium in case 8 cannot be sustained as a PBE.

Both types do not claim. Upon observing the equilibrium message, the consumer cannot further update her beliefs about the firm's type, yielding $\mu_{NoClaim} = q$ in equilibrium, and $\mu_s = \{\mu_{Cert}, \mu_{UnCert}\} \in [0, 1]$ off-the-equilibrium. After observing no claim, the consumer buys if and only if $q \leq \bar{\mu}$; after observing a certified claim the consumer buys if and only if $\mu_{Cert} \leq \bar{\mu}$; and after observing an uncertified claim, she buys if and only if $\mu_{UnCert} \leq \bar{\mu}$.

We analyze eight different cases depending on consumers' beliefs:

Case 1: Consumer's beliefs are $q, \mu_{Cert}, \mu_{UnCert} \leq \bar{\mu}$, that is, she buys regardless of the message she observes. Anticipating consumer's response, the I-type chooses no claim rather than a certified claim if and only if $p - C_I \geq p - C_I - C_I^{Cert} - K$, implying $C_I^{Cert} + K \geq 0$, and the I-type firm does not deviate. The I-type chooses no claim rather than an uncertified claim if and only if $p - C_I \geq p - C_I - C^{UnCert} - K$, yielding $C^{UnCert} + K \geq 0$, and the I-type does not deviate. Similarly, the NI-type chooses no claim rather than a certified claim if and only if $p - C_{NI} \geq p - C_{NI} - C_{NI}^{Cert}$, implying $C_{NI}^{Cert} \geq 0$, and the NI-type firm does not deviate. The NI-type chooses no claim rather than an uncertified claim if and only if $p - C_{NI} \geq p - C_{NI} - C^{UnCert}$, yielding $C^{UnCert} \geq 0$, and the NI-type firm does not deviate. Therefore, this pooling equilibrium in case 1 can be supported as a PBE.

Case 2: Consumer's beliefs are $q, \mu_{UnCert} \leq \bar{\mu}$, and $\mu_{Cert} > \bar{\mu}$, that is, she buys after observing no claim or an uncertified claim, and does not buy otherwise. Anticipating consumer's response, the I-type chooses no claim rather than a certified claim if and only if $p - C_I \geq -C_I - C_I^{Cert}$, implying $p \geq -C_I^{Cert}$, and the I-type firm does not deviate. The I-type chooses no claim rather than an uncertified claim if and only if $p - C_I \geq p - C_I - C^{UnCert} - K$, yielding $C^{UnCert} + K \geq 0$, and the I-type firm does not deviate. Similarly, the NI-type chooses no claim rather than a certified claim if and only if $p - C_{NI} \geq -C_{NI} - C_{NI}^{Cert}$, implying $p \geq -C_{NI}^{Cert}$, and the NI-type firm does not deviate. The NI-type chooses no claim rather than an uncertified claim if and only if $p - C_{NI} \geq p - C_{NI} - C^{UnCert}$, yielding $C^{UnCert} \geq 0$, and the NI-type firm does not deviate. Hence, this pooling equilibrium in case 2 can be sustained as a PBE.

Case 3: Consumer's beliefs are $q > \bar{\mu}$, and $\mu_{Cert}, \mu_{UnCert} \leq \bar{\mu}$, that is, she does not buy after observing no claim, and buys otherwise. Anticipating consumer's response, the I-type chooses no claim rather than a certified claim if and only if $-C_I \geq p - C_I - C_I^{Cert} - K$, which is satisfied if $p \leq C_I^{Cert} + K$. The I-type chooses no claim rather than an uncertified claim if and only if $-C_I \geq p - C_I - C^{UnCert} - K$, which holds if $p \leq C^{UnCert} + K$. Similarly, the NI-type chooses no claim rather than a certified claim if and only if $-C_{NI} \geq p - C_{NI} - C_{NI}^{Cert}$, implying $p \leq C_{NI}^{Cert}$, which is not satisfied by definition; and the NI-type firm deviates towards a certified claim. Therefore, this pooling equilibrium in case 3 cannot be supported as a PBE.

Case 4: Consumer's beliefs are $q, \mu_{Cert} > \bar{\mu}$, and $\mu_{UnCert} \leq \bar{\mu}$, that is, she does not buy after observing no claim or a certified claim, and buys otherwise. Anticipating the consumer's response, the I-type chooses no claim rather than a certified claim if and only if $-C_I \geq -C_I - C_I^{Cert}$, yielding $C_I^{Cert} \geq 0$, and the I-type firm does not deviate. The I-type chooses no claim rather than an uncertified claim if and only if $-C_I \geq p - C_I - C^{UnCert} - K$, which is satisfied if $p \leq C^{UnCert} + K$. Similarly, the NI-type chooses no claim rather than a certified claim if and only if $-C_{NI} \geq -C_{NI} - C_{NI}^{Cert}$, implying $C_{NI}^{Cert} \geq 0$, and the NI-type firm does not deviate. The NI-type chooses no claim rather than an uncertified claim if and only if $-C_{NI} \geq p - C_{NI} - C^{UnCert}$, yielding $p \leq C^{UnCert}$, which is not satisfied by definition; and the NI-type firm deviates towards an uncertified claim. Hence, this pooling equilibrium in case 4 cannot be sustained as a PBE.

Case 5: Consumer's beliefs are $q, \mu_{Cert} \leq \bar{\mu}$, and $\mu_{UnCert} > \bar{\mu}$, that is, she buys after observing no claim or a certified claim, and does not buy otherwise. Anticipating consumer's response, the I-type chooses no claim rather than a certified claim if and only if $p - C_I \geq p - C_I - C_I^{Cert} - K$, yielding $C_I^{Cert} + K \geq 0$, and the I-type firm does not deviate. The I-type chooses no claim rather than an uncertified claim if and only if $p - C_I \geq -C_I - C^{UnCert}$, implying $p \geq -C^{UnCert}$, and the I-type firm does not deviate. Similarly, the NI-type chooses no claim rather than a certified claim if and only if $p - C_{NI} \geq p - C_{NI} - C_{NI}^{Cert}$, yielding $C_{NI}^{Cert} \geq 0$, and the NI-type firm does not deviate. The NI-type chooses no claim rather than an uncertified claim if and only if $p - C_{NI} \geq -C_{NI} - C^{UnCert}$, implying $p \geq -C^{UnCert}$, which holds by definition; and the NI-type firm does not deviate. Therefore, this pooling equilibrium in case 5 can be supported as a PBE.

Case 6: Consumer's beliefs are $q \leq \bar{\mu}$, and $\mu_{Cert}, \mu_{UnCert} > \bar{\mu}$, that is, she buys after observing no claim, and does not buy otherwise. Anticipating consumer's response, the I-type chooses no claim rather than a certified claim if and only if $p - C_I \geq -C_I - C_I^{Cert}$, yielding $p \geq -C_I^{Cert}$, and the I-type firm does not deviate. The I-type chooses no claim rather than an uncertified claim if and only if $p - C_I \geq -C_I - C^{UnCert}$, implying $p \geq -C^{UnCert}$, and the I-type firm does not deviate. Similarly, the NI-type chooses no claim rather than a certified claim if and only if $p - C_{NI} \geq -C_{NI} - C_{NI}^{Cert}$, yielding $p \geq -C_{NI}^{Cert}$, and the NI-type firm does not deviate. The NI-type chooses no claim rather than an uncertified claim if and only if $p - C_{NI} \geq -C_{NI} - C^{UnCert}$, implying $p \geq -C^{UnCert}$, and the NI-type firm does not deviate. Hence, this pooling equilibrium in case 6 can be sustained as a PBE.

Case 7: Consumer's beliefs are $q, \mu_{UnCert} > \bar{\mu}$, and $\mu_{Cert} \leq \bar{\mu}$, that is, she only buys after

observing a certified claim. Anticipating consumer's response, the I-type chooses no claim rather than a certified claim if and only if $-C_I \geq p - C_I - C_I^{Cert} - K$, which is satisfied if $p \leq C_I^{Cert} + K$. The I-type chooses no claim rather than an uncertified if and only if $-C_I \geq -C_I - C^{UnCert}$, yielding $C_I^{Cert} \geq 0$, and the I-type firm does not deviate. Similarly, the NI-type chooses no claim rather than a certified claim if and only if $-C_{NI} \geq p - C_{NI} - C_{NI}^{Cert}$, implying $p \leq C_{NI}^{Cert}$, which does not hold by definition; and the NI-type firm deviates towards a certified claim. Therefore, this pooling equilibrium in case 7 cannot be supported as a PBE.

Case 8: Consumer's beliefs are $q, \mu_{Cert}, \mu_{UnCert} > \bar{\mu}$, that is, she does not buy regardless of the message she observes. Anticipating consumer's response, the I-type chooses no claim rather than a certified claim if and only if $-C_I \geq -C_I - C_I^{Cert}$, yielding $C_I^{Cert} \geq 0$, and the I-type firm does not deviate. The I-type chooses no claim rather than an uncertified claim if and only if $-C_I \geq -C_I - C^{UnCert}$, implying $C^{UnCert} \geq 0$, and the I-type firm does not deviate. Similarly, the NI-type chooses no claim rather than a certified claim if and only if $-C_{NI} \geq -C_{NI} - C_{NI}^{Cert}$, yielding $C_{NI}^{Cert} \geq 0$, and the NI-type firm does not deviate. The NI-type chooses no claim rather than an uncertified claim if and only if $-C_{NI} \geq -C_{NI} - C^{UnCert}$, implying $C^{UnCert} \geq 0$, and the NI-type firm does not deviate. Hence, this pooling equilibrium in case 8 can be sustained as a PBE.

Intuitive Criterion (IC)

Case 1. In this pooling PBE, both firms choose to certify, so the most profitable deviation among all off-the-equilibrium messages is towards no claim. We next analyze which type of firm, if any, has incentives to deviate towards no claim. First, the I-type obtains, at most, a profit of $p - C_I$ from no claim, which exceeds its equilibrium payoff of $p - C_I - C_I^{Cert} - K$, since $C_I^{Cert} + K \geq 0$; implying that this type of firm deviates. Second, the NI-type obtains, at most, a profit of $p - C_{NI}$ from no claim, which exceeds its equilibrium payoff of $p - C_{NI} - C_{NI}^{Cert}$, since $C_{NI}^{Cert} \geq 0$; which implies that this type of firm deviates as well. In summary, both types of firm have incentives to deviate towards no claim, entailing that consumers cannot update their off-the-equilibrium beliefs $\mu_{NoClaim}$. Therefore, the pooling PBE in Proposition 4 (Case 1) survives the Cho and Kreps' Intuitive Criterion.

Case 2. In this pooling PBE, both firms choose an uncertified claim. Like in Case 1, the most profitable deviation among all off-the-equilibrium messages is towards no claim. We next study which type of firm, if any, has incentives to deviate towards no claim. First, the I-type obtains, at most, a profit of $p - C_I$ from no claim, which exceeds its equilibrium payoff of $p - C_I - C^{UnCert} - K$, since $C^{UnCert} + K \geq 0$; implying that this type of firm deviates. Second, the NI-type obtains, at most, a profit of $p - C_{NI}$ from no claim, which exceeds its equilibrium payoff of $p - C_{NI} - C^{UnCert}$, since $C^{UnCert} \geq 0$; which implies that this type of firm deviates too. Summarizing, both types of firm have incentives to deviate towards no claim, entailing that consumers cannot update their off-the-equilibrium beliefs $\mu_{NoClaim}$. Therefore, the pooling PBE in Proposition 4 (Case 2) survives the Intuitive Criterion.

Case 3i. In this pooling PBE, both firms make no claim about its product, which is responded with a purchase by consumers. The most profitable deviation among all off-the-equilibrium mes-

sages is, then, an uncertified claim. We next analyze which type of firm, if any, has incentives to deviate. First, the I-type obtains, at most, a profit of $p - C_I - C^{UnCert} - K$ from an uncertified claim, which cannot exceed its equilibrium payoff of $p - C_{NI}$, since $C^{UnCert} + K \geq 0$; implying that this type of firm has no incentives to deviate. Second, the NI-type obtains, at most, a profit of $p - C_{NI} - C^{UnCert}$ from an uncertified claim, which cannot exceed its equilibrium payoff of $p - C_{NI}$, since $C^{UnCert} \geq 0$; which implies that this type of firm does not deviate either. In summary, no type of firm deviates towards an uncertified claim, entailing that consumers cannot update their off-the-equilibrium beliefs μ_{UnCert} . Therefore, the pooling PBE in Proposition 4 (Case 3i) survives the Intuitive Criterion.

Case 3ii. In this pooling PBE, both firms make no claim about the product, which is now responded with no purchase by consumers. Like in Case 3i, the most profitable deviation among all off-the-equilibrium messages is towards an uncertified claim. We next examine which type of firm, if any, has incentives to deviate. First, the I-type obtains, at most, a profit of $p - C_I - C^{UnCert} - K$ from an uncertified claim, which exceeds its equilibrium payoff of $-C_I$ if $p \geq C^{UnCert} + K$. Second, the NI-type obtains, at most, a profit of $p - C_{NI} - C^{UnCert}$ from an uncertified claim, which exceeds its equilibrium payoff of $-C_{NI}$, since $p \geq C^{UnCert}$ by definition; implying that this type of firm deviates. Summarizing, when $p \geq C^{UnCert} + K$ holds, both types of firm have incentives to deviate towards an uncertified claim, entailing that consumers cannot update their off-the-equilibrium beliefs μ_{UnCert} , and the pooling PBE in Proposition 4 (Case 3ii) survives the Cho and Kreps' Intuitive Criterion. However, when $p < C^{UnCert} + K$ holds, only the I-type deviates, leading consumers to update their off-the-equilibrium beliefs to $\mu_{UnCert} = 1$. Since consumers exhibit a negative perception towards the innovation, they respond not purchasing the good, eliminating any incentives for the I-type firm to deviate towards an uncertified claim. Therefore, the pooling PBE in Proposition 4 (Case 3ii) survives the Intuitive Criterion.

Proof of Proposition 5

I-type firm chooses a certified claim and NI-type no claim. Consumer's beliefs are the same than in Proposition 1. The I-type chooses a certified claim if and only if $p - C_I - C_I^{Cert} \geq -C_I$, implying $p \geq C_I^{Cert}$, which holds by definition, and the firm does not deviate. If $\mu_{UnCert} \geq \bar{\mu}$, the I-type chooses a certified rather than an uncertified claim if and only if $p - C_I - C_I^{Cert} \geq p - C_I - C^{UnCert} - \tilde{F}$, which is satisfied if and only if $\tilde{F} \geq C_I^{Cert} - C^{UnCert}$. If $\mu_{UnCert} < \bar{\mu}$, the I-type chooses a certified rather than an uncertified claim if and only if $p - C_I - C_I^{Cert} \geq -C_I - C^{UnCert} - \tilde{F}$, implying $p \geq C_I^{Cert} - C^{UnCert} - \tilde{F}$, which holds by definition, and the I-type firm does not deviate. Similarly, the NI-type does not claim if and only if $-C_{NI} \geq p - C_{NI} - C_{NI}^{Cert} - K$, which is satisfied if $p \leq C_{NI}^{Cert} + K$. If $\mu_{UnCert} \geq \bar{\mu}$, the NI-type does not claim rather than choosing an uncertified claim if and only if $-C_{NI} \geq p - C_{NI} - C^{UnCert} - \tilde{F} - K$, which is satisfied if and only if $\tilde{F} \geq p - K - C^{UnCert}$. If $\mu_{UnCert} < \bar{\mu}$, the NI-type does not claim rather than choosing an uncertified claim if and only if $-C_{NI} \geq -C_{NI} - C^{UnCert} - \tilde{F}$, implying $C^{UnCert} + \tilde{F} \geq 0$, which holds by definition, and the NI-type firm does not deviate. Therefore, this separating equilibrium can be supported as a PBE if and only if $p \leq C_{NI}^{Cert} + K$, and $\tilde{F} \geq \max\{p - K - C^{UnCert}, C_I^{Cert} - C^{UnCert}\}$ when $\mu_{UnCert} \geq \bar{\mu}$.

Since both types of firm don't have incentives to deviate this separating PBE survives the Intuitive Criterion.

I-type firm chooses an uncertified claim and NI-type no claim. Consumer's beliefs are the same than in Proposition 1. The I-type chooses an uncertified claim if $p - C_I - C^{UnCert} - \tilde{F} \geq -C_I$, which is satisfied if $\tilde{F} \leq p - C^{UnCert}$. If $\mu_{Cert} \geq \bar{\mu}$, the I-type chooses an uncertified rather than a certified claim if $p - C_I - C^{UnCert} - \tilde{F} \geq p - C_I - C_I^{Cert}$, which holds if $\tilde{F} \leq C_I^{Cert} - C^{UnCert}$. If $\mu_{Cert} < \bar{\mu}$, the I-type chooses an uncertified rather than a certified claim if $p - C_I - C^{UnCert} - \tilde{F} \geq -C_I - C_I^{Cert}$, which is satisfied if $\tilde{F} \leq p + (C_I^{Cert} - C^{UnCert})$. Similarly, the NI-type does not claim if $-C_{NI} \geq p - C_{NI} - C^{UnCert} - \tilde{F} - K$, which holds if $p \leq C^{UnCert} + \tilde{F} + K$. If $\mu_{Cert} \geq \bar{\mu}$, the NI-type does not claim rather than choosing a certified claim if $-C_{NI} \geq p - C_{NI} - C_{NI}^{Cert} - K$, which is satisfied if $p \leq C_{NI}^{Cert} + K$. If $\mu_{Cert} < \bar{\mu}$, the NI-type does not claim rather than choosing a certified claim if $-C_{NI} \geq -C_{NI} - C_{NI}^{Cert}$, yielding $C_{NI}^{Cert} \geq 0$, which holds by definition, and the NI-type firm does not deviate. Therefore, this separating equilibrium can be supported as a PBE if and only if $p \leq C^{UnCert} + K$ and $\tilde{F} \leq C_I^{Cert} - C^{UnCert}$ for any off-the-equilibrium beliefs $\mu_{Cert} \in [0, 1]$. And since both types of firm do not have incentives to deviate this separating PBE survives the Intuitive Criterion.

Following the proof for Proposition 1 it is easy to check that the other four possible options for separating equilibrium cannot be supported as a PBE. The only difference in the proof is the penalty \tilde{F} when an uncertified claim is in place.

Proof of Proposition 6

Both types choose a certified claim. Consumer's beliefs are the same than in Proposition 2. We analyze the same eight cases and we check that two of them can be supported as a PBE.

Case 5: The I-type chooses a certified rather than an uncertified claim if $p - C_I - C_I^{Cert} \geq p - C_I - C^{UnCert} - \tilde{F}$, which is satisfied if $\tilde{F} \geq C_I^{Cert} - C^{UnCert}$. The I-type chooses a certified claim rather than no claim if $p - C_I - C_I^{Cert} \geq -C_I$, implying $p \geq C_I^{Cert}$, which holds by definition. Similarly, the NI-type chooses a certified rather than an uncertified claim if $p - C_{NI} - C_{NI}^{Cert} - K \geq p - C_{NI} - C^{UnCert} - \tilde{F} - K$, which is satisfied if $\tilde{F} \geq C_{NI}^{Cert} - C^{UnCert}$. The NI-type chooses a certified claim rather than no claim if $p - C_{NI} - C_{NI}^{Cert} - K \geq -C_{NI}$, which holds if $p \geq C_{NI}^{Cert} + K$. Hence, since $C_{NI}^{Cert} - C^{UnCert} > C_I^{Cert} - C^{UnCert}$, this pooling equilibrium in case 5 can be sustained as a PBE if and only if $p \geq C_{NI}^{Cert} + K$ and $\tilde{F} \geq C_{NI}^{Cert} - C^{UnCert}$.

Case 6: The I-type chooses a certified rather than an uncertified claim if $p - C_I - C_I^{Cert} \geq -C_I - C^{UnCert} - \tilde{F}$, yielding $p \geq (C_I^{Cert} - C^{UnCert}) - \tilde{F}$, which holds by definition. The I-type chooses a certified claim rather than no claim if $p - C_I - C_I^{Cert} \geq -C_I$, implying $p \geq C_I^{Cert}$, which is satisfied by definition. Similarly, the NI-type chooses a certified rather than an uncertified claim if $p - C_{NI} - C_{NI}^{Cert} - K \geq -C_{NI} - C^{UnCert} - \tilde{F}$, which is satisfied if $p \geq (C_{NI}^{Cert} - C^{UnCert}) + K - \tilde{F}$. The NI-type chooses a certified claim rather than no claim if $p - C_{NI} - C_{NI}^{Cert} - K \geq -C_{NI}$, which holds if $p \geq C_{NI}^{Cert} + K$. Hence, since $C_{NI}^{Cert} + K > (C_{NI}^{Cert} - C^{UnCert}) + K - \tilde{F}$, this pooling equilibrium in case 6 can be sustained as a PBE if and only if $p \geq C_{NI}^{Cert} + K$.

Both types choose an uncertified claim. Consumer's beliefs are the same than in Proposition 2.

We analyze the same eight cases and we check that two of them can be supported as a PBE.

Case 5: The I-type chooses an uncertified rather than a certified claim if $p - C_I - C^{UnCert} - \tilde{F} \geq p - C_I - C_I^{Cert}$, which is satisfied if $\tilde{F} \leq C_I^{Cert} - C^{UnCert}$. The I-type chooses an uncertified claim rather than no claim if $p - C_I - C^{UnCert} - \tilde{F} \geq -C_I$, which holds if $p \geq C^{UnCert} + \tilde{F}$. Similarly, the NI-type chooses an uncertified rather than a certified claim if $p - C_{NI} - C^{UnCert} - \tilde{F} - K \geq p - C_{NI} - C_{NI}^{Cert} - K$, which is satisfied if $\tilde{F} \leq C_{NI}^{Cert} - C^{UnCert}$. The NI-type chooses an uncertified claim rather than no claim if $p - C_{NI} - C^{UnCert} - \tilde{F} - K \geq -C_{NI}$, which holds if $p \geq C^{UnCert} + \tilde{F} + K$. Therefore, this pooling equilibrium in case 5 can be supported as a PBE if and only if $p \geq C^{UnCert} + K + \tilde{F}$.

Case 6: The I-type chooses an uncertified rather than a certified claim if $p - C_I - C^{UnCert} - \tilde{F} \geq -C_I - C_I^{Cert}$, which is satisfied if $p \geq (C^{UnCert} - C_I^{Cert}) + \tilde{F}$. The I-type chooses an uncertified claim rather than no claim if $p - C_I - C^{UnCert} - \tilde{F} \geq -C_I$, which holds if $p \geq C^{UnCert} + \tilde{F}$. Similarly, the NI-type chooses an uncertified rather than a certified claim if $p - C_{NI} - C^{UnCert} - \tilde{F} - K \geq -C_{NI} - C_{NI}^{Cert}$, which is satisfied if $p \geq (C^{UnCert} - C_{NI}^{Cert}) + \tilde{F} + K$. The NI-type chooses an uncertified claim rather than no claim if $p - C_{NI} - C^{UnCert} - \tilde{F} - K \geq -C_{NI}$, which holds if $p \geq C^{UnCert} + \tilde{F} + K$. Hence, this pooling equilibrium in case 6 can be sustained as a PBE if and only if $p \geq C^{UnCert} + \tilde{F} + K$.

Both types do not claim. Consumer's beliefs are the same than in Proposition 2. We analyze the same eight cases and we check that six of them can be supported as a PBE.

Case 1: The I-type chooses no claim rather than a certified claim if $p - C_I \geq p - C_I - C_I^{Cert}$, yielding $C_I^{Cert} \geq 0$, which is satisfied by definition. The I-type chooses no claim rather than an uncertified claim if $p - C_I \geq p - C_I - C^{UnCert} - \tilde{F}$, implying $C^{UnCert} + \tilde{F} \geq 0$. Similarly, the NI-type chooses no claim rather than a certified claim if $p - C_{NI} \geq p - C_{NI} - C_{NI}^{Cert} - K$, yielding $C_{NI}^{Cert} + K \geq 0$. The NI-type chooses no claim rather than an uncertified claim if $p - C_{NI} \geq p - C_{NI} - C^{UnCert} - \tilde{F} - K$, implying $C^{UnCert} + \tilde{F} + K \geq 0$. Therefore, this pooling equilibrium in case 1 can be supported as a PBE.

Case 2: The I-type chooses no claim rather than a certified claim if $p - C_I \geq -C_I - C_I^{Cert}$, implying $p \geq -C_I^{Cert}$. The I-type chooses no claim rather than an uncertified claim if $p - C_I \geq p - C_I - C^{UnCert} - \tilde{F}$, yielding $C^{UnCert} + \tilde{F} \geq 0$. Similarly, the NI-type chooses no claim rather than a certified claim if $p - C_{NI} \geq -C_{NI} - C_{NI}^{Cert}$, implying $p \geq -C_{NI}^{Cert}$. The NI-type chooses no claim rather than an uncertified claim if $p - C_{NI} \geq p - C_{NI} - C^{UnCert} - \tilde{F} - K$, yielding $C^{UnCert} + \tilde{F} + K \geq 0$. Hence, this pooling equilibrium in case 2 can be sustained as a PBE.

Case 4: The I-type chooses no claim rather than a certified claim if $-C_I \geq -C_I - C_I^{Cert}$, yielding $C_I^{Cert} \geq 0$. The I-type chooses no claim rather than an uncertified claim if $-C_I \geq p - C_I - C^{UnCert} - \tilde{F}$, which is satisfied if $p \leq C^{UnCert} + \tilde{F}$. Similarly, the NI-type does not claim rather than choosing a certified claim if $-C_{NI} \geq p - C_{NI} - C_{NI}^{Cert} - K$, which holds if $p \leq C_{NI}^{Cert} + K$. The NI-type does not claim rather than choosing a certified claim if $-C_{NI} \geq p - C_{NI} - C^{UnCert} - \tilde{F} - K$, which is satisfied if $p \leq C^{UnCert} + \tilde{F} + K$. Hence, this pooling equilibrium in case 4 can be supported as a PBE if and if $p \leq C^{UnCert} + \tilde{F}$.

Case 5: The I-type chooses no claim rather than a certified claim if and only if $p - C_I \geq$

$p - C_I - C_I^{Cert}$, yielding $C_I^{Cert} \geq 0$. The I-type chooses no claim rather than an uncertified claim if $p - C_I \geq -C_I - C^{UnCert} - \tilde{F}$, implying $p \geq -C^{UnCert} - \tilde{F}$. Similarly, the NI-type chooses no claim rather than a certified claim if $p - C_{NI} \geq p - C_{NI} - C_{NI}^{Cert} - K$, yielding $C_{NI}^{Cert} + K \geq 0$. The NI-type chooses no claim rather than an uncertified claim if $p - C_{NI} \geq -C_{NI} - C^{UnCert} - \tilde{F}$, implying $p \geq -C^{UnCert} - \tilde{F}$. Therefore, this pooling equilibrium in case 5 can be sustained as a PBE.

Case 6: The I-type chooses no claim rather than a certified claim if $p - C_I \geq -C_I - C_I^{Cert}$, yielding $p \geq -C_I^{Cert}$. The I-type chooses no claim rather than an uncertified claim if $p - C_I \geq -C_I - C^{UnCert} - \tilde{F}$, implying $p \geq -C^{UnCert} - \tilde{F}$. Similarly, the NI-type chooses no claim rather than a certified claim if $p - C_{NI} \geq -C_{NI} - C_{NI}^{Cert}$, yielding $p \geq -C_{NI}^{Cert}$. The NI-type chooses no claim rather than an uncertified claim if $p - C_{NI} \geq -C_{NI} - C^{UnCert} - \tilde{F}$, implying $p \geq -C^{UnCert} - \tilde{F}$. Hence, this pooling equilibrium in case 6 can be supported as a PBE.

Case 8: The I-type chooses no claim rather than a certified claim if $-C_I \geq -C_I - C_I^{Cert}$, implying $C_I^{Cert} \geq 0$. The I-type chooses no claim rather than an uncertified claim if $-C_I \geq -C_I - C^{UnCert} - \tilde{F}$, yielding $C^{UnCert} + \tilde{F} \geq 0$. Similarly, the NI-type chooses no claim rather than a certified claim if $-C_{NI} \geq -C_{NI} - C_{NI}^{Cert}$, implying $C_{NI}^{Cert} \geq 0$. The NI-type chooses no claim rather than an uncertified claim if $-C_{NI} \geq -C_{NI} - C^{UnCert} - \tilde{F}$, yielding $C^{UnCert} + \tilde{F} \geq 0$. Hence, this pooling equilibrium in case 8 can be supported as a PBE.

Following the same rationale than in the proof for Proposition 2, it is easy to check that these pooling PBEs survive the Intuitive Criterion. The only difference in the proof is the penalty \tilde{F} when an uncertified claim is in place.

Proof of Proposition 7

I-type firm chooses no claim and the NI-type a certified claim. Consumer's beliefs are the same than in Proposition 3. The I-type does not claim if $-C_I \geq p - C_I - C_I^{Cert} - K$, which is satisfied if $p \leq C_I^{Cert} + K$. If $\mu_{UnCert} \leq \bar{\mu}$, the I-type chooses no claim rather than an uncertified claim if $-C_I \geq p - C_I - C^{UnCert} - \tilde{F} - K$, which holds if $\tilde{F} \geq p - K - C^{UnCert}$. If $\mu_{UnCert} > \bar{\mu}$, the I-type chooses no claim rather than an uncertified claim if $-C_I \geq -C_I - C^{UnCert} - \tilde{F}$, yielding $C^{UnCert} + \tilde{F} \geq 0$. Similarly, the NI-type chooses a certified claim if $p - C_{NI} - C_{NI}^{Cert} \geq -C_{NI}$, yielding $p \geq C_{NI}^{Cert}$, which is satisfied by definition. If $\mu_{UnCert} \leq \bar{\mu}$, the NI-type chooses a certified rather than an uncertified claim if $p - C_{NI} - C_{NI}^{Cert} \geq p - C_{NI} - C^{UnCert} - \tilde{F}$, which holds if $\tilde{F} \geq C_{NI}^{Cert} - C^{UnCert}$. If $\mu_{UnCert} > \bar{\mu}$, the NI-type chooses a certified rather than an uncertified claim if $p - C_I - C_{NI}^{Cert} \geq -C_I - C^{UnCert} - C^{UnCert}$, implying $p \geq (C_{NI}^{Cert} - C^{UnCert}) - \tilde{F}$. Therefore, this separating equilibrium can be supported as a PBE if and only if $p \leq C_I^{Cert} + K$ and $\tilde{F} \geq \max\{p - K - C^{UnCert}, C_{NI}^{Cert} - C^{UnCert}\}$. And since both types of firm do not have incentives to deviate this separating PBE survives the Intuitive Criterion.

I-type firm chooses no claim and the NI-type an uncertified claim. Consumer's beliefs are the same than in Proposition 3. The I-type does not claim if $-C_I \geq p - C_I - C^{UnCert} - \tilde{F} - K$, which is satisfied if $p \leq C^{UnCert} + \tilde{F} + K$. If $\mu_{Cert} \leq \bar{\mu}$, the I-type does not claim rather than choosing a certified claim if $-C_I \geq p - C_I - C_I^{Cert} - K$, which holds if $p \leq C_I^{Cert} + K$. If $\mu_{Cert} > \bar{\mu}$, the I-type

does not claim rather than choosing a certified claim if $-C_I \geq -C_I - C_I^{Cert}$, yielding $C_I^{Cert} \geq 0$. Similarly, the NI-type chooses an uncertified claim if $p - C_{NI} - C^{UnCert} - \tilde{F} \geq -C_{NI}$, which holds if $\tilde{F} \leq p - C^{UnCert}$. If $\mu_{Cert} \leq \bar{\mu}$, the NI-type chooses an uncertified rather than a certified claim if $p - C_{NI} - C^{UnCert} - \tilde{F} \geq p - C_{NI} - C_{NI}^{Cert}$, which is satisfied if $\tilde{F} \leq C_{NI}^{Cert} - C^{UnCert}$. If $\mu_{Cert} > \bar{\mu}$, the NI-type chooses an uncertified rather than certified claim if $p - C_{NI} - C^{UnCert} - \tilde{F} \geq -C_{NI} - C_{NI}^{Cert}$, which implies $p \geq (C^{UnCert} - C_I^{Cert}) - \tilde{F}$, where $(C^{UnCert} - C_I^{Cert}) - \tilde{F} < 0$. Therefore, this separating equilibrium can be supported as a PBE if and only if $C^{UnCert} + \tilde{F} \leq p \leq C^{UnCert} + \tilde{F} + K$, and if $p \leq C_I^{Cert} + K$ and $\tilde{F} \leq C_{NI}^{Cert} - C^{UnCert}$. Since both types of firm do not have incentives to deviate this separating PBE survives the Intuitive Criterion.

Following the proof for Proposition 3 it is easy to check that the other four possible options for separating equilibrium cannot be supported as a PBE. The only difference in the proof is the penalty \tilde{F} when an uncertified claim is in place.

Proof of Proposition 8

Both types choose a certified claim. Consumer's beliefs are the same than in Proposition 4. We analyze the same eight cases and we check that two of them can be supported as a PBE.

Case 5: The I-type chooses a certified rather than an uncertified claim if $p - C_I - C_I^{Cert} - K \geq p - C_I - C^{UnCert} - \tilde{F} - K$, which is satisfied if $\tilde{F} \geq C_I^{Cert} - C^{UnCert}$. The I-type chooses a certified claim rather than no claim if $p - C_I - C_I^{Cert} - K \geq -C_I$, which holds if $p \geq C_I^{Cert} + K$. Similarly, the NI-type chooses a certified rather than an uncertified claim if $p - C_{NI} - C_{NI}^{Cert} \geq p - C_{NI} - C^{UnCert} - \tilde{F}$, which is satisfied if $\tilde{F} \geq C_{NI}^{Cert} - C^{UnCert}$. The NI-type chooses a certified claim rather than no claim if $p - C_{NI} - C_{NI}^{Cert} \geq -C_{NI}$, implying $p \geq C_{NI}^{Cert} + K$. Therefore, this pooling equilibrium in case 5 can be supported as a PBE if and only if $p \geq C_I^{Cert} + K$ and $\tilde{F} \geq C_I^{Cert} - C^{UnCert}$.

Case 6: The I-type chooses a certified rather than an uncertified claim if $p - C_I - C_I^{Cert} - K \geq -C_I - C^{UnCert} - \tilde{F}$, which is satisfied if $p \geq (C_I^{Cert} - C^{UnCert}) - \tilde{F} + K$. The I-type chooses a certified claim rather than no claim if $p - C_I - C_I^{Cert} - K \geq -C_I$, which is satisfied if $p \geq C_I^{Cert} + K$. Similarly, the NI-type chooses a certified rather than an uncertified claim if $p - C_{NI} - C_{NI}^{Cert} \geq -C_{NI} - C^{UnCert} - \tilde{F}$, implying $p \geq (C_{NI}^{Cert} - C^{UnCert}) - \tilde{F}$, which is satisfied by definition $p \geq C_{NI}^{Cert}$. The NI-type chooses a certified claim rather than no claim if $p - C_{NI} - C_{NI}^{Cert} \geq -C_{NI}$, yielding $p \geq C_{NI}^{Cert}$, which holds by definition. Hence, this pooling equilibrium in case 6 can be supported as a PBE if and only if $p \geq C_I^{Cert} + K$.

Both types choose an uncertified claim. Consumer's beliefs are the same than in Proposition 4. We analyze the same eight cases and show that two of them can be supported as a PBE.

Case 5: The I-type chooses an uncertified rather than a certified claim if $p - C_I - C^{UnCert} - \tilde{F} - K \geq p - C_I - C_I^{Cert} - K$, which is satisfied if $\tilde{F} \leq C_I^{Cert} - C^{UnCert}$. The I-type chooses an uncertified claim rather than no claim if $p - C_I - C^{UnCert} - \tilde{F} - K \geq -C_I$, which holds if $p \geq C^{UnCert} + \tilde{F} + K$. Similarly, the NI-type chooses an uncertified rather than a certified claim if $p - C_{NI} - C^{UnCert} - \tilde{F} \geq p - C_{NI} - C_{NI}^{Cert}$, which is satisfied if $\tilde{F} \leq C_{NI}^{Cert} - C^{UnCert}$. The NI-type chooses an uncertified claim rather than no claim if $p - C_{NI} - C^{UnCert} - \tilde{F} \geq -C_{NI}$, which holds if $p \geq C^{UnCert} + \tilde{F}$. Therefore, this pooling equilibrium in case 5 can be supported as a PBE if and

only if $p \geq C^{UnCert} + \tilde{F} + K$ and $\tilde{F} \leq C_{NI}^{Cert} - C^{UnCert}$.

Case 6: The I-type chooses an uncertified rather than a certified claim if $p - C_I - C^{UnCert} - \tilde{F} - K \geq -C_I - C_I^{Cert}$, which is satisfied if $p \geq (C^{UnCert} - C_I^{Cert}) + \tilde{F} + K$. The I-type chooses an uncertified claim rather than no claim if $p - C_I - C^{UnCert} - \tilde{F} - K \geq -C_I$, which holds if $p \geq C^{UnCert} + \tilde{F} + K$. Similarly, the NI-type chooses an uncertified rather than a certified claim if $p - C_{NI} - C^{UnCert} - \tilde{F} \geq -C_{NI} - C_{NI}^{Cert}$, which is satisfied if $p \geq (C^{UnCert} - C_{NI}^{Cert}) + \tilde{F}$. The NI-type chooses an uncertified claim rather than no claim if $p - C_{NI} - C^{UnCert} - \tilde{F} \geq -C_{NI}$, which holds if $p \geq C^{UnCert} + \tilde{F}$. Hence, this pooling equilibrium in case 6 can be sustained as a PBE if and only if $p \geq C^{UnCert} + \tilde{F} + K$ and $\tilde{F} \leq C_{NI}^{Cert} - C^{UnCert}$.

Both types do not claim. Consumer's beliefs are the same than in Proposition 4. We analyze the same eight cases and we show that six of them can be supported as a PBE.

Case 1: The I-type chooses no claim rather than a certified claim if $p - C_I \geq p - C_I - C_I^{Cert} - K$, implying $C_I^{Cert} + K \geq 0$. The I-type chooses no claim rather than an uncertified claim if $p - C_I \geq p - C_I - C^{UnCert} - \tilde{F} - K$, yielding $C^{UnCert} + \tilde{F} + K \geq 0$. Similarly, the NI-type chooses no claim rather than a certified claim if $p - C_{NI} \geq p - C_{NI} - C_{NI}^{Cert}$, implying $C_{NI}^{Cert} \geq 0$. The NI-type chooses no claim rather than an uncertified claim if $p - C_{NI} \geq p - C_{NI} - C^{UnCert} - \tilde{F}$, yielding $C^{UnCert} + \tilde{F} \geq 0$. Therefore, this pooling equilibrium in case 1 can be supported as a PBE.

Case 2: The I-type chooses no claim rather than a certified claim if $p - C_I \geq -C_I - C_I^{Cert}$, implying $p \geq -C_I^{Cert}$. The I-type chooses no claim rather than an uncertified claim if $p - C_I \geq p - C_I - C^{UnCert} - \tilde{F} - K$, yielding $C^{UnCert} + \tilde{F} + K \geq 0$. Similarly, the NI-type chooses no claim rather than a certified claim if $p - C_{NI} \geq -C_{NI} - C_{NI}^{Cert}$, implying $p \geq -C_{NI}^{Cert}$. The NI-type chooses no claim rather than an uncertified claim if $p - C_{NI} \geq p - C_{NI} - C^{UnCert} - \tilde{F}$, yielding $C^{UnCert} + \tilde{F} \geq 0$. Hence, this pooling equilibrium in case 2 can be sustained as a PBE.

Case 4: The I-type chooses no claim rather than a certified claim if $-C_I \geq -C_I - C_I^{Cert}$, yielding $C_I^{Cert} \geq 0$. The I-type chooses no claim rather than an uncertified claim if $-C_I \geq p - C_I - C^{UnCert} - \tilde{F} - K$, which is satisfied if $p \leq C^{UnCert} + \tilde{F} + K$. Similarly, the NI-type chooses no claim rather than a certified claim if $-C_{NI} \geq -C_{NI} - C_{NI}^{Cert}$, implying $C_{NI}^{Cert} \geq 0$. The NI-type chooses no claim rather than an uncertified claim if $-C_{NI} \geq p - C_{NI} - C^{UnCert} - \tilde{F}$, which is satisfied if $p \leq C^{UnCert} + \tilde{F}$. Hence, this pooling equilibrium in case 4 can be sustained as a PBE if and only if $p \leq C^{UnCert} + \tilde{F} + K$.

Case 5: The I-type chooses no claim rather than a certified claim if $p - C_I \geq p - C_I - C_I^{Cert} - K$, yielding $C_I^{Cert} + K \geq 0$. The I-type chooses no claim rather than an uncertified claim if $p - C_I \geq -C_I - C^{UnCert} - \tilde{F}$, implying $p \geq -C^{UnCert} - \tilde{F}$. Similarly, the NI-type chooses no claim rather than a certified claim if $p - C_{NI} \geq p - C_{NI} - C_{NI}^{Cert}$, yielding $C_{NI}^{Cert} \geq 0$. The NI-type chooses no claim rather than an uncertified claim if $p - C_{NI} \geq -C_{NI} - C^{UnCert} - \tilde{F}$, implying $p \geq -C^{UnCert} - \tilde{F}$, which holds by definition. Therefore, this pooling equilibrium in case 5 can be supported as a PBE.

Case 6: The I-type chooses no claim rather than a certified claim if $p - C_I \geq -C_I - C_I^{Cert}$, yielding $p \geq -C_I^{Cert}$. The I-type chooses no claim rather than an uncertified claim if $p - C_I \geq -C_I - C^{UnCert} - \tilde{F}$, implying $p \geq -C^{UnCert} - \tilde{F}$. Similarly, the NI-type chooses no claim rather

than a certified claim if and only if $p - C_{NI} \geq -C_{NI} - C_{NI}^{Cert}$, yielding $p \geq -C_{NI}^{Cert}$. The NI-type chooses no claim rather than an uncertified claim if $p - C_{NI} \geq -C_{NI} - C^{UnCert} - \tilde{F}$, implying $p \geq -C^{UnCert} - \tilde{F}$. Hence, this pooling equilibrium in case 6 can be sustained as a PBE.

Case 8: The I-type chooses no claim rather than a certified claim if $-C_I \geq -C_I - C_I^{Cert}$, yielding $C_I^{Cert} \geq 0$. The I-type chooses no claim rather than an uncertified claim if $-C_I \geq -C_I - C^{UnCert} - \tilde{F}$, implying $C^{UnCert} + \tilde{F} \geq 0$. Similarly, the NI-type chooses no claim rather than a certified claim if $-C_{NI} \geq -C_{NI} - C_{NI}^{Cert}$, yielding $C_{NI}^{Cert} \geq 0$. The NI-type chooses no claim rather than an uncertified claim if $-C_{NI} \geq -C_{NI} - C^{UnCert} - \tilde{F}$, implying $C^{UnCert} + \tilde{F} \geq 0$. Hence, this pooling equilibrium in case 8 can be sustained as a PBE.

Following the same rationale than in the proof for Proposition 4, it is easy to check that these pooling equilibria survive the Intuitive Criterion. The only difference in the proof is the penalty \tilde{F} when an uncertified claim is in place.

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