

Elections and Strategic Voting: Condorcet and Borda

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- Paper tries to answer question

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- profile $U.$ - - specification of each individual's utility function

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- so, correct definition:
 - for *generic* profile $U_.$ and all $Y \subseteq X$

$$F(U_., Y) \in Y$$

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 - invoked by both Arrow (1951) and Nash (1950)

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- no “spoilers” (e.g. Nader in 2000 U.S. presidential election, Le Pen in 2002 French presidential election)

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y	z	y	$F^P(U., \{x, y\}) = y$
z	x	x	

- rank-order voting

$\frac{.55}{x}$	$\frac{.45}{y}$	$F^B(U., \{x, y, z\}) = y$
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- the members of coalition C can't all gain from misrepresenting utility functions as U'_i

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overly pessimistic - - many cases in which some rankings
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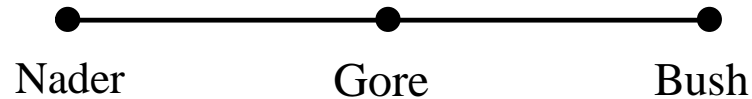
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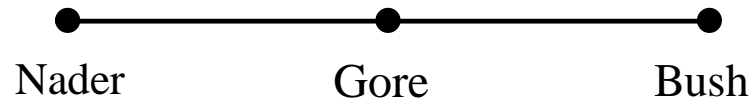
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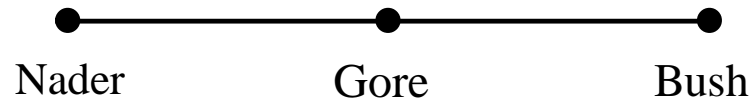
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 - in 2002 French election, 3 main candidates: Chirac, Jospin, Le Pen
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 - e.g., F^C works well when preferences single-peaked

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- shows that Theorem 1 holds when NM replaced by ordinality

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Condorcet cycle

x	y	z
y	z	x
z	x	y

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- Continuing in the same way, let $U^4 = \begin{array}{ccc} \underline{1} & \dots \underline{n-1} & \underline{n} \\ x & x & z \\ z & z & x \end{array}$

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- But not hard to show that F^C unique voting rule satisfying P,A,N, and NM when $|X| = 2$ - - contradiction

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