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Kiriti Kanjilal and Félix Muñoz-García

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Kiriti Kanjilal† and Félix Muñoz-García‡
School of Economic Sciences
Washington State University
Pullman, WA 99164

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Abstract

We consider a duopoly in which firms can strategically choose equity shares on their rival's profits before competing in quantities. We identify equilibrium equity shares, and then compare them against the socially optimal equity shares that maximize welfare. Most previous studies assume that equity shares are exogenous, and those allowing for endogenous shares do not evaluate if equilibrium shares are socially excessive or insufficient. Our results also help us identify subsidies and taxes on equity acquisition that induce firms to produce a socially optimal output.

KEYWORDS: Cournot duopoly; Endogenous equity shares; Social optimum; Equity share subsidies.

JEL CLASSIFICATION: L10, L2, D43, D6.

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†Address: 323F Hulbert Hall, Washington State University, Pullman, WA 99164. E-mail: kiriti.kanjilal@wsu.edu.

‡Address: 103G Hulbert Hall, Washington State University, Pullman, WA 99164. E-mail: fmunoz@wsu.edu.

1 Introduction

Partial cross ownership (PCO) across firms implies that two or more firms hold equity shares in each others' profits, while firms continue to operate independently. PCOs are common in several industries, such as automobiles, banks, energy, media, and financial institutions.[1] The literature has extensively analyzed the role that equity shares play in limiting firms' competition[2]; but mostly assuming that equity shares are exogenously given. For instance, Reynolds and Snapp (1986) study how equilibrium quantities in a Cournot oligopoly with symmetric firms decrease as the equity shares of any firm increase; and Fanti (2015) which, allowing for asymmetric production costs, considers that only one firm holds an exogenous participation on its rival's profit.[3]

Therefore, equity shares are exogenous in most studies. A few papers allow for endogenous equity shares. In a symmetric Cournot duopoly, Reitman (1994) shows that both firms have incentives to hold positive equity on each other's profits. His article, however, does not identify the equity shares that firms hold in equilibrium. Instead, Reitman (1994) checks if, for a given equity profile, firm i has incentives to unilaterally deviate by modifying its equity stake. In addition, the paper shows that, in oligopolies involving more than two firms, at least one firm finds it optimal to not hold equity on its rivals' profits.

Qin et al. (2017) finds the equilibrium equity shares that firms choose before competing a la Cournot. We also allow for endogenous equity acquisition, and show that our equilibrium equity shares reproduce those in Qin et al. (2017). However, we identify the socially optimal equity share that maximizes welfare, which we then compare against the equity share firms choose in equilibrium. This allows us to determine whether equilibrium equity shares are excessive, relative to the social optimum, or insufficient. We show that they are insufficient when: (1) a small proportion of output is sold domestically; and (2) the production process generates much pollution. In this setting, our results suggest that the equilibrium output in the second stage of the game is socially insufficient; as in standard Cournot models without pollution. Regulators can then induce firms to increase their equity shares, approaching them to the social optimum, by providing subsidies that lower their equity acquisition costs. In contrast, when (1) or (2) do not hold, we demonstrate that the equilibrium output that emerges during the second satge becomes socially excessive, as in models where firms' production generates a substantial pollution. In this context, equity shares that firms

[1] In the automobile industry, for example, Renault holds 44.3% equity shares in Nissan, while Nissan holds 15% in Renault; see Bárcena-Ruiz and Campo (2012) and www.nissan-global.com. Cross-ownership is also common in the financial sector, where Allianz AG owns 22.5% of Dresdner Bank, who owns 10% of Allianz AG; see La Porta et al. (1999). Other examples include only one firm holding equity shares on their rival's profits, such as Gillette, which owns 22.9% of the non-voting stock of Wilkison Sword, as reported in Gilo et al. (2006); and General Motors, which acquired 20% of Subaru's stock in 1999; see Ono et al. (2004).

[2] This result has been empirically confirmed in several industries where PCOs reduce output and increase prices, such as telecommunications, Parker and Roller (1997); energy sector in Northern Europe, Amundsen and Bergman (2002); and Italian banks, Trivieri (2007).

[3] The literature has also examined whether collusion becomes easier to sustain under PCOs. Specifically, Malueg (1992) considers a setting in which firms hold symmetric shares on each others profits, showing that collusive behavior becomes more difficult; whereas Gilo and Spiegel (2006) extend this model to a context of asymmetric equity shares, demonstrating that collusion can become easier to sustain under certain equity profiles.

choose in equilibrium are excessive; calling for a tax on share acquisition to increase firms' costs when purchasing equity. Our welfare analysis thus helps us examine a novel policy tool –subsidies and taxes on equity acquisition– which may be easier to implement than trying to observe output or sales when monitoring is imperfect.

In the field of environmental economics, Ellis and Nouweland (2006) and Kanjilal and Munoz-Garcia (2017) also allow for endogenous equity shares, but in contexts of common-pool resources (e.g., fishing grounds and forests) where fishing firms hold equity shares on each other's profits.[4] These studies, however, consider a given market price (i.e., fishing vessels sell their appropriation in a perfectly competitive international market), and allow for every firm's exploitation of the resource to generate a cost externality on its rivals. In contrast, we focus on a more standard Cournot competition, where market price is not given but decreasing in aggregate output.

In summary, previous studies analyzing firms' choice of profit-maximizing equity shares (endogenous equity acquisition) focus on how output and collusive practices are affected, but overlook their welfare implications. In contrast, our study examines the welfare consequences of endogenous equity shares, both in markets where firms' production is sold entirely in the domestic market, in those were a proportion is sold overseas, in industries where production does not generate environmental externalities (pollution), in industries where it does; and in combinations of the above. Furthermore, our findings also identify socially optimal subsidies and taxes that can be implemented to induce firms to hold welfare maximizing equity shares. For completeness, the appendix examines three extensions of our model where we allow for: (i) linear, rather than convex, cost of acquiring equity shares; (ii) convex production costs; and (iii) firms jointly choosing their equity shares in each others' profits. We show that our results are not qualitatively affected.

Subsidies and taxes on equity transactions are relatively small in percentage across countries, but they are not uncommon. 40 nations implement financial transaction taxes.[5] Our results in this paper suggest that such taxes can be used as a tool to induce a socially optimal output. While other policy tools like output subsidies and taxes can also be implemented, directly subsidizing (or taxing) equity is less costly to monitor and implement. Moreover, traditional taxes on output could reduce firm profits in equilibrium. This occurs, for instance, when equilibrium production is socially excessive, and a per unit tax is implemented, reducing profits. However, the optimal policy tool we suggest in this case, an equity subsidy, increases the seller's profits while helping to implement the socially optimal outcome.

The following section describes the model. Section 3 identifies firms' equilibrium output (in the second stage), and equilibrium equity shares (in the first stage). Section 4 finds which are the socially optimal equity shares maximizing welfare, and compares them against equilibrium equity

[4] Examples include the Northeast Tilefish fishery, Kitts et al. (2007); the Alaskan Chignik Salmon fishery, Deacon et al. (2008); and the Pacific Whiting fishery, Sullivan (2001).

[5] For instance, the US Section 31 fee imposes \$21.80 per million dollars for securities transactions; and the UK uses the Stamp Duty Reserve Tax at a rate of 0.5% on purchases of shares of companies headquartered in the UK, raising around \$US4.4 billion per year. Similar equity taxes exist in Japan, Singapore, India, France, and Sweden. Worldwide, financial transaction taxes raise more than \$US 38 billion. For a review of this taxation across different countries, see Anthony et al. (2012).

shares. Section 5 examines equity subsidies and taxes that induce firms to choose socially optimal equity shares. Finally, Section 6 discusses our results and offers policy implications.

2 Model

Consider a duopoly with two firms, 1 and 2, competing in quantities. They face an inverse demand function $p(q_i, q_j) = a - b(q_i + q_j)$, where $i = \{1, 2\}$, $j \neq i$, and $a, b > 0$. Firms have a common marginal cost c, where $a > c > 0$.[6] Every firm i's profit function is then

$$\pi_i \equiv [a - b(q_i + q_j)]\, q_i - cq_i.$$

We consider that firms i and j can hold shares in one another's profits. Shares held by firm i in firm j's profits are given by $\alpha_i \in \left[0, \frac{1}{2}\right]$. Similarly, $\alpha_j \in \left[0, \frac{1}{2}\right]$ is the share firm j holds in firm i's profits. Thus, firm i's objective function is given by:

$$V_i \equiv (1 - \alpha_j)\pi_i + \alpha_i \pi_j \tag{2.1}$$

When $\alpha_j = \alpha_i = 0$, firms do not share profits and the objective function in expression (1) collapses to π_i. In contrast, when $\alpha_i = \alpha_j = 1/2$, firm i's objective function coincides with that in a merger of symmetric firms $\frac{1}{2}(\pi_i + \pi_j)$. If $\alpha_j = 0$ but $\alpha_i > 0$, we firm i's objective function reduces to $\pi_i + \alpha_i \pi_j$, indicating that firm i obtains a share α_i on firm j's profits, whereas firm j does not receive any profits from firm i. The opposite argument applies if $\alpha_i = 0$ but $\alpha_j > 0$.

In the first stage, every firm i simultaneously and independently chooses its equity share on its rival's profit, α_i. In the second stage, firms observe the equity profile (α_i, α_j) chosen in the first period, every firm i responds selecting its output level q_i. We solve this game by backward induction.

3 Equilibrium analysis

3.1 Second stage - Optimal output

Differentiating expression (1) with respect to output q_i, yields the best response function

$$q_i(q_j) = \begin{cases} \frac{a-c}{2b} - \frac{1+\alpha_i-\alpha_j}{2(1-\alpha_j)} q_j & \text{if } q_j \leq \frac{(a-c)(1-\alpha_j)}{b(1+\alpha_i-\alpha_j)} \\ 0 \quad \text{otherwise.} \end{cases}.$$

Graphically, $q_i(q_j)$ originates at $\frac{a-c}{2b}$, which is constant in equity shares, and has a negative slope $\frac{1+\alpha_i-\alpha_j}{2(1-\alpha_j)}$, which is increasing in equity shares α_i and α_j. Therefore, $q_i(q_j)$ pivots inwards as either firm increases its equity share, i.e, as α_i and α_j increase, thus indicating that firms' output

[6] If firms exhibit convex production costs, Perry and Porter (1985) show that they may have stronger incentives to merge. In our setting, this could entail that firms have stronger incentives to acquire equity shares in each other's profits. For completeness, Appendix 2 examines how our results are affected by convex production costs.

become more intense strategic substitutes. When firms hold no equity shares, $\alpha_i = \alpha_j = 0$, the slope of best response function $q_i(q_j)$ collapses to $\frac{1}{2}$, as in standard Cournot models. When only firm i holds equity shares on firm j, $\alpha_i > 0$ but $\alpha_j = 0$, the slope becomes $\frac{1+\alpha_i}{2}$, implying that the best response function is steeper than when no firm holds equity shares. Finally, when both firms hold equity shares, the slope becomes $\frac{1+\alpha_i-\alpha_j}{2(1-\alpha_j)}$. Therefore, firm i's best response pivots inwards again.

Using $q_i(q_j)$ and $q_j(q_i)$ to simultaneously solve for q_i and q_j, we obtain the equilibrium output that firms choose in the second stage, as a function of the equity profile (α_i, α_j) chosen in the first period, as follows

$$q_i^*(\alpha_i, \alpha_j) = \frac{(a-c)(1-\alpha_i)}{(3-\alpha_i-\alpha_j)b}.$$

which is strictly positive since $a > c$ by definition. Equilibrium output is decreasing in firm i's equity share on its rival's profit, α_i, since firm i internalizes a larger portion of the price reduction that its output produces on firm j's revenue. However, equilibrium output $q_i^*(\alpha_i, \alpha_j)$ is increasing in firm j's equity share on firm i's profits, α_j. Intuitively, a larger α_j pivots firm j's best response function $q_j(q_i)$ inward, reducing this firm's equilibrium output while increasing firm i's given that best response functions are negatively sloped.

If firms choose symmetric equity shares during the first stage of the game, i.e., $\alpha_i = \alpha_j = \alpha$, optimal output collapses to $q_i^*(\alpha) = \frac{(a-c)(1-\alpha)}{(3-2\alpha)b}$, which is decreasing in α. In addition, when firms hold no equity shares on each other's profits, $\alpha_i = \alpha_j = 0$, this output collapses to $\frac{a-c}{3b}$, as in standard duopoly models.

3.2 First stage - Optimal equity shares

In the first stage, we use the output profile that arises during the second stage, $q_i^*(\alpha_i, \alpha_j)$ and $q_j^*(\alpha_i, \alpha_j)$, to find every firm i's equilibrium equity share α_i. For this, we first substitute these two terms into firm i's objective function, which yields[7]

$$\begin{aligned} & \max_{0 \leq \alpha_i \leq \frac{1}{2}} \; (1-\alpha_j)\pi_i(q_i^*, q_j^*) + \alpha_i \pi_j(q_j^*, q_i^*) - C(\alpha_i) \\ = \; & \frac{(a-c)^2(1-\alpha_j)}{(3-\alpha_i-\alpha_j)^2 b} - \delta\alpha_i^2 \end{aligned} \tag{3.1}$$

where the first term, $(1-\alpha_j)\pi_i + \alpha_i\pi_j = \frac{(a-c)^2(1-\alpha_j)}{(3-\alpha_i-\alpha_j)^2 b}$, collapses to $\frac{(a-c)^2}{9b}$ when firms hold no equity shares on each other's profits, $\alpha_i = \alpha_j = 0$. This term measures the profits that firm i obtains in the subsequent stage, from its share in its own profits and in its rival profits, and coincides with

[7] For completeness, Appendix 3 examines how our results are affected if firms jointly choose their equity shares (as in negotiations between both firms), as opposed to independently in the current setting.

that in Qin et al. (2017).[8] The second term in the firm's program, $C(\alpha_i) = \delta\alpha_i^2$, represents the cost of a acquiring equity, where $\delta \geq 0$. This cost is weakly increasing and convex in equity α_i, indicating that acquiring further equity on other firms becomes more costly as shares become more scarce. (As a robustness check, Appendix 1 shows that our results are qualitatively unaffected if we consider linear cost of acquiring equity, i.e., $C(\alpha_i) = c\alpha_i$.)

Differentiating with respect to α_i in problem (2), we obtain

$$MB_i \equiv \frac{2(a-c)^2(1-\alpha_j)}{(3-\alpha_i-\alpha_j)^3 b} = 2\alpha_i\delta \equiv MC_i \tag{3.2}$$

In addition, MB_i is increasing and convex in equity shares α_i.[9] Intuitively, if acquiring equity was costless, firms would hold as much equity as possible (as in a merger where $\alpha_i = 1/2$) given that a larger equity produces an unambigous increase in profits during the subsequent stage when firms compete a la Cournot; as recurrently shown, for the case of a duopoly, in studies evaluating firms' incentives to merge such as Perry and Porter (1985) and Levin (1990).

Since the equilibrium equity share that solves expression (3), α_i^*, is highly non-linear, Table I reports α_i^* for different cost of acquiring equity, δ (in rows), and marginal cost of production, c (in columns).[10] Intuitively, the equilibrium equity share α_i^* decreases both in the cost of acquiring equity δ, and in production cost c.

Cost of equity δ / Marginal cost c	$c = 0$	$c = 0.1$	$c = 0.3$	$c = 0.5$	$c = 0.7$	$c = 0.9$
$\delta = 0.1$	0.5	0.5	0.23	0.10	0.03	0.004
$\delta = 0.3$	0.14	0.11	0.06	0.03	0.01	0.001
$\delta = 0.5$	0.08	0.06	0.03	0.02	0.01	0.001
$\delta = 0.7$	0.06	0.04	0.02	0.01	0.005	0.001
$\delta = 0.9$	0.04	0.03	0.02	0.01	0.004	0.0004

Table I. Optimal equity share α_i^*.

When demand becomes stronger (higher a), the marginal benefit of acquiring equity increases, raising the equilibrium equity α_i^* that firms hold. When the demand function becomes steeper

[8] Expression (11) in Qin et al.'s (2017) paper, $\frac{\alpha_{ii}(a-c)^2}{\left(1+\sum_{j=1}^{n}\alpha_{jj}\right)^2}$ collapses to $\frac{\alpha_{ii}(a-c)^2}{\left(1+\alpha_{ii}+\alpha_{jj}\right)^2}$ in the case of $n = 2$ firms. Furthermore, α_{ii} in Qin et al. (2017) can be interpreted as the share that firm i holds on its own profits. In the case of $n = 2$ firms, share α_{ii} can then be rewritten as $(1-\alpha_j)$. Therefore, the term in their denominator, $(1+\alpha_{ii}+\alpha_{jj})^2$, can be expressed as $(1+(1-\alpha_j)+(1-\alpha_i))^2$ which simplifies to $(3-\alpha_i-\alpha_j)^2$. Finally, Qin et al. (2017) assume a demand function $p(Q) = a - Q$, where Q denotes aggregate output. As result, they consider that $b = 1$, ultimately implying that our expression coincides with theirs.

[9] Differentiating MB_i with respect to α_i, we find $\frac{\partial MB_i}{\partial \alpha_i} = \frac{6(1-\alpha_i)(a-c)^2}{b(3-\alpha_i-\alpha_j)^4} > 0$. Moreover, differentiating MB_i with respect to α_i again yields $\frac{\partial MB_i}{\partial \alpha_i^2} = \frac{24(1-\alpha_i)(a-c)^2}{b(3-\alpha_i-\alpha_j)^5} > 0$. In addition, evaluating MB_i at $\alpha_i = 0$, we obtain $MB_i = \frac{2(a-c)^2(1-\alpha_j)}{b(3-\alpha_j)^3}$, while evaluating MB_i at its upper bound, $\alpha_i = \frac{1}{2}$, yields $MB_i = \frac{2(a-c)^2(1-\alpha_j)}{b\left(\frac{5}{2}-\alpha_j\right)^3}$.

[10] For convenience, Table I assumes that $a = b = 1$, but other parameter combinations yield similar results, and can be provided by the authors upon request.

(higher parameter b), consumers are less price sensitive, which allows every firm to capture a larger profit margin for given equity shares. In this context, the marginal benefit from acquiring equity decreases, reducing firms' incentives to hold shares on each others' profits. In contrast, when parameter b decreases (approaching zero), competition in the Cournot duopoly becomes tougher, increasing firms' incentives to acquire equity shares to ameliorate posterior competition.

4 Welfare analysis

The social planner considers a welfare function

$$\max_{q_i, q_j} W = \gamma CS + PS - Env(q_i, q_j) \tag{4.1}$$

where $CS = \frac{b(q_i+q_j)^2}{2}$ denotes consumer surplus. For generality, parameter $\gamma \in [0, 1]$ represents the share of aggregate production $q_i + q_j$ sold domestically, which allows for $\gamma = 0$ and $\gamma = 1$ as special cases. The second term captures the producer surplus $PS = [V_i - C(\alpha_i)] + [V_j - C(\alpha_j)]$, which simplifies to $[\pi_i - C(\alpha_i)] + [\pi_j - C(\alpha_j)]$. Finally, $Env(q_i, q_j) = d(q_i + q_j)^2$ represents the environmental damage that firms' pollution generates, where $d \geq 0$. When $d = 0$, our setting collapses to the welfare function in standard duopoly models. In this case, the regulator only deals with one output distortion (insufficient production in duopoly), whereas when $d > 0$ he also faces the output distortion stemming from pollution (excessive production).

Differentiating with respect to q_i we obtain that socially optimal output is[11]

$$q_i^{SO} = \frac{a-c}{4(b+d) - 2b\gamma}$$

which is positive for all admissible parameter values.[12] We can now compare equilibrium and socially optimal output, q_i^* and q_i^{SO}, by setting $q_i^* - q_i^{SO} = 0$, and solving for α. We find that $q_i^* > q_i^{SO}$ if and only if $\alpha < \alpha^{SO}$, where

$$\alpha^{SO} \equiv 1 - \frac{b}{4d + 2b(1-\gamma)}$$

As depicted in Figure 1, when firms hold equity shares below cutoff α^{SO}, they produce a socially excessive amount of output.[13] However, when firms hold equity shares above cutoff α^{SO} (which may include a total merger, where $\alpha = 1/2$, as a special case), equilibrium output becomes socially insufficient. In addition, cutoff α^{SO} decreases in γ, implying that the region of socially excessive production shrinks as firms sell a larger share of output domestically. However, cutoff α^{SO} shifts

[11] Differentiating with respect to q_i in problem (4.1), we find that $q_i(q_j) = \frac{a-c}{2(b+d)-b\gamma} - q_j$. A symmetric expression applies when differentiating with respect to q_j. In a symmetric output profile, we obtain socially optimal output q_i^{SO}.

[12] The numerator of q_i^{SO} is positive since $a > c$ by definition. The denominator is positive for all $\gamma < 2 + \frac{2d}{b}$, which holds for all $d, b \geq 0$ since $\gamma \in [0, 1]$ by definition.

[13] Like in Table I, we consider $a = b = 1$. We also assume now an environmental damage of $d = 0.2$. Other parameter values yield similar results and can be provided by the authors upon request.

upwards when the environmental damage from production, d, increases, thus expanding the region of socially excessive production; as illustrated in Figure 1b.[14] For instance, when production does not generate pollution, $d = 0$, cutoff α^{SO} simplifies to $1 - \frac{1}{2(1-\gamma)}$; as depicted in Figure 1b. When firms sell no output domestically, $\gamma = 0$ (in the vertical intercept of cutoff α^{SO} in Figure 1b), $\alpha^{SO} = 1/2$, which implies that output is socially excessive unless firms merge, i.e., $\alpha = 1/2$. However, when firms sell more than 50% of their output domestically, equilibrium output becomes socially insufficient for all parameter values. As a reference, this includes that in which $\gamma = 1$ as a special case, where cutoff α^{SO} becomes zero.[15]

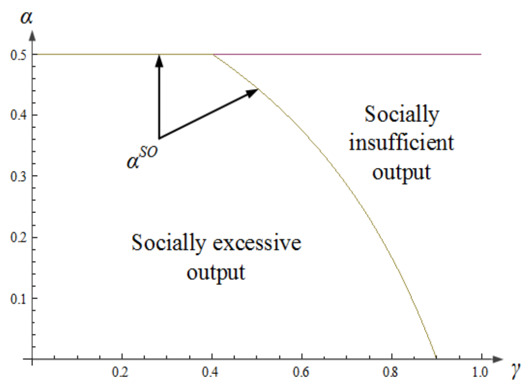

Fig. 1a. Cutoff α^{SO}.

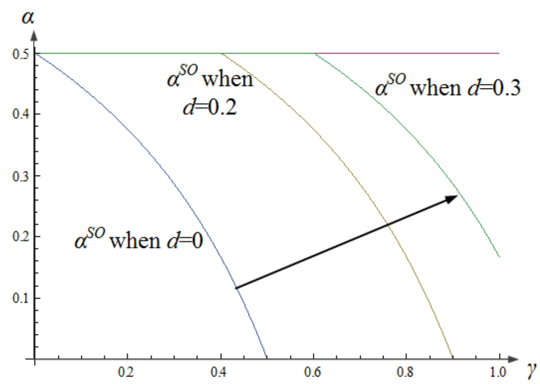

Fig. 1b. Cutoff α^{SO} when d increases.

In addition, cutoff α^{SO} satisfies $\alpha^{SO} > 0$ if and only if $\gamma < \gamma_1 \equiv \frac{1}{2} + \frac{2d}{b}$, and $\alpha^{SO} < 1/2$ if and only if $\gamma > \gamma_2 \equiv \frac{2d}{b}$. Furthermore, $\gamma_1 > \gamma_2$ since $\gamma_1 - \gamma_2 = \frac{1}{2} > 0$. Therefore, the proportion of output sold domestically must take intermediate values for cutoff α^{SO} to lie strictly inside its admissible range $[0, 1/2]$. However, when γ satisfies $\gamma > \gamma_1$, $\alpha^{SO} = 0$; whereas when $\gamma < \gamma_2$, cutoff α^{SO} becomes maximal at $\alpha^{SO} = 1/2$.[16]

[14] Furthermore, cutoff α^{SO} shifts downward as b increases, which captures the magnitude of the slope of the demand curve; producing a shrink in the region where socially excessive production can be sustained. Differentiating cutoff α^{SO} with respect to the parameter values, we obtain that $\frac{\partial \alpha^{SO}}{\partial d} = \frac{b}{(b+2d-b\gamma)^2} > 0$, $\frac{\partial \alpha^{SO}}{\partial b} = -\frac{d}{(b+2d-b\gamma)^2} < 0$, and $\frac{\partial \alpha^{SO}}{\partial \gamma} = -\frac{b^2}{(b+2d-b\gamma)^2} < 0$.

[15] Our results then connect with Levin (1990), who considers an industry with N firms, and examines their incentives to merge as well as the resulting social welfare; where his welfare function only considers consumer and producer surplus (i.e., he assumes $\gamma = 1$ and $d = 0$). If we evaluate Levin's (1990) results in the context of two firms, he shows that mergers are either profit enhancing or welfare increasing, but not both. As discussed above, in the case that $\gamma = 1$ and $d = 0$ our results show that mergers are profit enhancing but welfare reducing, in line with Levin (1990). However, we also demonstrate that equity share acquisition (and mergers) can lead to an increase in both profits and social welfare when assumptions $\gamma = 1$ and $d = 0$ are relaxed.

[16] In addition, note that cutoff γ_1 is positive for all parameter values, and satisfies $\gamma_1 < 1$ for all $d < \frac{b}{4}$. Similarly, cutoff γ_2 is positive for all parameter values, and satisfies $\gamma_2 < 1$ for all $d < \frac{b}{2}$. Therefore, three cases can arise depending on the size of environmental damages. First, when environmental damages are low, $d < \frac{b}{4}$, cutoffs γ_1 and γ_2 satisfy $\gamma_1, \gamma_2 < 1$. In this case, the admissible range of $\gamma \in [0, 1]$ is divided into three regions: $\alpha^{SO} = 1/2$ for all $\gamma < \gamma_2$, $\alpha^{SO} \in (0, 1/2)$ for all $\gamma_2 \leq \gamma < \gamma_1$, and $\alpha^{SO} = 0$ otherwise. Second, when environmental damages are

Comparing equilibrium and socially optimal equity shares, we obtain that they do not necessarily coincide, thus yielding a socially excessive equity acquisition if $\alpha_i^* > \alpha^{SO}$, or socially insufficient equity holdings if $\alpha_i^* < \alpha^{SO}$. Figure 2 superimposes equilibrium equity $\alpha_i^* = 0.23$ on Figure 1a, which occurs when $c = 0.3$ and $\delta = 0.1$; as reported in the upper row of Table I.

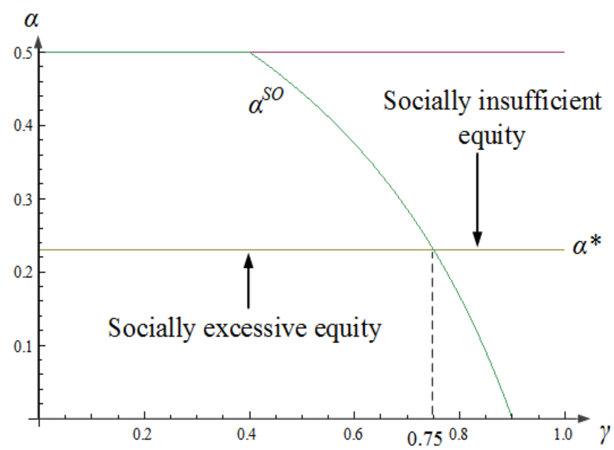

Fig. 2. Comparing α_i^* and α^{SO}.

5 Equity share subsidies and taxes

In this section, we examine how government agencies can design subsidies (or taxes) inducing every firm i to hold an equilibrium equity share α_i^* that coincides with the socially optimal equity share α^{SO} found above. Since equilibrium equity is a function of the cost of acquiring shares, we can express it as $\alpha_i^*(\delta)$, which is decreasing in δ. Hence, the social planner can identify the subsidy s that solves $\alpha_i^*(\delta - s) = \alpha^{SO}$, i.e., a subsidy if $s > 0$ or a tax otherwise. Monitoring equity transactions is often done for legal and accounting reasons, thus being a policy easier to implement than monitoring output or sales. Table II reports, for different values of γ (in rows), the socially optimal equity, α^{SO}, equity shares without subsidies, α_i^*, and the subsidy s that induces firms to

intermediate, $\frac{b}{4} \leq d < \frac{b}{2}$, cutoffs γ_1 and γ_2 satisfy $\gamma_1 > 1$ and $\gamma_2 < 1$. In this case, the admissible range of $\gamma \in [0,1]$ is divided into two regions alone: $\alpha^{SO} = 1/2$ for all $\gamma < \gamma_2$, $\alpha^{SO} \in (0, 1/2)$ otherwise. Finally, when environmental damages are large, $d \geq \frac{b}{2}$, cutoffs γ_1,$\gamma_2 > 1$, implying that $\alpha^{SO} = 1/2$ for all values of γ.

choose α^{SO} in equilibrium.[17]

Domestic sales γ	Optimal equity α^{SO}	Equil. equity α_i^*	Subsidy/tax s
$\gamma = 0$	0.5	0.23	0.04
$\gamma = 0.1$	0.5	0.23	0.04
$\gamma = 0.5$	0.44	0.23	0.03
$\gamma = 0.7$	0.29	0.23	0.02
$\gamma = 0.8$	0.17	0.23	-0.03
$\gamma = 1$	0	0.23	-18.07

Table II. Optimal equity subsidies.

When γ is low (i.e., most output is sold overseas), we have that $\alpha^{SO} > \alpha_i^*$ (see first rows). In this context, the social planner can subsidize firms so they acquire more equity. In contrast, when γ is relatively high, we obtain that $\alpha^{SO} < \alpha_i^*$, and the social planner can tax equity acquisition. For instance, when $\gamma = 0.7$, socially optimal equity is $\alpha^{SO} = 0.29$, whereas equilibrium equity is only $\alpha_i^* = 0.23$. A subsidy solving $\alpha_i^*(\delta - s) = 0.29$ induces firms to choose α^{SO}, which yields $s = 0.02$.

If the cost of acquiring equity, δ, increases, equilibrium equity α_i^* decreases (as shown in Table I) while optimal equity α^{SO} is unaffected. In this setting, the subsidy s that the planner offers to induce optimal equity acquisition must become more generous. A similar argument applies when production cost c increases since α_i^* decreases whereas optimal equity α^{SO} is unchanged.

In contrast, when demand becomes stronger (higher a), firms increase their equilibrium equity α_i^* since sharing profits becomes more attractive, but optimal equity α^{SO} remains unaffected relative to Table I since α^{SO} is not a function of parameter a . Table III considers that a increases from $a = 1$ in Tables I-II to $a = 2$. In this setting, the regulator finds that $\alpha_i^* = \alpha^{SO}$ when most output is sold overseas, but otherwise sets a tax to curb equity acquisition.

Domestic sales γ	Optimal equity α^{SO}	Equil. equity α_i^*	Subsidy/tax s
$\gamma = 0$	0.5	0.5	0
$\gamma = 0.1$	0.5	0.5	0
$\gamma = 0.5$	0.44	0.5	-0.29
$\gamma = 0.7$	0.29	0.5	-0.40
$\gamma = 0.8$	0.17	0.5	-0.65
$\gamma = 1$	0	0.5	-107.04

Table III. Optimal equity subsidies with stronger demand.

[17] Like in Table I, we consider $a = b = 1$. We also assume now an environmental damage $d = 0.2$, production cost $c = 0.3$ and a cost of acquiring equity of $\delta = 0.1$. Other parameter values yield similar results and can be provided by the authors upon request.

When demand becomes steeper (higher b), firms reduce their equilibrium equity α_i^* (from $\alpha_i^* = 0.23$ in Table II to $\alpha_i^* = 0.10$ in Table IV), and so does optimal equity α^{SO} (compare the second column in Tables II and IV). As a consequence, $\alpha^{SO} > \alpha_i^*$ when most output is sold overseas, leading the regulator to offer a subsidy on equity acquisition; whereas $\alpha^{SO} < \alpha_i^*$ when most output is sold domestically, which yields a tax on equity acquisition.

Domestic sales γ	Optimal equity α^{SO}	Equil. equity α_i^*	Subsidy/tax s
$\gamma = 0$	0.5	0.10	0.07
$\gamma = 0.1$	0.5	0.10	0.07
$\gamma = 0.5$	0.28	0.10	0.06
$\gamma = 0.7$	0	0.10	-8.99
$\gamma = 0.8$	0	0.10	-8.99
$\gamma = 1$	0	0.10	-8.99

Table IV. Optimal equity subsidies with steeper demand.

6 Discussion

An alternative policy. Governments commonly use price-based policy tools, such as per-unit subsidies and taxes to alter firms' production decisions. Output subsidies, for instance, induce firms to increase their production, which is a common policy in industries where few companies compete, and thus unregulated output is naturally low. In contrast, per-unit taxes seek to curb excessive production, and thus are often introduced in polluting industries (e.g., emission fees). Alternatively, regulatory agencies can set quantity-based systems, such as production quotas, that firms must comply with. While these policy tools are often effective, they entail monitoring and supervision costs, which are often substantial.[18] Our paper suggests an alternative policy tool, subsidies or taxes on equity share acquisition. Since firms must regularly inform about their equity holdings on other firms, this policy can be easier to monitor than price or quantity-based policies. Importantly, equity share policies operate before firms choose their equity and, as a consequence, prior to their competition with other firms in subsequent periods. In addition, this policy does not require ex-post monitoring of firms' output or prices. Intuitively, the policy provides every firm with the incentives to acquire the socially optimal level of equity shares, α^{SO}, which implies that firms' production decisions in subsequent stages are socially optimal as well.

High or low equity subsidies? Our results also help us understand the size of equity share subsidies in different settings. First, when a large proportion of the good is sold domestically, the social planner seeks a higher output level which, in turn, entails a lower level of socially optimal equity. In this case, equity subsidies must be low, and may even become taxes if firms' cost of

[18] For empirical estimations of the monitoring costs of production quotas in Canada, see Grafton (1996). Hatcher (1998) provides a similar estimation for quotas in the UK, and Milazzo (1998) for the U.S.

acquiring equity is low and/or their production does not generates large negative externalities (e.g., pollution). Conversely, when a small proportion of the good is sold domestically, subsidies need to be larger; an effect that is augmented if pollution is substantial and/or if demand is relatively weak.

Pollution effects. More damaging pollution produces the opposite effect than a larger proportion of goods being sold domestically. Intuitively, since pollution reduces social welfare, the social planner seeks to induce lower production levels, which in our findings can be done via subsidies on equity shares. Conversely, if the production process does not generate much pollution (relative to its consumer and producer surplus), the regulator seeks to induce a larger production by taxing equity share acquisition.

7 Appendix

7.1 Appendix 1 - Extension to linear cost of equity

In this appendix, we examine how our results are affected if we consider linear cost of acquiring equity, $C(\alpha_i) = c\alpha_i$, which indicates that firm i spends c in every unit of equity regardless of the equity share it owns on its rival's profit. In this context, equation (3.2) in the main body of the paper becomes

$$MB_i \equiv \frac{2(a-c)^2(1-\alpha_j)}{(3-\alpha_i-\alpha_j)^3 b} = \delta \equiv MC_i.$$

In this setting, the MB_i coincides with that in expression (3.2), but the MC_i curve is now flat. As a result, an increase in c produces an increase in equilibrium equity, as illustrated in Table A-I. For comparison purposes, the table considers the same parameter values as Table I in the main text.

Cost of equity δ / Marginal cost c	$c = 0$	$c = 0.1$	$c = 0.3$	$c = 0.5$	$c = 0.7$	$c = 0.9$
$\delta = 0.1$	0.29	0.49	0.5	0.5	0.5	0.5
$\delta = 0.3$	0.5	0.5	0.5	0.5	0.5	0.5
$\delta = 0.5$	0.5	0.5	0.5	0.5	0.5	0.5
$\delta = 0.7$	0.5	0.5	0.5	0.5	0.5	0.5
$\delta = 0.9$	0.5	0.5	0.5	0.5	0.5	0.5

Table A-I. Optimal equity share α_i^*.

We also provide next Table A-II, which considers the same parameter values as Table II in the main text, in order to examine how optimal subsidies are affected by a linear cost of equity function. Optimal equity α^{SO} is unaffected relative to Table II, but equilibrium equity α^* increases since acquiring equity is now cheaper than in Table II. Since α^* increases in δ in this context, the regulator needs to set a tax when $\alpha^* < \alpha^{SO}$, and a subsidy otherwise.

Domestic sales γ	Optimal equity α^{SO}	Equil. equity α_i^*	Subsidy/tax s
$\gamma = 0$	0.5	0.5	0
$\gamma = 0.1$	0.5	0.5	0
$\gamma = 0.5$	0.44	0.5	0.07
$\gamma = 0.7$	0.29	0.5	0.08
$\gamma = 0.8$	0.17	0.5	0.08
$\gamma = 1$	0	0.5	0.08

Table A-II. Optimal equity subsidies.

7.2 Appendix 2 - Extension to convex production costs

In this appendix, we explore how our results change when firms face a convex production cost $C(q_i) = c\,(q_i)^2$, where $c > 0$. For simplicity, we assume $a = b = 1$. In this context, every firm i's profit π_i is

$$\pi_i \equiv [1 - (q_i + q_j)]\, q_i - c\,(q_i)^2 \,.$$

Equilibrium output. Using this definition of π_i in problem (3.1), and differentiating with respect to q_i, we obtain best response function

$$q_i(q_j) = \begin{cases} \frac{1}{2(1+c)} - \frac{(1+\alpha_i-\alpha_j)}{2(1-\alpha_j)(1+c)} q_j & \text{if } q_j \leq \frac{(1-\alpha_j)}{(1+\alpha_i-\alpha_j)} \\ 0 \quad \text{otherwise.} \end{cases} .$$

where the vertical axis is $\frac{1}{2(1+c)}$ thus being unaffected by firms' equity shares. Like in the main body of the paper, when firms hold no equity shares, $\alpha_i = \alpha_j = 0$, the best response function collapses to $\frac{1}{2(1+c)} - \frac{1}{2(1+c)} q_j$. When only firm i holds equity shares on firm j's profits, $\alpha_i > 0$ but $\alpha_j = 0$, the best response function pivots inwards becoming $\frac{1}{2(1+c)} - \frac{(1+\alpha_i)}{2(1+c)} q_j$. Finally, when both firms sustain positive equity shares, $\alpha_i, \alpha_j > 0$, the best response function pivots inwards even further.

Simultaneously solving for output levels q_i and q_j, we find

$$q_i^* = \frac{(1-\alpha_i)\left[(1-\alpha_i-\alpha_j) + 2(1-\alpha_j)c\right]}{(3-\alpha_i-\alpha_j)(1-\alpha_i-\alpha_j) + 8(1-\alpha_i)(1-\alpha_j)c + 4(1-\alpha_i)(1-\alpha_j)c^2}$$

which is positive under all parameter values.

Equilibrium equity. We now substitute the solution for q_i^* and q_j^* into the profit function of firm i, π_i, and obtain $\pi_i(\alpha_i, \alpha_j)$, which represents the profit that firm i earns during the second stage as a function of equity shares α_i and α_j. We can now insert profit $\pi_i(\alpha_i, \alpha_j)$ into firm i's equity choice in the first-period game, as follows.

$$\max_{0 \leq \alpha_i \leq \frac{1}{2}} \; (1-\alpha_j)\pi_i + \alpha_i \pi_j - C\,(\alpha_i)$$

where, similarly as in expression (3.2), $C(\alpha_i) = \delta\alpha_i^2$ captures the cost of acquiring equity. Differentiating the above expression with respect to equity share α_i to obtain firm i's marginal benefit of acquiring equity, MB_i. This marginal benefit is, however, very large in this setting of convex production costs. For tractability, we do not provide the expression of MB_i here. However, we set $MB_i = MC_i$, and Table A-III below provides an analogous simulation of optimal equity share α_i^* as in Table I of the paper using the same parameter values.

Cost of equity δ / Marginal cost c	$c = 0$	$c = 0.1$	$c = 0.3$	$c = 0.5$	$c = 0.7$	$c = 0.9$
$\delta = 0.1$	0.5	0.5	0.5	0.5	0.47	0.45
$\delta = 0.3$	0.14	0.15	0.16	0.15	0.14	0.14
$\delta = 0.5$	0.08	0.09	0.09	0.09	0.09	0.08
$\delta = 0.7$	0.06	0.06	0.06	0.06	0.06	0.06
$\delta = 0.9$	0.04	0.05	0.05	0.05	0.05	0.04

Table A-III. Optimal equity share α_i^* with convex production costs.

Overall, equilibrium equity share α_i^* is larger when firms face convex than linear production costs. Intuitively, firms have stronger incentives to acquire equity on their rivals' profits when facing convex production costs, since this allows the firm to reduce its total costs. In addition, equilibrium equity α_i^* decreases as the cost of acquiring additional equity, δ, increases.

Welfare analysis. Like in the paper, we next identify the socially optimal output in this context with convex production costs, q^{SO}, and find the corresponding cutoff α^{SO}. The welfare function is given by expression (4.1), but where firm i's profit is now $\pi_i \equiv [1 - (q_i + q_j)] q_i - c(q_i)^2$. This yields a socially optimal output of

$$q^{SO} = \frac{1}{2(2 + c + 2d - \gamma)}$$

which is positive under all parameter values. As a next step, we evaluate equilibrium output q_i^* in the case of a symmetric equilibrium in the first stage, $\alpha_i = \alpha_j = \alpha$, as described above. This yields an equilibrium output

$$q_i^* = \frac{(1 - \alpha_i)}{3 + 2c - 2\alpha_i(1 + c)}$$

Thus, the condition for which equilibrium output is socially excessive, $q_i^* > q^{SO}$, is

$$\alpha < \alpha^{SO} \equiv 1 - \frac{1}{2(1 + 2d - \gamma)}$$

Cutoff α^{SO} is identical to the cutoff we found with linear production costs and $b = 1$. Intuitively, the welfare function in expression (4.1) considers consumer surplus, producer surplus, and environmental damage from production. However, producer surplus collapses to $\pi_i + \pi_j$, which coincides with $[V_i - C(\alpha_i)] + [V_j - C(\alpha_j)]$, where $V_i = (1 - \alpha_j)\pi_i + \alpha_i\pi_j$ for every firm i. Therefore, convex costs symmetrically affect the firm's and the social planner's problem. In contrast, consumer surplus and environmental damage are not affected by the convexity of production costs, ultimately implying that cutoff α^{SO} is unaffected by the type of production costs (linear or convex) that the firm faces. Therefore, the comparative statics of this cutoff remain the same.

Domestic sales γ	Optimal equity α^{SO}	Equil. equity α_i^*	Subsidy/tax s
$\gamma = 0$	0.5	0.5	0
$\gamma = 0.1$	0.5	0.5	0
$\gamma = 0.5$	0.44	0.5	-0.02
$\gamma = 0.7$	0.29	0.5	-0.08
$\gamma = 0.8$	0.17	0.5	-0.18
$\gamma = 1$	0	0.5	-43.38

Table A-IV. Optimal equity subsidies with convex production costs.

Relative to the setting with linear production costs (Table II in the main body of the paper), socially optimal equity α^{SO} does not change, while equilibrium equity α_i^* is higher, ultimately yielding lower equity subsidies when γ is relatively low, or more stringent taxes when γ is high.

7.3 Appendix 3 - Extension to joint equity share acquisition

Previous sections considered that every firm independently chooses its equity shares. In some settings, however, firms may negotiate with each other their equity holding. In this appendix, we explore how our findings are affected when firms jointly choose their equity shares in each other's profits, α_i and α_j, solving the following joint-maximization problem

$$\begin{aligned} \max_{\alpha_i,\alpha_j \geq 0} \; & (1-\alpha_j)\pi_i\left(q_i^*, q_j^*\right) + \alpha_i \pi_j\left(q_i^*, q_j^*\right) - (F + 2\delta\alpha_i^2) \\ & +(1-\alpha_i)\pi_j\left(q_i^*, q_j^*\right) + \alpha_j \pi_i\left(q_i^*, q_j^*\right) - (F + 2\delta\alpha_j^2) \end{aligned}$$

Differentiating with respect to α_i we obtain

$$MB_i \equiv \frac{(a-c)^2(1-\alpha_i-\alpha_j)}{(3-\alpha_i-\alpha_j)^3 b} = 2\delta\alpha_i \equiv MC_i$$

and when differentiating with respect to α_j yields

$$MB_j \equiv \frac{(a-c)^2(1-\alpha_i-\alpha_j)}{(3-\alpha_i-\alpha_j)^3 b} = 2\delta\alpha_j \equiv MC_j$$

where $MB_i = MB_j$.[19] In a symmetric equilibrium, $\alpha_i^* = \alpha_j^* = \alpha^*$. Since the above first-order conditions are highly nonlinear, we cannot obtain an explicit solution for equilibrium equity share α^*. Following a similar approach as in Section 3.2, Table A-V below numerically solves for the equilibrium equity share α^*. For comparison purposes, we consider the same parameter values as in Table I.

Cost of equity δ / Marginal cost c	$c = 0$	$c = 0.1$	$c = 0.3$	$c = 0.5$	$c = 0.7$	$c = 0.9$
$\delta = 0.1$	0.17	0.14	0.09	0.05	0.02	0(0.002)
$\delta = 0.3$	0.06	0.05	0.03	0.02	0.01	0(0.0006)
$\delta = 0.5$	0.04	0.03	0.02	0.01	0(0.003)	0(0.0004)
$\delta = 0.7$	0.03	0.02	0.01	0.01	0(0.002)	0(0.0003)
$\delta = 0.9$	0.02	0.02	0.01	0.01	0(0.002)	0(0.0002)

Table A-V. Equilibrium equity share α^*.

Therefore, equity shares are lower when firms jointly determine their equity holdings than when they independently select their equity, i.e., compare every cell in Table A-V against the same cell in Table I. In words, when every firm i independently increases its equity share on its rival, α_i, it ignores the profit reduction that such an increase produces on the other firm's objective function. However, firms internalize this external effect in the joint-maximization problem, leading them to acquire less equity on each other's profits.

For completeness, we also report Table A-VI (analogous to Table II in Section 5), which compares the equilibrium equity share α^*, the socially optimal equity share α^{SO}, and the subsidy s that lowers firms' cost of equity acquisition to induce firms to hold a socially optimal equity in equilibrium α^{SO}. Since equilibrium equity shares are lower than when firms independently choose α_i, the regulator needs to provide a more generous subsidy to induce the optimal equity share α^{SO}. Therefore, when firms jointly choose their equity shares on each other's profits, regulators can expect lower equity shares in equilibrium, implying that subsidies would need to be more generous to induce optimal outcomes.

[19] In addition, MB_i originates at $\frac{(a-c)^2(1-\alpha_j)}{(3-\alpha_j)^3 b}$ when $\alpha_i = 0$, and reaches a height of $\frac{(a-c)^2(\frac{1}{2}-\alpha_j)}{(\frac{5}{2}-\alpha_j)^3 b}$ when $\alpha_i = 1/2$. Furthermore, MB_i is decreasing in firm i's equity share since $\frac{\partial MB_i}{\partial \alpha_i} = -\frac{2(a-c)^2(\alpha_i+\alpha_j)}{(3-\alpha_i-\alpha_j)^4 b} < 0$, and convex in firm i's equity share since $\frac{\partial^2 MB_i}{\partial \alpha_i^2} = -\frac{6(a-c)^2(1+\alpha_i+\alpha_j)}{(3-\alpha_i-\alpha_j)^5 b} < 0$.

Domestic sales γ	α^*	α^{SO}	Subsidy s
$\gamma = 0$	0.5	0.09	0.1
$\gamma = 0.1$	0.5	0.09	0.1
$\gamma = 0.5$	0.44	0.09	0.09
$\gamma = 0.7$	0.29	0.09	0.07
$\gamma = 0.8$	0.17	0.09	0.05
$\gamma = 1$	0	0.09	-8.97

Table A-VI. Optimal equity share α^*.

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