Common Pool Resources with Endogenous Equity Shares*

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Abstract

We consider a common pool resource (CPR) where, in the first stage, every firm chooses an equity share on its rivals' profits (cross-ownership) and, in the second stage, firms compete for the resource. We identify equilibrium equity shares in this setting, and compare them against the socially optimal shares that maximize welfare. Our results show that equity shares are welfare improving under certain conditions, but can lead to a socially insufficient exploitation of the CPR if shares are large enough; as in a merger where firms equally share equity. We also find that, as the number of firms exploiting the resource increases, socially excessive exploitation occurs under larger parameter combinations. We analyze common policy tools, such as quotas and emission fees, evaluating how they are affected by equity shares; and then compare them against a novel policy tool: optimal equity subsidies.

Keywords: Common pool resources; Endogenous equity shares; Social optimum; Emission fees; Equity share subsidies.
JEL classification: D21, D62, Q5.

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1 Introduction

Firms hold equity shares on each other's profits in common pool resources (CPRs), especially in fisheries where cross-ownership is often maximal. Firms equally share profits in this context, and their behavior coincides with that under a merger (or a multi-plant monopoly) where firms coordinate their appropriation levels.[1] In the case of U.S. "corporate-cooperative management" fisheries, for instance, companies exploiting the resource coordinate their appropriation decisions as a single entity. Examples include the Northeast Tilefish fishery, Kitts et al. (2007); the Alaskan Chignik Salmon fishery, Deacon et al. (2008); the Pacific Whiting fishery, Sullivan (2001); and the Bearing Sea Pollock fishery, Kitts and Edwards (2003).[2,3] The Whiting and Pollock conservation cooperatives, for instance, significantly reduced catches, as reported by the private fishery harvest monitoring service SeaState, Inc. and the U.S. National Marine Fisheries Service; see Sullivan (2001). In this paper, we study in which cases equity shares can help to protect the CPR by reducing appropriation and when, instead, they lead to an under-exploitation of the CPR, thus becoming socially damaging.

Our model considers that, in the first stage, every firm chooses its equity share on the other firm's profits; and, in the second stage, observing the profile of equity shares, every firm selects its appropriation level of the resource. First, we identify the equity shares that firms choose in equilibrium, anticipating how their share will affect subsequent competition for the resource. Second, we find the equity shares that maximize social welfare. We then identify in which cases equilibrium equity shares fall below the socially optimal level, and when they exceed this level. This comparison helps us explore a novel policy tool —subsidies (or taxes) on equity acquisition— which induces firms to hold socially optimal levels of equity before competing for the resource. Unlike

[1] Equity shares across rival firms are also common in other sectors, and are often referred to as "partial cross ownership." In the automobile industry, for instance, Renault currently holds 44.3% equity shares in Nissan, while Nissan holds 15% in Renault; see Bárcena-Ruiz and Campo (2012) and www.nissan-global.com. Cross-ownership is also common in the financial sector, where Allianz AG holds 5% of Deutsche Bank, while Deutsche Bank holds 10% in Allianz AG. Similarly, Allianz AG owns 22.5% of Dresdner Bank, who owns 10% of Allianz AG; see La Porta et al. (1999). In the U.S. mutual funds industry, State Street Corporation (STT) owns minority shares in other funds, such as 4.77% of T. Rowe Price and 3.05% in Black Rock. Similarly, T. Rowe Price Group owns 3.28% of STT, and Black Rock Inc. owns 2.68% of T. Rowe Price Group; see Levy and Szafarz (2017). Other examples include only one firm holding equity shares on their rival's profits, such as Gillette, which owns 22.9% of the non-voting stock of Wilkison Sword, Gilo et al. (2006); Ford, which purchased 25% of Mazda's shares in 1979; and General Motors, which acquired 20% of Subaru's stock in 1999; see Ono et al. (2004). Finally, this type of partial ownership is also common in the media sector, where Comcast owns multiple large media outlets including NBC, The Weather Channel, and CNBC; Time Warner owns CNN, HBO and Cartoon Network; and News Corp, which owns the Wall Street Journal and the New York Post.

[2] In "corporate management" systems, however, firms transfer their appropriation decisions to a separate corporation, which centrally determines the appropriation levels for each member. While these systems have not been fully implemented yet, some fisheries have adopted variants of this approach since 1995, such as New Zealand's Bluff Oyster and Challenger Scallop fisheries, Yang et al. (2014); and Australia's Exmouth Gulf Prawn fishery, Rogers (2009).

[3] While cooperatives of individuals or firms exploiting a CPR fit our model, "catch share" programs do not. In catch shares, such as those supported by NOAA, a portion of the catch for a species of fish is allocated to individual fishermen. Some programs allow every fisherman to purchase a larger catch share from other fishermen, which lets the fisherman increase his individual appropriation. The catch share program, however, does not provide the fisherman with a proportion of other fishermen profits.

common policies, such as quotas or emission fees, equity share subsidies do not require posterior supervision or monitoring of firms' appropriation (e.g., fish catches at ports and vessel inspections). If monitoring costs are large enough, equity subsidies can become a preferred policy tool.

We first identify firms' equilibrium appropriation in the second stage of the game. When equity shares are absent, every firm considers only its own profit when choosing its appropriation level; and when equity shares are present, every firm considers both companies' profits while choosing appropriation, which reduces its exploitation of the resource.[4] We then compare equilibrium and socially optimal levels of exploitation, finding under which parameter combinations: (i) a socially excessive exploitation of the resource emerges (overexploitation, as in standard CPRs problems); (ii) a socially insufficient appropriation arises (underexploitation); and (iii) for which parameters firms' exploitation exactly coincides with the social optimum.

Intuitively, when every firm shares a high proportion of its profits with other companies, its profit-maximization problem resembles that of a merged firm. Each firm reduces its appropriation since it now internalizes the cost externality that its exploitation imposes on its rival. We demonstrate that, if such a reduction in output is relatively small, the presence of equity shares can help equilibrium appropriation approach its socially optimal level. In other words, equity shares can ameliorate the over-exploitation of the stock that arises in the commons. Specifically, the welfare benefits from the output reduction due to equity shares (larger profits and a lower environmental damage) exceed the welfare loss (reduction in consumer surplus), ultimately increasing overall welfare. In this context, regulators can provide subsidies that lower firms' cost of equity acquisition, inducing them to increase their equity on rivals' profits, ultimately increasing welfare. If equity shares are significant, however, the decrease in appropriation can be severe, leading firms to exploit the resource below what the social planner would recommend. In this case, firms' equilibrium behavior changes from an overexploitation of the stock (under no equity shares) to an underexploitation (when equity shares are significant). In extreme settings where firms equally share profits (such as in a merger like those discussed above in fishing grounds), our results find that underexploitation can be sustained under large parameter combinations. In this context, the welfare benefits from equity shares do not offset its welfare loss, thus providing regulators with incentives to tax equity shares in such a CPR.

Specifically, we show that the presence of equity shares produces a small reduction in output, thus helping ameliorate overexploitation, when the following conditions hold: (1) firms sell most of their appropriation overseas; (2) the resource is abundant; (3) firms' exploitation of the resource generates large environmental externalities (e.g., pollution from fishing vessels such as oil and water discharges);[5] (4) demand is relatively low; and (5) several firms exploit the commons. In these

[4] This result has been empirically confirmed in several industries where cross-ownership reduces output, such as telecommunications, Parker and Roller (1997); Italian banks, Trivieri (2007); and energy industry in Northern Europe, Amundsen and Bergman (2002).

[5] For instance, Albert et al. (2014) tested wastewater from fishing boats finding that, out of 48 substances found in hold water, 19 exceeded safety benchmarks. Fifteen out of 35 exceeded the benchmarks in cleaning water, including arsenic, dissolved copper, and ammonia. Similarly, Topping et al. (1997) found that 75.2% of vessels operating on Canada's coast threw debris into the sea. Another example is in Australia, where debris found in beaches (60 items

cases, the social planner's problem becomes similar to that of firms, as consumer surplus and environmental damage are both minor. In contrast, when one or more conditions of (1)-(5) do not hold, firms decrease their equilibrium appropriation below the social optimum, ultimately yielding an under-exploitation of the resource.

We identify the equity share that firms choose in equilibrium during the first stage, and compare it against the social optimum. We demonstrate that equilibrium equity can be insufficient when firms face large costs in acquiring it, and when conditions (1)-(5) hold. Intuitively, the socially optimal appropriation is relatively low in this setting, which can only be reached with significant equity shares. Our results also identify the optimal equity subsidy which decreases equity acquisition costs enough to induce firms to hold equity shares at the socially optimal level. In contrast, when equity acquisition costs are low, or conditions (1)-(5) do not hold, firms acquire excessive equity shares.

Our results suggest that equity subsidies can be an alternative policy tool in some CPRs where common policies, such as quotas or emission fees, require costly ex-post monitoring (e.g., in the case of fisheries, supervising catches at port, searching vessels, and setting fines if necessary). In contrast, equity subsidies only require firms to report information about their equity acquisitions, which is often collected for accounting and tax purposes anyway, without the need to conduct ex-post monitoring. Equity subsidies are, thus, especially attractive when monitoring costs are relatively high; as empirically shown in fisheries by Grafton (1996) for Canada, Hatcher (1998) for the UK, Milazzo (1998) for the U.S., and Arnason (2000) for Norway.[6] Equity subsidies can, nonetheless, be combined with other policies. Indeed, our setting also allows for a policy mix between equity subsidies (before firms choose their equity shares) and fees (after selecting their equity shares). In this case, equity subsidies are lower than in a setting with equity subsidies alone, while emission fees are less stringent than in a context with emission fees alone. As such, the policy mix can provide two benefits: subsidies would be less expensive to fund (an important point if tax collection produces large market distortions), and less stringent emission fees would probably face less political resistance.

Subsidies on equity acquisition are relatively minor in most countries, although 40 nations use different forms of taxes or subsidies on financial transactions.[7] However, our results show that the implementation of these subsidies could be attractive to firms in affected industries. To see this point, we first demonstrate that, in all settings under which the exploitation of the CPR

per km) was identified as garbage from fishing boats in the sea; see Jones (1995). Finally, Perez et al. (2017) found that fishing vessels deliver a larger quantity of oily waste than cruise ships.

[6] In particular, monitoring costs in the UK for fisheries were estimated at 7.5% of the landings in 1996/97, Hatcher (1998); 15% of the landings in the U.S., Milazzo (1998); 8% in Norway and at least 15% in Newfoundland, Arnason (2000). Similarly, a report by MRAG (2007) states that the monitoring of quotas in the Northertn Prawn fishery is approximately 2 million AUD a year.

[7] Worldwide, financial transaction taxes raise more than $US 38 billion. For instance, the US Section 31 fee imposes $21.80 per million dollars for securities transactions; and the UK uses the Stamp Duty Reserve Tax at a rate of 0.5% on purchases of shares of companies headquartered in the UK, raising around $US4.4 billion per year. Similar equity taxes exist in France, Sweden, Taiwan, Singapore, Japan, and India. For a detailed review of this type of taxation across different countries, see Anthony et al. (2012).

is socially excessive, a traditional fee on appropriation would be positive, or a quota limiting exploitation would be binding, thus reducing firms' profits in equilibrium. In contrast, under the same parameter conditions, we identify that a subsidy on equity acquisition is profit-enhancing and socially optimal. Firms would then have incentives to a lobby in favor of a change in policy tools (moving from current emission fees and quotas to equity subsidies), as their equilibrium profits would be larger while reducing overexploitation.

Related literature. Several studies have analyzed the overexploitation of the commons; for a detailed review of the literature see Ostrom (1990), Ostrom et al. (1994) and Faysee (2005). In order to reduce this excessive appropriation, Ostrom (1999) suggested that CPRs can be managed by local governance structures. Kirkley et al. (2003) examined the importance of preserving CPRs for the long term, especially in developing countries where there is excess capacity, Hackett et al. (1994) analyzed equal appropriation rules in irrigation in India, and Coward (1979) discusses water assignments as a function of land held in the Phillipines.[8]

In our paper, we consider an alternate policy tool, and evaluate its effectiveness in helping avoid overexploitation. In particular, we allow firms to hold equity shares on each others' profits. Ellis (2001) presents a similar model, but he takes equity shares as exogenously given, and assumes that welfare coincides with the sum of firms' profits, thus ignoring the role of consumer surplus and environmental damage from the exploitation of the CPR.[9] We show that our model can reproduce Ellis' results in the special case in which all appropriation is sold overseas (no consumer surplus) and exploitation does not cause any environmental damage. In that setting, firms only behave optimally if they all hold maximal equities. However, when consumer surplus and/or environmental damage are considered, this finding no longer applies. Instead, firms should optimally hold lower equity shares, even converging to zero in some specific cases.

Our paper connects with the literature analyzing the effect of equity shares in industrial organization. In particular, Reynolds and Snapp (1986) examines a standard Cournot model when firms hold exogenous shares in each other's profits, showing that equilibrium quantities decrease as equity shares increase, regardless of which company shares increases.[10] While our paper considers a similar model, it extends their setting along several dimensions: it allows for cost externalities, thus helping understand how their results apply to CPRs; considers a polluting industry and its

[8] Other articles consider uncertainty in the resource's stock, and how such uncertainty affects individual appropriation levels in the commons, approaching them to socially optimal levels; see Suleiman and Rapoport (1988), Suleiman et al. (1996), and Apesteguia (2006).

[9] His model was extended in Ellis and Nouweland (2006) which considers that every individual firm exploits the resource in the first stage, and invests in equity shares during the second stage; earning profits only at the end of the game. Anticipating the equilibrium profile of shares at the second stage, every firm's appropriation during the first stage approaches the cooperative solution. While their model endogenizes equity acquisition, it assumes that it happens at the last stage of the game; as opposed to our setting where equity acquisition is chosen during the first stage.

[10] Farrell and Shapiro (1990) consider a Cournot oligopoly in which firms buy new capital either by acquiring it from a rival, from a third party (not a rival), or buying shares from a rival. Fanti (2015) modifies this setting, by considering that only one firm holds an exogenous participation on its rival's profit. In addition, the paper allows for asymmetries in production costs, showing that the output reduction arising from cross-ownership can be welfare improving if the firm holding stock on its rival's profits is less efficient than the rival.

optimal environmental regulation; and endogenizes equity share acquisition. Dietzenbacher et al. (1999) use data from the Dutch financial sector, empirically confirming that output is lower when firms hold shares on each other than otherwise.[11]

Qin et al. (2017) consider a Cournot game which, like in our paper, allows for firms to select equity shares before their subsequent choice of output.[12] In particular, the authors characterize 'pairwise stability' wherein no two firms can make themselves better off by trading any further shares. They also find that the equity shares that firms choose increase (approaching collusive outcomes) when only a few firms compete in the industry. Unlike our paper, their setting does not consider cost externalities, thus not allowing for the analysis of CPRs; and does not compare equilibrium equity shares against the social optimum, which helps us provide more direct policy implications.

Finally, Bárcena-Ruiz and Campo (2012) investigate a country's optimal emission fee on a polluting firm when this company holds equity shares on another polluting firm located in a different country. The article then compares emission fees in a non-cooperative setting (independent environmental policies across countries) and a cooperative context (coordination of environmental policies). However, it assumes a single firm in each jurisdiction, exogenous and symmetric equity shares across firms, and that firms do not impose cost externalities on one another since they exploit different CPRs. We relax all three assumptions.

The following section describes the model. Section 3 identifies equilibrium appropriation, Section 4 finds socially optimal appropriation, and compares it against equilibrium values. At the end of the section, we examine under which parameter conditions overexploitation or underexploitation can more likely arise. Section 5 tests the robustness of our results under different modelling assumptions, while Section 6 discusses our results offering policy implications.

2 Model

Consider two firms exploiting a CPR of size $\theta \in (0, 1]$. Every firm $i = \{1, 2\}$ simultaneously and independently chooses its appropriation level q_i. For simplicity, we assume that firms sell their appropriation in a perfectly competitive market, facing a price normalized to 1.[13] In addition, firm i's cost function is

$$C_i(q_i, q_j) = \frac{q_i(q_i + q_j)}{\theta},$$

[11] Malueg (1992) considers a setting in which firms hold exogenous symmetric shares on each others profits, showing that collusive behavior becomes more difficult to sustain than when firms do not own equity shares. Gilo and Spiegel (2006) extend this model to a context in which firms are allowed to hold asymmetric equity shares, but still exogenously given shares, demonstrating that collusion can become easier to sustain under certain equity profiles.

[12] For empirical studies analyzing firms' motivations to hold equity on their rivals' profits, see Demsetz and Lehn (1985), which considers 511 U.S. firms; and Bishop et al. (2002), which examines the 162 largest Hungarian firms.

[13] For instance, firms sell all their fish captures in an international market for that fish variety, where they compete against many other fishermen, each of them representing a negligible proportion of aggregate sales.

entailing that total and marginal costs, $\frac{\partial C_i(q_i,q_j)}{\partial q_i} = \frac{2q_i+q_j}{\theta}$, decrease in the available stock, θ, but increase in the firm's appropriation, q_i. Intuitively, firm i finds the resource easier to exploit as it becomes more abundant, but more difficult to capture as it increases its own appropriation (i.e., convex cost). In addition, total and marginal costs are also increasing in firm j's appropriation, q_j, indicating that, the resource is more difficult to exploit as firm j increases its appropriation.[14] (As a robustness check, Appendix 1 extends all our results in a setting where we allow for the cost function to exhibit different degrees of cost externalities, i.e., $C_i(q_i, q_j) = \frac{q_i(q_i+\lambda q_j)}{\theta}$, where parameter $\lambda \in [0, 1]$ denotes the severity of the cost externality that firm j's appropriation produces on firm i. Our findings are qualitatively unaffected.)

The time structure of the game is as follows:

1. In the first stage, every firm i chooses its equity share on its rival's profit; and
2. In the second stage, observing the equity shares selected during the first stage by all firms, every firm i chooses its appropriation level q_i.

3 Equilibrium Analysis

3.1 Second stage - Equilibrium appropriation

In the second stage, for every equity share profile (α_i, α_j) chosen during the first stage of the game, every firm i selects its appropriation level q_i to solve

$$\max_{q_i \geq 0} \; V_i = (1 - \alpha_j)\pi_i + \alpha_i \pi_j \tag{1}$$

where $\pi_i \equiv q_i - \frac{q_i(q_i+q_j)}{\theta}$ denotes firm i's profit, $j \neq i$, and prices are normalized to one. Therefore, each firm i has two components in its objective function: (1) the share that firm i keeps in its own profit π_i, after subtracting the share that firm j holds, $\alpha_j \in [0, 1/2]$; and (2) a share $\alpha_i \in [0, 1/2]$ on the other firm's profits π_j.[15] Intuitively, parameter α_i represents firm i's equity share on its rival's profits. Specifically, when firms hold no equity shares, $\alpha_i = \alpha_j = 0$, the above objective function collapses to π_i, indicating that every firm only considers its own profit when choosing its individual appropriation level q_i; as in standard CPR models. When $\alpha_i = \alpha_j = 1/2$, firms equally share their profits (as in a merger of symmetric firms). In that setting, the above objective function simplifies to $\frac{\pi_i+\pi_j}{2}$, leading every firm i to fully consider the profits of its rival in its individual appropriation decisions. Finally, when $\alpha_i > 0$ but $\alpha_j = 0$, the objective function in (1) becomes

[14] Firm i's marginal cost is $\frac{2q_i+q_j}{\theta}$, while a marginal increase in firm j's appropriation causes an increase of $\frac{q_i}{\theta}$, which is smaller than the marginal cost for all admissible parameter values. Hence, a marginal increase in its own appropriation produces a larger increase in firm i's costs than a marginal increase in its rival's appropriation, i.e., $\frac{\partial C_i(q_i,q_j)}{\partial q_i} > \frac{\partial C_i(q_i,q_j)}{\partial q_j}$.

[15] Allowing for equity shares above 1/2 would entail that a firm holds more equity on its rival than the rival holds in its own company; as in an acquisition. For simplicity, we do not consider these cases in our analysis.

$\pi_i + \alpha_i\pi_j$, indicating that firm i fully retains its profit and receives a share α_i of firm j's profits.[16]

We start our equilibrium analysis by describing firms' best response function in the second stage. (All proofs are relegated to the appendix.)

Lemma 1. *Every firm i's best response function is given by*

$$q_i(q_j) = \begin{cases} \frac{\theta}{2} - \frac{1+\alpha_i-\alpha_j}{2(1-\alpha_j)}q_j & \text{if} \quad q_j \leq \frac{\theta(1-\alpha_j)}{1+\alpha_i-\alpha_j} \\ 0 & \text{otherwise.} \end{cases}$$

As depicted in Figure 1, the vertical intercept of the best response function, $\frac{\theta}{2}$, is only affected by the CPR's stock, θ; while its slope, $\frac{1+\alpha_i-\alpha_j}{2(1-\alpha_j)}$, depends on firms' equity shares. For presentation purposes, we next examine the best response function in different settings.

Case 1: CPR models without equity shares. When firms hold no shares on their rivals' profits, $\alpha_i = \alpha_j = 0$, every firm i's best response function collapses to

$$q_i(q_j) = \frac{\theta}{2} - \frac{1}{2}q_j \tag{BRF$_1$}$$

which is positive for all $q_j \leq \theta$; as in standard CPR models. In words, firm i appropriates $\frac{\theta}{2}$ when its rival does not exploit the resource, $q_j = 0$, but decreases its appropriation as q_j increases, i.e., firms' exploitation levels are strategic substitutes.

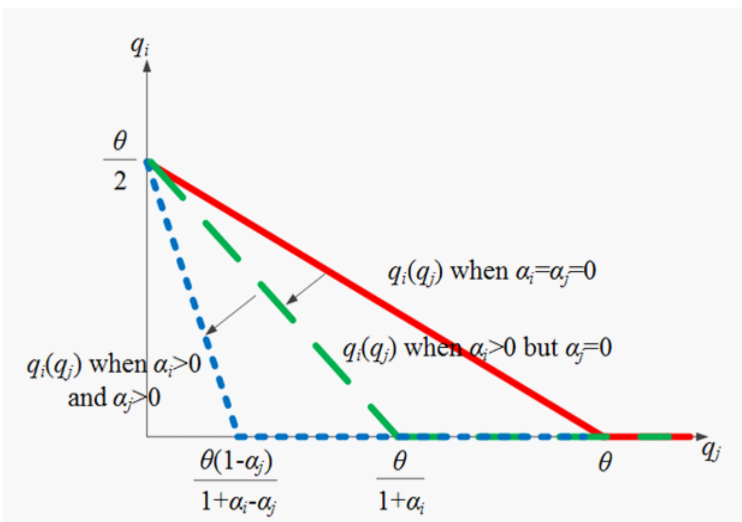

Figure 1. Firm i's best response function.

[16] Alternatively, parameter α_i could represent firms' altruistic concerns. In that case, $\alpha_i = 0$ reflects a selfish agent who only cares about its own payoff, whereas $\alpha_i = 1/2$ indicates a fully altruistic agent. See Velez et al. (2009) for controlled experiments in artisanal fisheries in Colombia, reporting that individuals who exploit a fishery display altruism and other socially favorable behaviors. Our subsequent analysis applies, nonetheless, to both interpretations.

Case 2: CPR with firm i holding equity shares. When firm i is the only company holding a positive equity on its rival's profits, $\alpha_i > 0$ but $\alpha_j = 0$, its best response function becomes

$$q_i(q_j) = \frac{\theta}{2} - \frac{1+\alpha_i}{2}q_j, \tag{BRF$_2$}$$

thus pivoting inwards relative to BRF_1, as depicted in Figure 1.[17] Intuitively, firm i internalizes a share of the external effect that its appropriation causes on its rival's profit, and thus reduces its own exploitation of the resource. Since the best response function is now steeper, firms' appropriation becomes strategic substitutes to a greater extent.

Case 3: CPR with both firms holding equity shares. If both companies hold equity shares, $\alpha_i, \alpha_j > 0$, we obtain the best response function in Lemma 1, which experiences a further pivoting effect inwards relative to BRF_2 (where $\alpha_i > 0$ but $\alpha_j = 0$); as illustrated in Figure 1. In this case, exploitation also decreases, which is now due to the fact that firm i retains a smaller share of its own profits when $\alpha_j > 0$.

The following Proposition analyzes equilibrium appropriation levels.

Proposition 1. *Every firm i's equilibrium appropriation is $q_i^* = \frac{\theta(1-\alpha_i)}{3-\alpha_i-\alpha_j}$, which is strictly positive for all admissible parameter values. In addition, q_i^* is increasing in θ and in α_j, but decreasing in α_i. Furthermore, $q_i^* \geq q_j^*$ if and only if $\alpha_i \leq \alpha_j$.*

Confirming our discussion of firms' best response function in Lemma 1, equilibrium appropriation q_i^* is increasing in the CPR stock, θ, but decreasing in firm i's equity share on firm j, α_i, since firm i internalizes the cost externality that it imposes on its rival to a larger extent. Firm i's exploitation increases in the share that its rival holds in its profit, α_j. Intuitively, firm i retains a smaller share of its own profit, providing it with more incentives to increase its own appropriation. Finally, firm i exploits the resource more intensively than its rival if it holds a smaller share of equity, $\alpha_i \leq \alpha_j$.

The following corollary evaluates equilibrium appropriation at special cases.

Corollary 1. *Equilibrium appropriation q_i^* becomes*

1. *CPR model without equity shares: $q_i^* = \frac{\theta}{3}$, i.e., $\alpha_i = 0$ for every firm i;*
2. *CPR model with symmetric equity shares: $q_i^* = \frac{\theta(1-\alpha)}{3-2\alpha}$, i.e., $\alpha_i = \alpha_j = \alpha$;*
3. *CPR model with equally shared equity: $q_i^* = \frac{\theta}{4}$ when firms equally share equity, i.e., $\alpha_i = \alpha_j = \frac{1}{2}$.*

[17] In this setting, the horizontal intercept of the best response function is $q_j = \frac{\theta}{1+\alpha_i}$, where $\frac{\theta}{1+\alpha_i} < \theta$ since $\alpha_i > 0$.

In the standard CPR model without equity shares, appropriation is $q_i^* = \frac{\theta}{3}$; which decreases to $q_i^* = \frac{\theta(1-\alpha)}{3-2\alpha}$ when firms hold symmetric equity shares, $\alpha_i = \alpha_j = \alpha$. In addition, $q_i^* = \frac{\theta(1-\alpha)}{3-2\alpha}$ decreases in α, reaching its lowest level when firms equally share profits, $\alpha = 1/2$, as in a merger, where their equilibrium appropriation becomes $q_i^* = \frac{\theta}{4}$.

3.2 First stage - Equilibrium equity shares

In the first stage, every firm i anticipates the equilibrium appropriation pair $\left(q_i^*, q_j^*\right)$ in the subsequent stage of the game. Evaluating its profits during the second stage, we obtain $\pi_i\left(q_i^*, q_j^*\right)$, while those of firm j are $\pi_j\left(q_i^*, q_j^*\right)$. Therefore, every firm i simultaneously and independently chooses the equity share it acquires on its rival's profits, α_i, to solve[18]

$$\max_{\alpha_i \geq 0} \ (1-\alpha_j)\pi_i\left(q_i^*, q_j^*\right) + \alpha_i \pi_j\left(q_i^*, q_j^*\right) - C(\alpha_i) \tag{2}$$

where $C(\alpha_i) = F + c\alpha_i^2$ represents the cost of acquiring equity α_i. For generality, this cost includes a fixed cost $F \geq 0$ (e.g., broker fees), and a convex cost of equity, $c\alpha_i^2$, where $\infty > c \geq 0$. Intuitively, as firm i purchases more shares from firm j, the cost of additional equity increases, i.e., firm j's equity becomes more scarce, and firm i's opportunity cost of capital goes up. Most studies analyzing endogenous equity assume that equity acquisition is costless, which our model allows as a special case when $F = c = 0$; but lets more general cost structures. (As a robustness check, Appendix 3 examines other cost of equity functions, such as constant or concave in equity acquisition α_i, showing that our results are qualitatively unaffected.)

Proposition 2. *In the first stage of the game, every firm i chooses the equity share α_i^* that solves $\frac{2\theta(1-\alpha_j)}{(3-\alpha_i-\alpha_j)^3} = 2c\alpha_i$.*

Proposition 2 states that, differentiating firm i's problem in (2) with respect to its equity share on firm j, α_i, yields the first-order condition

$$MB_i \equiv \frac{2\theta(1-\alpha_j)}{(3-\alpha_i-\alpha_j)^3} = 2c\alpha_i \equiv MC_i.$$

The left-hand side captures the marginal benefit from acquiring equity shares MB_i, indicating that firms better coordinate their appropriation decisions, thus increasing profits; while the right-hand side represents the marginal cost of additional equity. As depicted in Figure 2, the marginal benefit of equity shares, MB_i, originates at a strictly positive height, $\frac{2\theta(1-\alpha_j)}{(3-\alpha_j)^3}$ when $\alpha_i = 0$, and monotonically increases in α_i as firm i holds more equity on its rival's profit; reaching $\frac{16\theta(1-\alpha_j)}{(5-2\alpha_j)^3}$ when firm i holds the maximal equity, $\alpha_i = 1/2$. The marginal benefit is, therefore, positive, increasing,

[18] In the extensions section, we examine how our results are affected when firms jointly determine their equity shares, as in explicit negotiations between both firms.

and convex in equity, for all parameter values.[19] Intuitively, firm i's second-period profits are the lowest when it does not hold equity on its rival's profits, but become maximal when it holds the highest possible equity on its rival's profits.

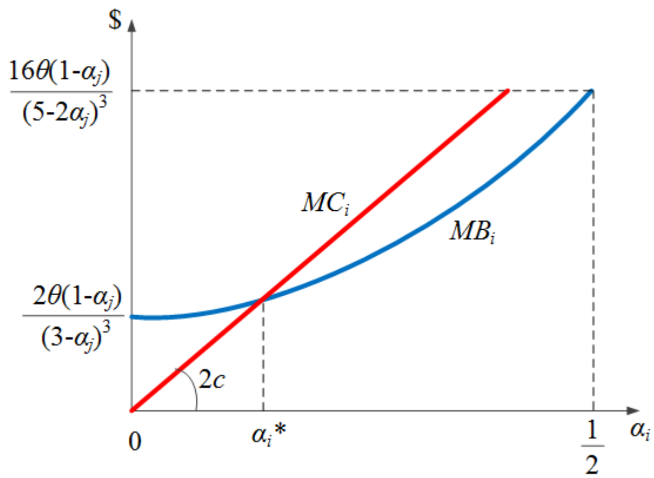

Figure 2. MB_i and MC of increasing α_i.

In addition, firm i's marginal benefit of acquiring equity, MB_i, is increasing in its rival's equity, α_j, if and only if $\frac{\partial MB_i}{\partial \alpha_j} = \frac{2\theta(\alpha_i - 2\alpha_j)}{(3-\alpha_i-\alpha_j)^4} > 0$, or if firm i's equity, α_i, satisfies $\alpha_i > 2\alpha_j$.[20] In words, firm i has further incentives to increase its stake at firm j (shifting MB_i upwards) when it holds twice as much equity on firm j than the latter holds on i.

When c is sufficiently low, $c \leq \frac{3\theta(1-\alpha_j)}{(3-\alpha_i-\alpha_j)^4}$ (or $c = 0$ as a special case), firm i finds that the marginal benefit of equity lies above its marginal cost for all values of α_i,[21] thus increasing its stock on firm j as much as possible, i.e., $\alpha_i^* = 1/2$ in a corner solution. This case can arise, for instance, in the fisheries mentioned in the Introduction, where participants equally share profits. The opposite corner solution, where the firm acquires no equity on its rival's profit, i.e., $\alpha_i^* = 0$, could only arise, however, if $c \to \infty$, making the MC_i curve completely vertical; which cannot be sustained given that $\infty > c \geq 0$ by definition. Finally, when the marginal cost of acquiring equity is intermediate, $\frac{3\theta(1-\alpha_j)}{(3-\alpha_i-\alpha_j)^4} < c < \infty$, the firm holds an interior equity share $\alpha_i^* \in (0, 1/2)$ that solves $\frac{2\theta(1-\alpha_j)}{(3-\alpha_i-\alpha_j)^3} = 2c\alpha_i$. Solving for equity share α_i^* in equation $\frac{2\theta(1-\alpha_j)}{(3-\alpha_i-\alpha_j)^3} = 2c\alpha_i$, however,

[19] In particular, the marginal benefit $MB_i = \frac{2\theta(1-\alpha_j)}{(3-\alpha_i-\alpha_j)^3}$ satisfies $\frac{\partial MB_i}{\partial \alpha_i} = \frac{6\theta(1-\alpha_j)}{(3-\alpha_i-\alpha_j)^4}$, which is positive for all admissible parameter values, and $\frac{\partial^2 MB_i}{\partial \alpha_i^2} = \frac{24\theta(1-\alpha_j)}{(3-\alpha_i-\alpha_j)^5}$, which is also positive. Therefore, MB_i is increasing and convex in equity α_i.

[20] Therefore, firm i's best response function during the first-stage game, $\alpha_i(\alpha_j)$, is decreasing in its rival's equity α_j when $\alpha_i \leq 2\alpha_j$; but becomes increasing otherwise. Graphically, in the (α_i, α_j) −quadrant, firm i decreases its choice of α_i for all $\alpha_i \leq 2\alpha_j$, indicating that firms regard their cross-ownerships as strategic subsitutes, but firm i increases α_i for all $\alpha_i > 2\alpha_j$, reflecting that equity shares become strategic substitutes.

[21] Graphically, this occurs when MC_i lies below MB_i, which is equivalent to the slope of MC_i being lower than that of MB_i. In particular, this entails $\frac{\partial MC_i}{\partial \alpha_i} \leq \frac{\partial MB_i}{\partial \alpha_i}$, or $2c \leq \frac{6\theta(1-\alpha_j)}{(3-\alpha_i-\alpha_j)^4}$, which collapses to $c \leq \frac{3\theta(1-\alpha_j)}{(3-\alpha_i-\alpha_j)^4}$.

produces an intractable root. For illustration purposes, Table I evaluates this equilibrium equity α_i^* at different parameter combinations of θ and c (the only two parameters affecting the firm's decision on equity).[22]

Cost c / Stock θ	$\theta = 0.3$	$\theta = 0.5$	$\theta = 0.7$	$\theta = 1$
$c = 1/40$	0.5	0.5	0.5	0.5
$c = 1/25$	0.44	0.5	0.5	0.5
$c = 1/20$	0.31	0.5	0.5	0.5
$c = 1/15$	0.20	0.44	0.5	0.5
$c = 1/10$	0.12	0.23	0.39	0.5
$c = 1/5$	0.06	0.10	0.15	0.23

Table I. Optimal equity share α_i^*.

For instance, in the column where stock satisfies $\theta = 0.5$, the equilibrium equity share reaches its highest admissible value, $\alpha_i^* = 0.5$ when costs are relatively low, $c = 1/40$ and $c = 1/20$; but decreases to $\alpha_i^* = 0.44$ when equity costs increase to $c = 1/15$, and further reduces to $\alpha_i^* = 0.23$ and $\alpha_i^* = 0.10$ when c increases to $c = 1/10$ and $c = 1/5$. A similar argument applies for other stocks, such as $\theta = 1$, where equilibrium equity is maximal for most values of c; or at $\theta = 0.3$, where equilibrium equity is closer to zero for a larger range of c.

4 Welfare analysis

We next compare the equilibrium results against the social optimum. The social planner solves

$$\max_{q_i, q_j} W = \gamma CS(q_i, q_j) + PS(q_i, q_j) - Env(q_i, q_j). \tag{3}$$

where $\gamma \in [0, 1]$ denotes the proportion of goods sold domestically, and the inverse demand function is given by $p(q_i, q_j) = a - b(q_i + q_j)$ with parameters $a > 1$ and $b > 0$. This domestic demand function is then evaluated at the internationally given price $p = 1$ to find consumer surplus $CS(q_i, q_j) = \frac{(a-1)(q_i+q_j)}{2}$. Producer surplus $PS(q_i, q_j) = [V_i - C(\alpha_i)] + [V_j - C(\alpha_j)]$ sums the objective functions of both firms, and $Env(q_i, q_j) = d(q_i + q_j)^2$ denotes the environmental damage, which is convex in aggregate appropriation $(q_i + q_j)$, and $d \in [0, 1]$. Intuitively, exploiting the resource can affect the food chain connected to a fishing ground, damaging biodiversity in the area, or be due to the pollution from fishing vessels. Note that producer surplus $PS(q_i, q_j)$ collapses to $[\pi_i - C(\alpha_i)] + [\pi_j - C(\alpha_j)]$. Therefore, after differentiating with respect to output q_i, the social planner's problem in (3) does not contain equity shares.

The next proposition identifies social optimum appropriation levels.

[22] Table I restricts the values of α_i^* to its admissible range $\alpha_i^* \in [0, 0.5]$. In addition, in case of multiple roots, we report the only root that lies in the admissible range.

Proposition 3. *The socially optimal appropriation for every firm i is*

$$q_i^{SO} = \frac{\theta\left[2 + (a-1)\gamma\right]}{8(1 + d\theta)}$$

which increases in θ, γ, and a, but decreases in d.

Therefore, the social planner seeks to implement a larger appropriation level when the stock is more abundant, a larger share of the appropriation is sold domestically, and demand is strong. However, he reduces socially optimal output when the exploitation of the resource generates a more intense environmental damage. In the case that appropriation generates no environmental damages ($d = 0$), our results collapse to $q_i^{SO} = \frac{\theta[2+(a-1)\gamma]}{8}$, which further simplifies to $q_i^{SO} = \frac{\theta}{4}$ if all production is sold overseas ($\gamma = 0$).

We next compare socially optimal appropriation and the exploitation levels that firms choose in equilibrium, q_i^{SO} and q_i^*, which helps us identify the socially optimal equity shares firms should hold to maximize social welfare, α^{SO}. We will then compare α^{SO} against the equity shares firms choose in equilibrium, α_i^*, as identified in Proposition 2. Since equilibrium equity shares α_i^* were symmetric, Corollary 3 finds symmetric socially optimal equity shares.

Corollary 2. *Socially optimal output q_i^{SO} satisfies $q_i^{SO} \leq q_i^*$ if and only if $\alpha \leq \alpha^{SO}$, where*

$$\alpha^{SO} \equiv 1 + \frac{2 + \gamma(a-1)}{2\gamma(a-1) - 4(1 + 2d\theta)}.$$

Cutoff $\alpha^{SO} \geq 0$ if and only if $d \geq d_1 \equiv \frac{3\gamma(a-1)-2}{8\theta}$, and $\alpha^{SO} \leq \frac{1}{2}$ if and only if $d \leq d_2 \equiv \frac{\gamma(a-1)}{2\theta}$, where $d_2 > d_1$.

Corollary 2 states that equilibrium appropriation is socially excessive, $q_i^{SO} \leq q_i^*$, if firms share relatively low equities, i.e., $\alpha \leq \alpha^{SO}$, which includes the setting in which they ignore each other's profits, $\alpha = 0$, as a special case. In addition, when the exploitation of the resource does not generate large environmental damages, $d < d_1$, condition $\alpha \leq \alpha^{SO}$ cannot hold for any admissible $\alpha \in [0, 1/2]$. Intuitively, when environmental damage is small, the social planner would recommend even more exploitation than firms choose in equilibrium. As a consequence, equilibrium appropriation becomes socially insufficient for all values of the remaining parameters (d, θ, γ).

In contrast, cutoff α^{SO} lies above its upper bound, 1/2, when the exploitaiton of the CPR generates a large environmental damage, $d > d_2$. In that context, condition $\alpha \leq \alpha^{SO}$ holds for all admissible α's. In words, the social planner would like to reduce appropriation substantially given its large damage, yielding a socially excessive exploitation of the resource for all values of the remaining parameters (d, θ, γ). Overall, condition $\alpha \leq \alpha^{SO}$ becomes binding when cutoff α^{SO} satisfies $\alpha^{SO} \in [0, 1/2]$, which holds if environmental damage is intermediate, i.e., $d \in [d_1, d_2]$.[23]

[23] Cutoff d_1 decreases in the proportion of output sold domestically, γ, in the strength of demand, a, but decreases in the abundance of the CPR, θ. Similar comparative statics apply to cutoff d_2. In addition, the distance $d_2 - d_1 =$

In the case of a CPR model without equity shares $\alpha = 0$, condition $\alpha \leq \alpha^{SO}$ in Corollary 3 collapses to $0 \leq \alpha^{SO}$ in this context, which is satisfied for all $a \leq a_1$. We next evaluate the comparative statics of cutoff α^{SO}.

Corollary 3. *Cutoff α^{SO} is*

1. *decreasing in γ and a for all parameter values,*

2. *increasing in d and θ for all parameter values.*

3. *When environmental externalities are absent, $d = 0$, cutoff α^{SO} reduces to $\alpha^{SO} \equiv 1 + \frac{2+\gamma(a-1)}{2\gamma(a-1)-4}$; which further simplifies to $\alpha^{SO} \equiv \frac{1}{2}$ when $\gamma = 0$.*

Therefore, cutoff α^{SO} increases in the environmental damage of appropriation, d; as illustrated in Figure 3.[24] The region of (α, d)-pairs below cutoff α^{SO} indicate a socially excessive exploitation of the resource, i.e., $q_i^{SO} \leq q_i^*$, while a socially insufficient exploitation occurs at points above cutoff α^{SO}. When appropriation generates a small environmental externality, cutoff α^{SO} lies in the admissible range (between zero and $1/2$), breaking the quadrant into two regions: one in which the resource is overexploited, which occurs when firms hold too much equity shares on each other, $\alpha > \alpha^{SO}$ (since in this setting firms restrict production too much, relative to the social optimum); and another region in which the resource is overexploited because firms hold a too small equity share on each other's profits, $\alpha < \alpha^{SO}$. When environmental damage is large enough (d lies to the right-hand side of the kink in α^{SO}, which in our parametric example occurs at $d = 0.5$), only overexploitation of the resource emerges in equilibrium. Intuitively, the environmental damage is sufficiently intense to induce a relatively low q^{SO}, which the firms do not produce in equilibrium even if they were to sustain the largest equity shares (as in a merger, where $\alpha = 1/2$).

$\frac{2+\gamma(a-1)}{8\theta}$, which measures the range of environmental damages supporting a strictly interior equity share α^{SO}, is increasing in γ and a, but decreasing in θ.

[24] Figure 1 assumes, for simplicity, $a = 2$, and $\theta = \gamma = 1/2$. Other parameters produce similar results, and can be provided by the authors upon request.

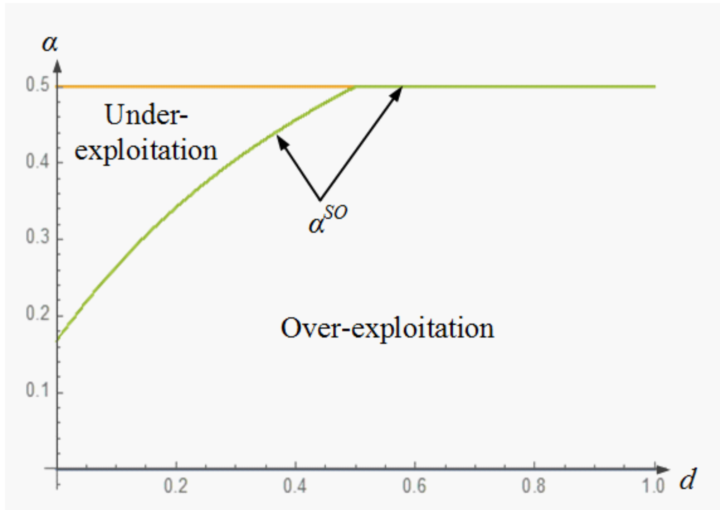

Figure 3. Cuoff α^{SO} as a function of d.

From Corollary 3, cutoff α^{SO} shifts downwards as a increase. In words, the resource increases consumer surplus thus increasing the social optimum, while leaving the firm's equilibrium appropriation unchanged, thus enlarging the region of under-exploitation. A similar argument applies for γ, which also shifts cutoff α^{SO} downwards. Intuitively, as a larger proportion of output is sold domestically, the social planner seeks firms to increase appropriation, whereas individual firms' exploitation remains unaffected. As a result, the region of under-exploitation expands.

Cutoff α^{SO} shifts upwards when the environmental damage d increases. In this case, optimal appropriation would be lower, while equilibrium appropriation is unaffected by d, entailing that the region of where over-exploitation arises expands. Similarly, cutoff α^{SO} shifts upwards when the size of the stock θ increases, indicating that socially excessive exploitation becomes more likely.[25]

Last, when environmental externalities are absent and firms do not sell products domestically (i.e., $d = \gamma = 0$), the above cutoff reduces to $\alpha^{SO} = 1/2$. Intuitively, equilibrium appropriation is only socially optimal when firms equally share their profits; as shown in Ellis (2001, Proposition 1). Otherwise, the resource is overexploited.[26]

Comparing equilibrium and socially optimal equity shares. Comparing the equilibrium equity share found in Proposition 2, α^*, and the socially optimal share identified in Proposition 3, α^{SO}, we can evaluate whether the equity shares firms choose during the first stage of the game are socially excessive or not. Unfortunately, the non-linearities in the expression of α^* does not allow

[25] In particular, a given increase in θ produces a larger increase in equilibrium appropriation q_i^* than in the socially optimal appropriation q_i^{SO}, since the every firm i only internalizes a share α_i of the cost externality that its appropriation imposes on its rival's profits. In contrast, the social planner fully internalizes this cost externality.

[26] This result can be alternatively understood by noticing that when $d = \gamma = 0$ the objective function in problem (3) simplifies to $\pi_i + \pi_j$; as in a merger. Therefore, firm i's problem in (1) coincides with (3), $\pi_i + \pi_j$, if and only if equity shares are maximal, i.e., $\alpha_i = \alpha_j = 1/2$; as in a merger between firms i and j.

for an evaluation of the difference $\alpha^* - \alpha^{SO} = 0$, to subsequently solve for a parameter value in order to identify an explicit condition for $\alpha^* > \alpha^{SO}$. To illustrate our results, Figure 4 depicts α^{SO} using the same parameter values as in Figure 3, and superimposes equilibrium equity share $\alpha^* = 0.44$ which arises from $c = 1/15$. Intuitively, when environmental damage is relatively low ($d < 0.36$ for the parameter values in Figure 4), firms choose a higher equity share than the social planner would, i.e., $\alpha^* > \alpha^{SO}$. In this context, appropriation does not need to be reduced significantly given its minor environmental damage, but firms decrease their exploitation of the resource when acquiring a relatively high equity share on each other's profits. In contrast, when environmental damage is larger ($d > 0.36$ in Figure 4), firms choose a lower equity share than the planner would, $\alpha^* < \alpha^{SO}$, leading them to over-exploit the resource relative to the social optimum. Finally, when $d = 0.36$, equilibrium equity coincides with the social optimal equity level, $\alpha^* < \alpha^{SO}$.

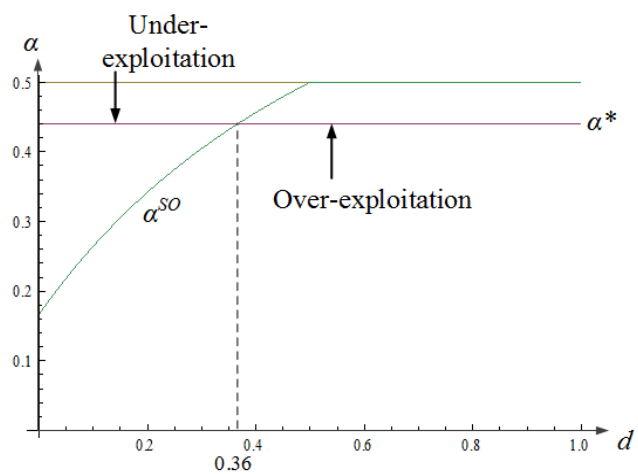

Figure 4. Comparing α^* and α^{SO}.

When c decreases to $c = 1/20$, equilibrium equity increases to its maximum $\alpha^* = 1/2$. In Figure 4, the flat line representing α^* shifts upwards until reaching the upper bound $\alpha^* = 1/2$. In words, equilibrium equity is socially excessive $\alpha^* > \alpha^{SO}$ for all values of $d < 1/2$ (to the left-hand side of the kink in cutoff α^{SO}, which happens at $d = 1/2$), but becomes socially optimal for all $d \geq 1/2$. In contrast, when c increases to $c = 1/5$, equilibrium equity decreases to $\alpha^* = 0.10$; thus lying below curve α^{SO} for all values of d. In words, equilibrium equity is socially insufficient, $\alpha^* < \alpha^{SO}$, for all values of d.

4.1 Policy tools with equity shares

In this section, we consider regulation that induces the socially optimal appropriation level, q_i^{SO}. We assume that the regulator chooses a policy tool before firms select their equity shares in the first stage of the game, so he can anticipate the equilibrium equity shares firms choose and their subsequent appropriation levels in equilibrium. For completeness, we analyze two commonly used policy tools, quotas and emission fees. However, we also explore an alternative policy tool, an "equity incentive," which induces firms to acquire the socially optimal equity level α^{SO}, and then, unlike the previous two instruments, lets firms exploit the CPR with no further regulation or supervision.

Proposition 4. *When firms hold equity shares:*

1. *The optimal quota is set at q_i^{SO} for every firm i;*
2. *The optimal emission fee is*

$$t^* = 1 - \frac{(3 - 2\alpha^*)\,[2 + \gamma(a-1)]}{8(1-\alpha^*)(1+d\theta)},$$

which is positive if and only if α satisfies $\alpha \leq \alpha^{SO}$. In addition, emission fee t^* decreases in the equity share that firms hold, α.

3. *The optimal equity incentive s solves $\alpha^*(c - s) = \alpha^{SO}$, which is a subsidy if $s > 0$ or a tax otherwise.*

When the regulator uses a quota, he only needs to set it equal to the socially optimal appropriation level, q_i^{SO}, and then monitor whether firms comply with it. If, instead, he uses an emission fee inducing firms to choose q_i^{SO}, the severity of the fee is affected by equity shares. In particular, when equity shares are sufficiently low, the regulator sets an emission fee to induce firms reduce their exploitation of the resource. This encompasses the standard CPR models, where $\alpha = 0$, as a special case. In contrast, if firms hold a substantial proportion of equity shares, $\alpha > \alpha^{SO}$, regulatory agencies find that equilibrium appropriation is socially insufficient, driving them to set a negative emission fee $t^* < 0$ (that is, a subsidy per unit of appropriation) which induces firms to increase their exploitation of the resource until reaching the socially optimal level. When firms hold the maximum amount of equity shares ($\alpha = 1/2$, as in a merger), condition $\alpha > \alpha^{SO}$ is likely satisfied, leading regulators to offer production subsidies rather than taxes.[27]

A similar argument explains why emission fee t^* is decreasing in the equity share that firms hold, α, i.e., $\frac{\partial t^*}{\partial \alpha} < 0$. Intuitively, as firms hold a larger equity on their rival's profits, they reduce their equilibrium appropriation, approaching it to the social optimal. As a result, the regulator does not need to set stringent fees to induce firms exploit the resource at the socially optimal level.

Finally, when using the optimal equity incentive, the regulator anticipates the equilibrium equity function $\alpha^*(c)$, and compares it against the socially optimal equity, α^{SO}. In order to make them coincide, the social planner offers a subsidy (if $s > 0$) or a tax (if $s < 0$) to alter the marginal cost

[27] In the standard Cournot model where firms hold no equity shares and cost externalities are absent ($\lambda = \alpha = 0$, as in Case 1), the above emission fee collapses to $t^* = \frac{2[1-(a-1)\gamma]+8d\theta}{(1+2d\theta)}$; which further simplifies to $2\,[1-(a-1)\gamma]$ when exploitation of the resource does not generate environmental damage, $d = 0$. This fee can become a subsidy if $2\,[1-(a-1)\gamma] < 0$, or $\gamma > \frac{1}{a-1}$, i.e., when a sufficiently large share of output is sold domestically.

of equity, ultimately making the equilibrium and socially optimal equities coincide, i.e., s solves $\alpha^*(c-s) = \alpha^{SO}$ where α^* is now a function of the difference $c-s$. For instance, starting from one of the cases we discussed in Table I, $c = 1/15$, if the environmental damage is $d = 0.7$, the unregulated equilibrium yields $\alpha^* = 0.44$ while $\alpha^{SO} = 0.5$. Therefore, the regulator can offer a subsidy $s > 0$ that increases equilibrium equity shares towards its socially optimal level. Specifically, subsidy s must solve $\alpha^*\left(\frac{1}{15} - s\right) = 0.5$, which yields $s^* = 0.004$. A similar approach applies to other environmental damages, as reported in the left panel of Table II. Recall that $\alpha^*\left(\frac{1}{15} - s\right)$ is only a function of θ and c, thus being unaffected by changes in d; whereas α^{SO} increases in d. When $d < 0.36$, equilibrium equities are socially excessive, $\alpha^* > \alpha^{SO}$, and the regulator sets a tax on equity acquisition, $s < 0$, which induces firms to choose α^{SO}, i.e., $\alpha^*\left(\frac{1}{15} - s\right) = \alpha^{SO}$. When $d = 0.36$, equilibrium equities are optimal, $\alpha^* = \alpha^{SO}$, entailing a zero subsidy $s = 0$. Last, when $d > 0.36$, equilibrium equities are socially insufficient, $\alpha^* < \alpha^{SO}$, and the regulator provides a subsidy $s > 0$ to increase equity holdings towards α^{SO}.

Damage d	α^*	α^{SO}	Subsidy s
$d = 0.1$	0.44	0.26	-0.03
$d = 0.2$	0.44	0.34	-0.01
$d = 0.7$	0.44	0.5	0.004
$d = 0.9$	0.44	0.5	0.004

Stock θ	α^*	α^{SO}	Subsidy s
$\theta = 0.3$	0.2	0.23	0.01
$\theta = 0.5$	0.44	0.26	-0.03
$\theta = 0.7$	0.5	0.30	-0.05
$\theta = 1$	0.5	0.34	-0.09

Table II. Optimal equity subsidies/taxes.

The right panel of Table II reports a similar evaluation of α^* and α^{SO}, but now fixing d at $d = 0.1$ and varying the stock parameter θ. Equilibrium equity α^* increases in θ, and so does α^{SO}. However, equilibrium equity increases faster than α^{SO}, reducing the amount of tax that the planner needs to offer firms to induce them to select α^{SO} in equilibrium.

5 Extensions

5.1 Changes in the cost externality

Appendix 1 tests the robustness of our results when we consider a more general cost function $C_i(q_i, q_j) = \frac{q_i(q_i+\lambda q_j)}{\theta}$, where parameter $\lambda \in [0,1]$ denotes the severity of the cost externality that firm j's appropriation produces on firm i. Our model in previous sections considered, for simplicity, that $\lambda = 1$, but we now allow for smaller cost externalities, $\lambda < 1$. In the appendix we identify every firm i's best response function which, as expected, pivots inwards as cost externality parameter λ increases. We then characterize equilibrium appropriation levels in this context. For instance, when firms share symmetric equity, equilibrium appropriation becomes $q_i^* = \frac{(1-\alpha)\theta}{2+\lambda-2\alpha}$, which is decreasing in cost externalities. We also identify the socially optimal exploitation level,

$q_i^{SO} = \frac{\theta[2+(a-1)\gamma]}{4(1+\lambda+2d\theta)}$, which is also decreasing in λ, and then compare q_i^{SO} and q_i^* to find under which parameter conditions exploitation is socially excessive. In particular, $q_i^{SO} \leq q_i^*$ if $\alpha \leq \alpha^{SO}$, where

$$\alpha^{SO}(\lambda) \equiv 1 + \frac{\lambda\left[2+\gamma(a-1)\right]}{2(a-1)\gamma - 4(\lambda+2d\theta)}$$

which decreases in λ if and only if $d > \frac{\gamma(a-1)}{4\theta}$. Intuitively, when exploitation generates large environmental damage, an increase in cost externalities leads the planner to reduce q_i^{SO} more significantly than firms decrease q_i^*, thus shrinking the region of over-exploitation. In contrast, when environmental damage is minor, an increase in cost externalities induces firms to reduce q_i^* more substantially than the social planner decreases q_i^{SO}, expanding as a result the region where over-exploitation can be sustained.

5.2 Allowing for N firms

For completeness, Appendix 2 extends our model to a setting with N firms, showing that our above results are qualitatively unaffected. As expected, we demonstrate that equilibrium appropriation and socially optimal appropriation are both decreasing in the number of firms, N. In addition, we show that equilibrium appropriation is socially excessive, $q_i^{SO}(N) \leq q_i^*(N)$, if and only if $\alpha < \alpha^{SO}(N)$, where

$$\alpha^{SO}(N) \equiv \frac{1}{N-1}\left[1 - \frac{2+\gamma(a-1)}{N(2-\gamma(a-1)+4d\theta)}\right].$$

We are interested in evaluating cutoff $\alpha^{SO}(N)$ at different values of N, to examine how less concentrated markets affect the region of (α, d)-pairs for which socially excessive exploitation can be sustained. Figure 3 depicts cutoff $\alpha^{SO}(N)$ assuming the same parameter values as Figure 2, at $N = 2$, $N = 5$ and $N = 10$ firms.

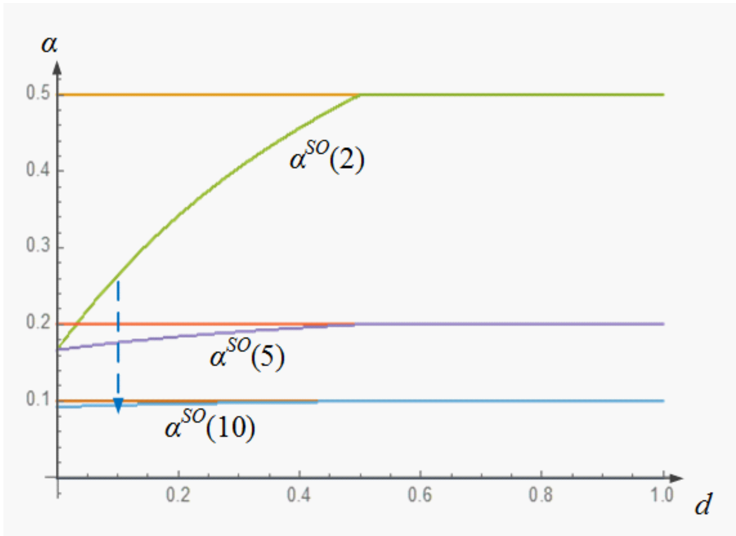

Figure 3. Cutoff $\alpha^{SO}(N)$ for different values of N.

Cutoff $\alpha^{SO}(N)$ is decreasing in the number of firms. However, the set of parameter values for which socially excessive exploitation holds, $\alpha \leq \alpha^{SO}(N)$, needs to be compared against the upper bound on α, $1/N$. When comparing them, it is clear that the region below cutoff $\alpha^{SO}(N)$ spans over a larger share of the area below $1/N$ as more firms compete for the resource.[28] Intuitively, as more firms enter into the CPR, their appropriation becomes socially excessive under most parameter conditions. Indeed, while individual appropriation q_i^* decreases in N, aggregate appropriation $Q^* = Nq_i^*$ increases, which explains socially excessive exploitation arises under larger conditions or, alternatively, why firms' exploitation is only socially optimal if equity shares are close to $1/2$.

5.3 Allowing for different costs of equity

Appendix 3 analyzes how equilibrium equity is affected if firms face a linear cost of equity function $C(\alpha_i) = c\alpha_i$, or a concave function $C(\alpha_i) = c\sqrt{\alpha_i}$, where $c > 0$. Intuitively, the first case could arise if firms pay the same amount per share when acquiring further equity on their rivals' profits, while the second case could exist if firms see their cost per share decrease as they increase their equity on other companies. (While these cases may be rare in some industries, we consider them here as

[28] Formally, we first need to find the crossing point between the horizontal line $1/N$ and cutoff $\alpha^{SO}(N)$. (Considering the parameter values in Figure 3, this crossing point occurs at $\widehat{d} = \frac{1}{2}$.) We then evaluate: (a) the area to the right-hand side of $\widehat{d}$, a rectangle with height $1/N$ and base $\widehat{d}$; and (b) the area below cutoff $\alpha^{SO}(N)$ to the left-hand side of crossing point $\widehat{d}$, i.e., between $\widehat{d}$ and 0, given by the integral $\int_0^{\widehat{d}} \alpha^{SO}(N)dd$. Adding up the areas in (a) and (b), we obtain an expression that increases in the number of firms, N. In particular, we can compute the percentage that areas (a) and (b) represent out of the rectangle with height $1/N$ and base 1, $SER \equiv \frac{(a)+(b)}{1/N}$, where SER denotes the percentage of admissible (α, d)-pairs for which socially excessive exploitation emerges in equilibrium. For our parameter values in Figure 3, we find that $SER = 0.86$ in the case of $N = 2$ firms, increases to $SER = 0.96$ in the case of $N = 5$ firms, and to $SER = 0.98$ in the case of $N = 10$ firms.

a robustness check.) Equilibrium appropriation during the second stage is, of course, unaffected by the cost of equity function; an argument that applies to the socially optimal output q_i^{SO} and to the optimal cutoff α^{SO}. Equilibrium equity α_i^* is, however, affected by cost function $C(\alpha_i)$, thus affecting the equity subsidy/tax that the social planner needs to set in order to induce firms to move from equity level α_i^* to α^{SO}.

The marginal cost of equity identified in Proposition 3 becomes a constant in the case that firms face a linear cost of equity function, $MC_i = c$. In terms of Figure 2, this entails that MC_i is flat, implying that a marginal increase in c now increases the firm's equilibrium equity α_i^*; a result that is due to the increasing and convex shape of MB_i. When firms face a concave cost of equity functions, marginal cost of equity becomes $MC_i = \frac{c}{2\sqrt{\alpha_i}}$, thus being decreasing in equity α_i. Like in the case of linear costs, concave costs also entail that a marginal increase in parameter c yield an increase in firms' equilibrium equity α_i^*. For illustration purposes, Appendix 3 reproduces Table I for these two alternative cost of equity functions, indicating that equilibrium equity α_i^* increases in c but decreases in θ; as well as Table II, which indicates how optimal subsidies are affected.

5.4 Jointly-determined equity shares

In previous sections, we considered that every firm independently chooses its equity share on its rival's profits. In some settings, however, firms may explicitly negotiate with each other their equity holding. In this subsection, we examine how our above results are affected when firms jointly choose their equity shares in each other's profits, α_i and α_j. In this case, firms solve the following joint-maximization problem

$$\max_{\alpha_i,\alpha_j \geq 0} (1-\alpha_j)\pi_i\left(q_i^*, q_j^*\right) + \alpha_i \pi_j\left(q_i^*, q_j^*\right) - (F + 2c\alpha_i^2)$$
$$+(1-\alpha_i)\pi_j\left(q_i^*, q_j^*\right) + \alpha_j \pi_i\left(q_i^*, q_j^*\right) - (F + 2c\alpha_j^2)$$

Differentiating with respect to α_i yields

$$MB_i \equiv \frac{\theta(1-\alpha_i-\alpha_j)}{(3-\alpha_i-\alpha_j)^3} = 2c\alpha_i \equiv MC_i$$

and, similarly, when differentiating with respect to α_j we obtain

$$MB_j \equiv \frac{\theta(1-\alpha_i-\alpha_j)}{(3-\alpha_i-\alpha_j)^3} = 2c\alpha_j \equiv MC_j$$

where $MB_i = MB_j$.[29] In a symmetric equilibrium, $\alpha_i^* = \alpha_j^* = \alpha^*$. Since the above first-order conditions are highly nonlinear, we cannot obtain an explicit solution for equilibrium equity share

[29] In addition, MB_i originates at $\frac{\theta(1-\alpha_j)}{(3-\alpha_j)^3}$ when $\alpha_i = 0$, and reaches a height of $\frac{\theta(\frac{1}{2}-\alpha_j)}{(\frac{5}{2}-\alpha_j)^3}$ when $\alpha_i = 1/2$. Furthermore, MB_i is decreasing in firm i's equity share since $\frac{\partial MB_i}{\partial \alpha_i} = -\frac{2\theta(\alpha_i+\alpha_j)}{(3-\alpha_i-\alpha_j)^4}$ is negative for all parameter values, and convex in firm i's equity share since $\frac{\partial^2 MB_i}{\partial \alpha_i^2} = -\frac{6\theta(1+\alpha_i+\alpha_j)}{(3-\alpha_i-\alpha_j)^5}$ is also negative for all parameter conditions.

α^*. Following a similar approach as in Section 3.2, Table III below numerically solves for the equilibrium equity share α^*. For comparison purposes, we consider the same list of parameter values as in Table I in that section.

Cost c / Stock θ	$\theta = 0.3$	$\theta = 0.5$	$\theta = 0.7$	$\theta = 1$
$c = 1/40$	0.2	0.29	0.35	0.39
$c = 1/25$	0.13	0.21	0.27	0.33
$c = 1/20$	0.11	0.17	0.23	0.29
$c = 1/15$	0.08	0.13	0.18	0.24
$c = 1/10$	0.06	0.09	0.13	0.17
$c = 1/5$	0.03	0.05	0.06	0.09

Table III. Equilibrium equity share α^*.

Equity shares are therefore lower when firms jointly choose their equity holdings than when they independently do (relative to Table I in Section 3.2). Intuitively, when every firm independently increases its equity share α_i, it ignores the profit reduction that such an increase produces on the other firm's objective function. However, in the joint-maximization problem firms internalize this effect, leading them to acquire less equity on each other's profits.

For completeness, we also report Table IV (analogous to Table II in Section 4.1), which compares the equilibrium equity share α^*, the socially optimal equity share α^{SO}, and the subsidy that lowers firms' cost of equity acquisition to induce them to hold a socially optimal equity in equilibrium. Since equilibrium equity shares are now lower than when firms independently choose α_i, the regulator needs to provide a more generous subsidy to induce socially optimal equity share α^{SO}, which holds under all parameter conditions. Therefore, when firms jointly choose their equity shares on each other's profits (as those emerging from explicit negotiations between firms), our results suggest that regulators can anticipate lower equity shares in equilibrium. In these settings, subsidies would need to be more generous to induce socially optimal outcomes.

Damage d	α^*	α^{SO}	Subsidy s
$d = 0.1$	0.13	0.26	0.03
$d = 0.2$	0.13	0.34	0.05
$d = 0.7$	0.13	0.5	0.07
$d = 0.9$	0.13	0.5	0.07

Stock θ	α^*	α^{SO}	Subsidy s
$\theta = 0.3$	0.08	0.23	0.04
$\theta = 0.5$	0.13	0.26	0.03
$\theta = 0.7$	0.18	0.30	0.03
$\theta = 1$	0.24	0.34	0.03

Table IV. Optimal equity subsidies/taxes.

6 Discussion

Our results identify the equity share that firms exploiting a CPR hold in equilibrium, and the equity level that guarantees that equilibrium appropriation coincides with the social optimum. Starting from the most basic setting, where all appropriation is sold overseas and exploiting the resource does not generate environmental externalities (i.e., $\gamma = d = 0$), the socially optimal equity share (as captured by cutoff α^{SO}) lies at its highest: firms must equally share profits for them to produce socially optimal amounts. We next discuss other, less extreme, settings.

Taxes on equity acquisition. When a positive share of output is sold domestically, $\gamma > 0$, cutoff α^{SO} decreases. In words, equilibrium appropriation can be socially optimal, even if firms do not equally share their profits. Interestingly, their exploitation of the resource can become socially insufficient if they sustain sufficiently large equity shares. In that setting firms' output would approach that of a merged company, while the social planner would prefer a higher production given its positive effect on consumer surplus. A similar argument applies when demand becomes stronger (higher a), since the planner seeks a larger appropriation level under larger parameter conditions. In settings where both γ and a are sufficiently high, it may become optimal for firms to not sustain equity shares on each other's profits; as otherwise their output would be socially insufficient. In such a case, the social planner can either: (1) set a lower bound on equity shares that firms can hold; (2) subsidize the exploitation of the resource to ameliorate underexploitation; or (3) impose a negative subsidy (a tax) on equity share acquisition. While the first two policy tools require costly monitoring and supervision, the third policy only needs information about firms' equity shares, which is generally requested by government authorities anyway.

Subsidies on equity acquisition. The opposite effect emerges when environmental damage is positive, $d > 0$, shifting cutoff α^{SO} upwards. In this case, equilibrium equities are likely lower than socially optimal equities, entailing that equilibrium appropriation becomes socially excessive under larger parameter combinations. In this setting, policy makers would want firms to hold a larger equity share on their rival's profits, which would reduce their equilibrium appropriation, ultimately curbing their associated environmental damage (e.g., pollution and biodiversity loss). A similar argument applies when the stock becomes more abundant (higher θ), which also shifts cutoff α^{SO} upward, making socially excessive exploitation more likely. In extreme settings where the stock is abundant, but its exploitation generates substantial environmental damages, firms would only have incentives to produce socially optimal levels if they equally share profits.

When overexploitation emerges in equilibrium, the social planner could set an upper bound on equity shares that firms can hold, which may not be feasible. Alternatively, regulators can set an emission fee on the exploitation of the resource itself (approaching equilibrium appropriation to its socially optimal level), or subsidize equity acquisition, inducing firms to choose the socially optimal level of equity, α^{SO}. As discussed in previous sections, emission fees are often costly to monitor and implement, thus making the alternative policy (equity subsidies and taxes) more attractive in overexploited CPRs.

Welfare improving policies. Our findings can be used by regulatory agencies to set equity bounds (both upper and lower) on the equity shares that firms exploiting CPRs can hold. However, setting a lower bound on equity shares may not even be politically feasible. We then characterize an alternative policy tool –subsidies and taxes on equity acquisition– which induces firms to hold the socially optimal amount of equity shares, thus exploiting the resource at an welfare-maximizing level. Unlike traditional policies, however, equity subsidies benefit from its easy implementation and low monitoring costs. Our findings also allow for a policy mix between emission fees and equity subsidies.

Further research. Our model can be extended to consider settings in which each firm exploits a different CPR, rather than both appropriating from the same commons; and to industries in which firms do not perfectly observe the extent to which one firm places a cost externality on another. Alternatively, the model can also be applied to a repeated game, where the resource depletes as time progresses; or a setting where firms sequentially choose their equity shares. Additionally, our results can be empirically estimated, and tested in field experiments, to evaluate if firms' observed exploitation of the CPR approaches our theoretical predictions.

7 Appendix

7.1 Appendix 1 - Extension to different cost externalities

In this appendix, we allow firms' appropriation to have different cost externalities, so every firm i's cost function becomes $C_i(q_i, q_j) = \frac{q_i(q_i+\lambda q_j)}{\theta}$. Parameter $\lambda \in [0, 1]$ indicates how firm j's appropriation affects firm i's costs, i.e., cost externalities. When $\lambda = 0$, every firm's costs are unaffected by its rival's appropriation; whereas when $\lambda = 1$, every unit of firm j's appropriation increases firm i's marginal costs by one dollar. The model in the paper is then a special case of this general setting, where $\lambda = 1$, collapsing the above cost function to $C_i(q_i, q_j) = \frac{q_i(q_i+q_j)}{\theta}$.

Equilibrium appropriation. Every firm i solves

$$\max_{q_i \geq 0} \quad V_i = (1 - \alpha_j)\pi_i + \alpha_i \pi_j$$

where $\pi_i \equiv q_i - \frac{q_i(q_i+\lambda q_j)}{\theta}$ denotes firm i's profit, and $j \neq i$. Differentiating with respect to q_i yields best response function

$$q_i(q_j) = \begin{cases} \frac{\theta}{2} - \frac{\lambda(1+\alpha_i-\alpha_j)}{2(1-\alpha_j)} q_j & \text{if} \quad q_j \leq \frac{\theta(1-\alpha_j)}{\lambda(1+\alpha_i-\alpha_j)} \\ 0 & \text{otherwise.} \end{cases}$$

which coincides with that in Lemma 1 when $\lambda = 1$; as expected. Simultaneously solving for appropriation levels q_i and q_j in best response functions $q_i(q_j)$ and $q_j(q_i)$, we obtain equilibrium appropriation

$$q_i^* = \frac{\theta(1 - \alpha_i)\left[2 - \alpha_j(2 - \lambda) - \lambda(1 + \alpha_i)\right]}{4(1 - \alpha_i)(1 - \alpha_j) - \left[1 - (\alpha_i - \alpha_j)\right]\lambda^2}$$

which collapses to $q_i^* = \frac{\theta(1-\alpha_i)}{(3-\alpha_i-\alpha_j)}$ when $\lambda = 1$, as in our paper. To show this appropriation is unambiguously positive, we consider the numerator and denominator separately. Term $\theta(1 - \alpha_i)$ is always positive, while $[2-\alpha_j(2-\lambda)-\lambda(1+\alpha_i)]$ is non-negative. To see this, note that the term is strictly decreasing in α_i and α_j. Therefore, evaluating this term at the upper bound of both α_i and α_j, i.e., $\alpha_i = \alpha_j = \frac{1}{2}$, we obtain that we need $\lambda > 1$ for the term to be negative, which is impossible by definition. For the denominator, we follow similar steps to demonstrate that is is positive. Specifically, term $4(1-\alpha_i)(1-\alpha_j)$ is positive, and its lower bound is 1, which occurs when $\alpha_i = \alpha_j = \frac{1}{2}$. Term $[1-(\alpha_i-\alpha_j)]\,\lambda^2$ reaches its upper bound when $\alpha_i = \alpha_j$ and $\lambda = 1$. Therefore, the denominator is weakly positive.

Finally, subtracting equilibrium terms of q_j^* from q_i^*, we obtain

$$q_i^* - q_j^* = \frac{(\alpha_j-\alpha_i)(1-\alpha_i-\alpha_j)\lambda\theta}{4(1-\alpha_i)(1-\alpha_j)-[1-(\alpha_i-\alpha_j)^2]\,\lambda^2}$$

which, solving for α_i, is positive if only if $\alpha_i \leq \alpha_j$.

We next examine equilibrium appropriation q_i^* in special cases, as in Corollary 1 in the main body of the paper.

1. No equity shares, and no cost externalities: $q_i^* = \frac{\theta}{2}$, i.e., $\alpha_i = 0$ for every firm i, and $\lambda = 0$;
2. CPR model without equity shares: $q_i^* = \frac{\theta}{2+\lambda}$, i.e., $\alpha_i = 0$ for every firm i;
3. CPR model with symmetric equity shares: $q_i^* = \frac{(1-\alpha)\theta}{2+\lambda-2\alpha}$, i.e., $\alpha_i = \alpha_j = \alpha$;
4. CPR model with equally shared equity: $q_i^* = \frac{\theta}{2+2\lambda}$, i.e., $\alpha_i = \alpha_j = \frac{1}{2}$.

Optimal equity shares We first evaluate firm i's equilibrium profits in the second stage of the game, $\pi_i\left(q_i^*, q_j^*\right)$, by inserting equilibrium appropriation levels q_i^* and q_j^*, which yields

$$\pi_i\left(q_i^*, q_j^*\right) = \frac{A\theta\left[2(1-\alpha_i)(1-\alpha_j)-(1-\alpha_i)(1-\alpha_i-\alpha_j)\lambda-\alpha_i(1-\alpha_i+\alpha_j)\lambda^2\right]}{\{4(1-\alpha_i)(1-\alpha_j)-[1-(\alpha_i-\alpha_j)^2]\,\lambda^2\}^2}$$

where $A \equiv (1-\alpha_i)\left[2-\alpha_j(2-\lambda)-\lambda(1+\alpha_i)\right]$. Note that $\pi_i\left(q_i^*, q_j^*\right)$ collapses to $\pi_i\left(q_i^*, q_j^*\right) = \frac{\theta(1-\alpha_i)}{(3-\alpha_i-\alpha_j)^2}$ when $\lambda = 1$, as in our paper. Operating similarly for the equilibrium profits of firm j, we obtain

$$\pi_j\left(q_i^*, q_j^*\right) = \frac{B\theta(1-\alpha_j)\{[2-(1-\alpha_j(1-\lambda))]\,\lambda-\alpha_i\left[2-\lambda-\alpha_j(2-\lambda+\lambda^2)\right]\}}{\{4(1-\alpha_i)(1-\alpha_j)-[1-(\alpha_i-\alpha_j)^2]\,\lambda^2\}^2}$$

where $B \equiv (1-\alpha_j)(2-\alpha_i(2-\lambda)-\lambda(1+\alpha_j))$. This term collapses to $\frac{\theta(1-\alpha_j)}{(3-\alpha_i-\alpha_j)^2}$ when $\lambda = 1$, as in our paper. .

Therefore, every firm i solves

$$\max_{\alpha_i \geq 0} \; (1-\alpha_j)\pi_i\left(q_i^*, q_j^*\right) + \alpha_i\pi_j\left(q_i^*, q_j^*\right) - (F + c\alpha_i^2)$$

Differentiating with respect to α_i yields

$$\frac{1}{C}(1-\alpha_j)\left[2D(4(1-\alpha_j)-2(\alpha_i-\alpha_j)\lambda^2)+E(4(1-\alpha_i)(1-\alpha_j)-(1-(\alpha_i-\alpha_j)^2)\lambda^2)\right]\theta=2c\alpha_i$$

where terms $C-E$ are

$$C\equiv\left[4(1-\alpha_i)(1-\alpha_j)-\left[1-(\alpha_i-\alpha_j)^2\right]\lambda^2\right]^3,$$

$$\begin{aligned}D\ \equiv\ & 4(1-\alpha_i)^2(1+\alpha_i-\alpha_j)(\alpha_j-1)+4(\alpha_i-1)^2(1+\alpha_i-\alpha_j)(\alpha_j-1)\lambda\\ &+\left[(\alpha_j-1)^2-\alpha_i^4+\alpha_i^3(1+3\alpha_j)+\alpha_i(\alpha_j+\alpha_j^2+\alpha_j^3-3)+\alpha_i^2\left[2-\alpha_j(2+3\alpha_j)\right]\right]\lambda^2\\ &+\alpha_i(1+\alpha_i-\alpha_j)(1-\alpha_i+\alpha_j)^2\lambda^3.\end{aligned}$$

$$\begin{aligned}E\ \equiv\ & 4\alpha_i^3(\lambda-1)\lambda^2-(\alpha_j-1)(\left[8\alpha_j-4+(4-8\alpha_j)\lambda-\left[3+\alpha_j(2+\alpha_j)\right]\lambda^2+(1+\alpha_j)^2\lambda^3\right]\\ &-3\alpha_i^2(\lambda-1)\left[4+\lambda^2+\alpha_j(3\lambda^2-4)\right]\\ &+\alpha_i\left[4\alpha_j(\lambda-1)\lambda^2-8+2\lambda\{4+(2-\lambda)\lambda\}+2\alpha_j^2(\lambda-1)(3\lambda^2-4)\right].\end{aligned}$$

The left-hand side represents the marginal benefit from equity acquisition. When evaluated at $\lambda=1$, the above expression collapses to $\frac{2\theta(1-\alpha_j)}{(3-\alpha_i-\alpha_j)^3}=2c\alpha_i$, as in the main body of the paper. If we invoke symmetry in shares, i.e., $\alpha_i=\alpha_j=\alpha$, we obtain,

$$\frac{(1-\alpha_i)\left[4(1-\alpha_i)^3+4(1-\alpha_i)^2\alpha_i\lambda-\left[1+(4\alpha_i-3)\alpha_i\right]\lambda^2-(1-\alpha_i)\lambda^3\right]\theta}{(2-2\alpha_i+\lambda)^3(2-2\alpha_i-\lambda)}=2c\alpha_i$$

Social optimum The social planner solves (3), but with out new cost function. Differentiating with respect to q_i yields

$$q_i(q_j)=\frac{\left[2+(a-1)\gamma\right]\theta-4(\lambda+d\theta)q_j}{4(1+d\theta)}$$

and a symmetric expression for $q_j(q_i)$. Simultaneously solving for $q_i(q_j)$ and $q_j(q_i)$, we find that the social optimum is

$$q_i^{SO}=\frac{\theta\left[2+(a-1)\gamma\right]}{4(1+\lambda+2d\theta)}$$

This collapses to $q_i^{SO}=\frac{\theta[2+(a-1)\gamma]}{8(1+d\theta)}$ when $\lambda=1$, as in our paper.

Evaluating the difference between the socially optimal appropriation and the equilibrium appropriation, $q_i^{SO}-q_i^*$, we obtain that such difference is negative (i.e., $q_i^{SO}\leq q_i^*$) if and only if α_i satisfies $\alpha\leq\alpha^{SO}$ (where we assumed symmetric equity shares, $\alpha_i=\alpha_j=\alpha$ in equilibrium), and

$$\alpha^{SO}\equiv1+\frac{\lambda\left[2+\gamma(a-1)\right]}{2(a-1)\gamma-4(\lambda+2d\theta)}.$$

which becomes $1+\frac{2+\gamma(a-1)}{2(a-1)\gamma-4(1+2d\theta)}$ when $\lambda=1$, as in our paper. Since $\alpha\in\left[0,\frac{1}{2}\right]$ by definition, we need first need that cuotff α^{SO} satisfies $\alpha^{SO}\geq 0$ which, solving for d, yields $d\leq\frac{\gamma(1+\lambda)(a-1)}{4\theta}$. Second, we need that cutoff α^{SO} satisfies $\alpha^{SO}\leq\frac{1}{2}$ which, solving for a, entails $d\geq\frac{\gamma(2+\lambda)(a-1)-2\lambda}{8\theta}$. In addition,

$$\frac{\gamma(1+\lambda)(a-1)}{4\theta}\geq\frac{\gamma(2+\lambda)(a-1)-2\lambda}{8\theta}$$

holds for all admissible parameter values. Therefore, cutoff α^{SO} lies in $\alpha^{SO}\in\left[0,\frac{1}{2}\right]$ if and only if $d\in\left[\frac{\gamma(2+\lambda)(a-1)-2\lambda}{8\theta},\frac{\gamma(1+\lambda)(a-1)}{4\theta}\right]$. This range collapses to $\alpha^{SO}\in\left[0,\frac{1}{2}\right]$ if and only if parameter d satisfies $d\in\left[\frac{3(a-1)\gamma-2}{8\theta},\frac{(a-1)\gamma}{2\theta}\right]$ when $\lambda=1$, as in our paper.

Comparative statics on socially optimal equity shares We next differentiate cutoff α^{SO} with respect to parameters. First,

$$\frac{\partial\alpha^{SO}}{\partial a}=-\frac{\gamma\lambda(1+\lambda+2d\theta)}{(\gamma-a\gamma+2\lambda+4d\theta)^2}$$

which is negative for all parameter values, implying that cutoff α^{SO} is decreasing in a. Second,

$$\frac{\partial\alpha^{SO}}{\partial\lambda}=\frac{(2+(a-1)\gamma)((a-1)\gamma-4d\theta)}{2(\gamma-a\gamma+2\lambda+4d\theta)^2}$$

which is negative if $d>\frac{\gamma(a-1)}{4\theta}$. In such a case, cutoff α^{SO} is decreasing in λ. Third,

$$\frac{\partial\alpha^{SO}}{\partial\gamma}=-\frac{(a-1)\lambda(1+\lambda+2d\theta)}{[(1-a)\gamma+2(\lambda+2d\theta)]^2}$$

which is negative for all parameter values, entailing that cutoff α^{SO} is decreasing in γ. Fourth,

$$\frac{\partial\alpha^{SO}}{\partial\theta}=\frac{2d(2+(a-1)\gamma)\lambda}{[(1-a)\gamma+2(\lambda+2d\theta)]^2}$$

which is positive for all parameter values. Therefore, cutoff α^{SO} is increasing in θ. Fifth,

$$\frac{\partial\alpha^{SO}}{\partial d}=\frac{2(2+(a-1)\gamma)\lambda\theta}{[(1-a)\gamma+2(\lambda+2d\theta)]^2}$$

which is positive for all parameter values. Therefore, cutoff α^{SO} is increasing in d.

Optimal fees When firm i faces an emission fee t, it solves problem (1) where now its profit is given by $\pi_i=q_i-\frac{q_i(q_i+\lambda q_j)}{\theta}-tq_i$. Differentiating with respect to q_i, and simultaneously solving, yields

$$q_i^*(t)=\frac{\theta(1-\alpha)\left[2(1-\alpha)-\lambda\right](1-t)}{4-\lambda^2},$$

which coincides with the equilibrium appropriation q_i^* when emission fees are absent, $t=0$. When fees are positive, $t>0$, equilibrium appropriation becomes lower.

In order to set the optimal emission fee, the regulator finds the fee t that solves $q_i^*(t) = q_i^{SO}$, that is

$$t^* = \frac{(a-1)\gamma\left[2(1-\alpha)+\lambda\right] - 2(1-2\alpha)\lambda - 8(1-\alpha)d\theta}{(\alpha-1)(1+\lambda+2d\theta)},$$

which is positive as long as α satisfies $\alpha \leq \alpha^{SO}$. This term collapses to $1 - \frac{(3-2\alpha)(2+(a-1)\gamma)}{8(1-\alpha)(1+d\theta)}$ when $\lambda = 1$, as in our paper. Finally, differentiating emission fee t^* with respect to α yields

$$\frac{\partial t^*}{\partial \alpha} = -\frac{\left[2+\gamma(a-1)\right]\lambda}{4(1-\alpha)^2(1+\lambda+2d\theta)} < 0.$$

7.2 Appendix 2 - Extension to N firms

In this appendix, we extend our model to a setting with N firms. For compactness, we focus on the second stage alone, and assume symmetric equity shares $\alpha_i = \alpha_j = \alpha$ for every two firms i and j. Following the same approach as in the main text, we first identify equilibrium appropriation in this context, then the socially optimal appropriation, and finally compare these two findings.

Equilibrium appropriation. Every firm i solves

$$\max_{q_i \geq 0} V_i = \left[1-(N-1)\,\alpha\right]\pi_i + \sum_{j\neq i} \alpha \pi_j$$

where $\pi_i \equiv q_i - \frac{q_i(q_i+Q_{-i})}{\theta}$ denotes firm i's profit, $Q_{-i} \equiv \sum_{j\neq i} q_j$ represents the aggregate appropriation from all firms other than i, and $\pi_j \equiv q_j - \frac{q_j(q_j+Q_{-j})}{\theta}$ denotes firm j's profit. Differentiating with respect to q_i yields best response function

$$q_i(Q_{-i}) = \frac{\theta}{2} - \frac{\left[(N-2)\alpha - 1\right]}{2\left[(N-1)\alpha - 1\right]} Q_{-i}.$$

Relative to the best response function identified in Lemma 1 for two firms, $q_i(Q_{-i})$ has the same vertical intercept, $\frac{\theta}{2}$, and decreases in its rivals' appropriation, Q_{-i}; but has a different slope, $\frac{[(N-2)\alpha-1]}{2[(N-1)\alpha-1]}$. In particular, the slope becomes more negative as the number of firms N increases. Intuitively, competition becomes tougher, and every individual firm reduces its own appropriation more significantly, i.e., firms' exploitation of the resource are strategic substitutes to a greater extent. Graphically, the best response function rotates inwards.

Invoking symmetry in equilibrium, $q_i^* = q_j^* = q^*$, we obtain that $Q_{-i}^* = (N-1)q^*$, which helps us simplify the above expression to

$$q^* = \frac{\theta}{2} - \frac{\left[(N-2)\alpha - 1\right]}{2\left[(N-1)\alpha - 1\right]}(N-1)q^*$$

Solving for q^*, yields the equilibrium appropriation in this N-firm setting

$$q_i^*(N) = \frac{\theta[\alpha(N-1)-1]}{N[\alpha(N-1)-1]-1}$$

When only two firms operate in the commons, $N = 2$, equilibrium appropriation simplifies to $q_i^*(2) = \frac{\theta(1-\alpha)}{3-2\alpha}$, which coincides with that in Corollary 1 when equity shares are equal across firms.

We next examine special cases, as in Corollary 1 in the main text, showing that equilibrium appropriation q_i^* becomes:

1. *CPR model without equity shares*: $q_i^* = \frac{\theta}{N+1}$, i.e., $\alpha_i = 0$ for every firm i.
2. *CPR model with equally shared equity*: $q_i^* = \frac{\theta}{2N}$, i.e., $\alpha_i = \alpha_j = \frac{1}{N}$.

Social optimum. The social planner solves

$$\begin{aligned}\max_{q_1,,,,q_N} W &= \gamma CS(Q) + PS(Q) - dQ^2 \\ &= \gamma \frac{(a-1)}{2} Q + \sum_{i=1}^{N} \pi_i - dQ^2\end{aligned}$$

where $Q \equiv \sum_{i=1}^{N} q_i$ denotes aggregate output. Note that, since equity shares are symmetric in this setting, they cancel out from the producer surplus, i.e., $PS(Q) = \sum_{i=1}^{N} V_i = \sum_{i=1}^{N} \pi_i$. Differentiating with respect to q_i, we obtain

$$q_i(Q_{-i}, Q) = \frac{[2 + (a-1)\gamma - 4dQ]\,\theta}{4} - Q_{-i}$$

Invoking symmetric appropriation, we find that the socially optimal exploitation level becomes

$$q_i^{SO}(N) = \frac{\theta\,[2 + (a-1)\gamma]}{4N(1 + d\theta)}$$

which collapses to $q_i^{SO}(2) = \frac{\theta[2+(a-1)\gamma]}{8(1+d\theta)}$ when only two firms operate, $N = 2$, thus coinciding with our result in Proposition 3. Socially optimal output $q_i^{SO}(N)$ decreases in the number of firms exploiting the resource, N since $\frac{\partial q_i^{SO}(N)}{\partial N} = -\frac{(2+(a-1)\gamma)\theta}{4N^2(1+d\theta)} < 0$. Finally, equilibrium appropriation is socially excessive, $q_i^{SO}(N) \leq q_i^*(N)$, if and only if

$$\alpha < \alpha^{SO}(N) \equiv \frac{1}{N-1}\left[1 - \frac{2 + \gamma(a-1)}{N(2 - \gamma(a-1) + 4d\theta)}\right]$$

In the case that only two firms compete in the commons, $N = 2$, cutoff $\alpha^{SO}(2)$ collapses to $\alpha^{SO}(2) \equiv 1 + \frac{2+\gamma(a-1)}{2(a-1)\gamma-4(1+2d\theta)}$, thus coinciding with cutoff α^{SO} in Corollary 2.

7.3 Appendix 3 - Extension to different cost of equity functions

In this appendix, we examine how our results are affected if we consider linear and concave cost of acquiring equity.

Linear cost of equity. First, we examine a linear cost of equity $C(\alpha_i) = c\alpha_i$, which implies that the first-order condition for equilibrium equity in Proposition 3 now becomes

$$MB_i \equiv \frac{2\theta(1-\alpha_j)}{(3-\alpha_i-\alpha_j)^3} = c \equiv MC_i.$$

In this setting, the MB_i coincides with that in Proposition 3 and Figure 2, but the MC_i curve is now flat. As a result, an increase in c produces an increase in equilibrium equity, as illustrated in Table A-I. For comparison purposes, the table considers the same parameter values as Table I in the main text.

Cost c / Stock θ	$\theta = 0.3$	$\theta = 0.5$	$\theta = 0.7$	$\theta = 1$
$c = 1/40$	0.12	0	0	0
$c = 1/25$	0.5	0.08	0	0
$c = 1/20$	0.5	0.29	0	0
$c = 1/15$	0.5	0.5	0.24	0
$c = 1/10$	0.5	0.5	0.5	0.29
$c = 1/5$	0.5	0.5	0.5	0.5

Table A-I. Equilibrium equity share α_i^*.

We also provide next Table A-II, which considers the same parameter values as Table II in the main text, in order to examine how optimal subsidies are affected by a linear cost of equity function. Since α^* increases in c in this context, the regulator needs to set a tax when $\alpha^* < \alpha^{SO}$, and a subsidy otherwise.

Damage d	α^*	α^{SO}	Subsidy s
$d = 0.1$	0.5	0.26	0.02
$d = 0.2$	0.5	0.34	0.01
$d = 0.7$	0.5	0.5	0
$d = 0.9$	0.5	0.5	0

Stock θ	α^*	α^{SO}	Subsidy s
$\theta = 0.3$	0.5	0.23	0.04
$\theta = 0.5$	0.5	0.26	0.02
$\theta = 0.7$	0.24	0.30	-0.004
$\theta = 1$	0	0.34	-0.04

Table A-II. Optimal equity subsidies/taxes.

Concave cost of equity. First, we examine a linear cost of equity $C(\alpha_i) = c\sqrt{\alpha_i}$, which implies that the first-order condition for equilibrium equity in Proposition 3 now becomes

$$MB_i \equiv \frac{2\theta(1-\alpha_j)}{(3-\alpha_i-\alpha_j)^3} = \frac{c}{2\sqrt{\alpha_i}} \equiv MC_i.$$

In this setting, the MB_i coincides with that in Proposition 3 and Figure 2, but the MC_i curve is now decreasing in equity α_i. Like in the cost of linear cost of equity, an increase in c produces an increase in equilibrium equity, as illustrated in Table A-III. For comparison purposes, the table considers the same parameter values as Table I in the main text. Unlike in the case of convex costs, the optimal shares a firm holds are now increasing in c and decreasing in θ; which is due to the difference in shape of the cost function.

Cost c / Stock θ	$\theta = 0.3$	$\theta = 0.5$	$\theta = 0.7$	$\theta = 1$
$c = 1/40$	0.21	0.09	0.05	0.03
$c = 1/25$	0.37	0.20	0.18	0.06
$c = 1/20$	0.47	0.26	0.17	0.09
$c = 1/15$	0.5	0.37	0.24	0.15
$c = 1/10$	0.5	0.5	0.4	0.26
$c = 1/5$	0.5	0.5	0.5	0.5

Table A-III. Equilibrium equity share α_i^*.

We also provide next Table A-IV, which considers the same parameter values as Table II in the main text, in order to examine how optimal subsidies are affected by a linear cost of equity function. Since α^* increases in c in this context, the regulator needs to set a tax when $\alpha^* < \alpha^{SO}$, and a subsidy otherwise. Our results in this case are, then, similar to those under linear costs, exhibiting the same comparative statics but with differences in magnitudes.

Damage d	α^*	α^{SO}	Subsidy s
$d = 0.1$	0.37	0.23	0.02
$d = 0.2$	0.37	0.26	0.01
$d = 0.7$	0.37	0.5	-0.02
$d = 0.9$	0.37	0.5	-0.02

Stock θ	α^*	α^{SO}	Subsidy s
$\theta = 0.35$	0.5	0.23	0.04
$\theta = 0.5$	0.37	0.26	0.02
$\theta = 0.7$	0.25	0.30	-0.01
$\theta = 1$	0.15	0.34	-0.06

Table A-IV. Optimal equity subsidies/taxes.

7.4 *Proof of Lemma 1*

Firm i solves problem (1). Differentiating with respect to q_i, we find $q_i(q_j) = \frac{\theta}{2} - \frac{(1+\alpha_i-\alpha_j)}{2(1-\alpha_j)}q_j$. Since firms appropriate weakly positive amounts, $\frac{\theta}{2} - \frac{(1+\alpha_i-\alpha_j)}{2(1-\alpha_j)}q_j \geq 0$ or, solving for q_j, $q_j \leq \frac{\theta(1-\alpha_j)}{(1+\alpha_i-\alpha_j)}$. Therefore, firm i's best response function is

$$q_i(q_j) = \begin{cases} \frac{\theta}{2} - \frac{(1+\alpha_i-\alpha_j)}{2(1-\alpha_j)}q_j & \text{if } q_j \leq \frac{\theta(1-\alpha_j)}{(1+\alpha_i-\alpha_j)} \\ 0 & \text{otherwise.} \end{cases}$$

7.5 *Proof for Proposition 1*

From Lemma 1, we found the best response function for firms i and j. Simultaneously solving for q_i and q_j in $q_i(q_j)$ and $q_j(q_i)$, we obtain that the optimal appropriation for every firm i is

$$q_i^* = \frac{\theta(1-\alpha_i)}{3-\alpha_i-\alpha_j}$$

which is positive for all admissible parameter values. We can now differentiate q_i^* with respect to parameters. First,

$$\frac{\partial q_i^*}{\partial \theta} = \frac{1-\alpha_i}{3-\alpha_i-\alpha_j} > 0$$

thus indicating that q_i^* is increasing in θ. Second,

$$\frac{\partial q_i^*}{\partial \alpha_i} = \frac{(\alpha_j-2)\theta}{(3-\alpha_i-\alpha_j)^2} < 0$$

which reflects that q_i^* is decreasing in α_i. Third,

$$\frac{\partial q_i^*}{\partial \alpha_j} = \frac{(1-\alpha_i)\theta}{(3-\alpha_i-\alpha_j)^2} > 0$$

which implies that q_i^* is increasing in α_j. Finally, the difference $q_i^* - q_j^* = \frac{\theta(\alpha_j-\alpha_i)}{3-\alpha_i-\alpha_j}$ is weakly positive if and only if $\alpha_i \leq \alpha_j$.

7.6 *Proof of Proposition 2*

We first evaluate equilibrium profits in the second stage of the game, $\pi_i\left(q_i^*, q_j^*\right)$, by inserting equilibrium appropriation levels found in Proposition 1, $q_i^* = \frac{\theta(1-\alpha_i)}{3-\alpha_i-\alpha_j}$ and $q_j^* = \frac{\theta(1-\alpha_j)}{3-\alpha_j-\alpha_i}$, which yields $\pi_i\left(q_i^*, q_j^*\right) = \frac{\theta(1-\alpha_i)}{(3-\alpha_i-\alpha_j)^2}$. Operating similarly for the equilibrium profit of firm j, we obtain $\pi_j\left(q_i^*, q_j^*\right) = \frac{\theta(1-\alpha_j)}{(3-\alpha_i-\alpha_j)^2}$. Therefore, every firm i solves

$$\begin{aligned} & \max_{\alpha_i \geq 0} \;\; (1-\alpha_j)\pi_i\left(q_i^*, q_j^*\right) + \alpha_i \pi_j\left(q_i^*, q_j^*\right) - c\alpha_i \\ = \;\; & \frac{\theta(1-\alpha_j)}{(3-\alpha_i-\alpha_j)^2} - c\alpha_i \end{aligned}$$

Differentiating with respect to α_i yields

$$\frac{2\theta(1-\alpha_j)}{(3-\alpha_i-\alpha_j)^3} - c \leq 0$$

with equality if $\alpha_i^* > 0$.

7.7 *Proof for Proposition 3*

The social planner solves problem in (3). Differentiating with respect to q_i yields

$$q_i(q_j) = \frac{[2+(a-1)\gamma]\,\theta - 4(1+d\theta)q_j}{4(1+d\theta)}$$

and a symmetric expression for $q_j(q_i)$. Simultaneously solving for $q_i(q_j)$ and $q_j(q_i)$, we find that the social optimum is

$$q_i^{SO} = \frac{\theta\,[2+(a-1)\gamma]}{8(1+d\theta)}$$

which is positive for all admissible parameter values.

Socially optimal output q_i^{SO} is increasing in a, in γ, but decreasing in d. In addition,

$$\frac{\partial q_i^{SO}}{\partial \theta} = \frac{2+(a-1)\gamma}{8(1+d\theta)^2}$$

which is positive for all admissible parameter values.

7.8 *Proof of Corollary 2*

Evaluating the difference between the socially optimal appropriation (from Proposition 3) and the equilibrium appropriation (from Proposition 1), $q_i^{SO} - q_i^* = \frac{\theta[2+(a-1)\gamma]}{8(1+d\theta)} - \frac{\theta(1-\alpha_i)}{3-\alpha_i-\alpha_j}$, we obtain that such difference is negative (i.e., $q_i^{SO} \leq q_i^*$) if and only if α_i satisfies $\alpha \leq \alpha_i^{SO}(\alpha_j)$, where cutoff $\alpha_i^{SO}(\alpha_j)$ is given by

$$\alpha_i^{SO}(\alpha_j) = \frac{2-3(a-1)\gamma + \alpha_j\,[2+(a-1)\gamma] + 8d\theta}{6-\gamma(a-1)+8d\theta}$$

In a symmetric equilibrium, equity shares satisfy $\alpha_i = \alpha_j = \alpha$, which collapses the above cutoff to

$$\alpha^{SO} \equiv 1 + \frac{2+\gamma(a-1)}{2(a-1)\gamma - 4(1+2d\theta)}.$$

Since $\alpha \in \left[0, \frac{1}{2}\right]$ by definition, we need first need that cuotff α^{SO} satisfies $\alpha^{SO} \geq 0$ which, solving for d, yields $d \geq d_1 \equiv \frac{3(a-1)\gamma-2}{8\theta}$. Second, we need that cutoff α^{SO} satisfies $\alpha^{SO} \leq \frac{1}{2}$ which, solving for d, entails $d \leq d_2 \equiv \frac{(a-1)\gamma}{2\theta}$. In addition, $d_2 > d_1$ since

$$\frac{(a-1)\gamma}{2\theta} > \frac{3(a-1)\gamma-2}{8\theta}$$

holds for all admissible parameter values. Therefore, cutoff α^{SO} lies in $\alpha^{SO} \in \left[0, \frac{1}{2}\right]$ if and only if parameter d satisfies $d \in \left[\frac{3(a-1)\gamma-2}{8\theta}, \frac{(a-1)\gamma}{2\theta}\right]$.

7.9 *Proof of Corollary 3*

We next differentiate cutoff α^{SO} (found in Corollary 2) with respect to parameters. First,

$$\frac{\partial \alpha^{SO}}{\partial a} = -\frac{2\gamma(1+d\theta)}{(2+\gamma-a\gamma+4d\theta)^2}$$

which is negative for all parameter values, implying that cutoff α^{SO} is decreasing in a. Second,

$$\frac{\partial \alpha^{SO}}{\partial \gamma} = -\frac{2(a-1)(1+d\theta)}{(2+\gamma-a\gamma+4d\theta)^2}$$

which is negative for all parameter values, entailing that cutoff α^{SO} is decreasing in γ. Fourth,

$$\frac{\partial \alpha^{SO}}{\partial \theta} = \frac{2d\,[2+(a-1)\gamma]}{(2+\gamma-a\gamma+4d\theta)^2}$$

which is positive for all parameter values. Therefore, cutoff α^{SO} is increasing in θ. Fifth,

$$\frac{\partial \alpha^{SO}}{\partial d} = \frac{2\theta\,[2+(a-1)\gamma]}{(2+\gamma-a\gamma+4d\theta)^2}$$

which is positive for all parameter values. Therefore, cutoff α^{SO} is increasing in d. Finally, Evaluating cutoff α^{SO} at $d=0$, we obtain $\alpha^{SO} = 1 + \frac{2+\gamma(a-1)}{2\gamma(a-1)-4}$, which further simplifies to $\alpha^{SO} = \frac{1}{2}$ when $\gamma = 0$.

7.10 *Proof of Proposition 4*

When firm i faces an emission fee t, it solves problem (1) where now its profit is given by $\pi_i = q_i - \frac{q_i(q_i+q_j)}{\theta} - tq_i$. Differentiating with respect to q_i, and simultaneously solving, yields

$$q_i^*(t) = \frac{\theta(1-\alpha)\,[2(1-\alpha)-1]\,(1-t)}{3},$$

which coincides with the equilibrium appropriation q_i^* when emission fees are absent, $t=0$. When fees are positive, $t>0$, equilibrium appropriation becomes lower.

In order to set the optimal emission fee, the regulator finds the fee t that solves $q_i^*(t) = q_i^{SO}$, that is

$$t^* = 1 - \frac{(3-2\alpha)(2+(a-1)\gamma)}{8(1-\alpha)(1+d\theta)}$$

which is positive as long as α satisfies $\alpha \leq \alpha^{SO}$. Finally, differentiating emission fee t^* with respect to α yields

$$\frac{\partial t^*}{\partial \alpha} = -\frac{2+\gamma(a-1)}{8(1-\alpha)^2(1+d\theta)} < 0.$$

References

[1] Ryan J.AlbertaChristineMcLaughlinaDebraFalatko

[2] Albert, Ryan J., McLaughlin, Christine, and Debra Falatko (2014) "Characterization of fish hold effluent discharged from commercial fishing vessels into harbor waters." Marine Pollution Bulletin 87(1-2): 29–38.

[3] Anthony, Jürgen, Michiel Bijlsma, Adam Elbourne, Marcel Lever, and Gijsbert Zwart (2012) "Financial transaction tax: review and assessment." The Netherlands Bureau for Economic Policy Analysis, Discussion Paper.

[4] Apesteguia, Jose (2006) "Does information matter in the commons?: Experimental evidence." Journal of Economic Behavior & Organization 60, no. 1: 55-69.

[5] Arnason, Ragnar, Rögnvaldur Hannesson, and William E. Schrank (2000) "Costs of fisheries management: the cases of Iceland, Norway and Newfoundland." Marine Policy 24, no. 3: 233-243.

[6] Bishop, Kate, Igor Filatotchev, and Tomasz Mickiewicz (2002) "Endogenous ownership structure." Acta Oeconomica 52, no. 4: 443-471.

[7] Bárcena-Ruiz, Juan Carlos, and María Luz Campo (2012) "Partial cross-ownership and strategic environmental policy." Resource and Energy Economics 34, no. 2: 198-210.

[8] Coward Jr, E. Walter (1979) "Principles of social organization in an indigenous irrigation system." Human Organization 38, no. 1: 28-36.

[9] Deacon, Robert T., Dominic P. Parker, and Christopher Costello (2008) "Improving efficiency by assigning harvest rights to fishery cooperatives: evidence from the Chignik salmon co-op." Arizona Law Review 50: 479.

[10] Demsetz, Harold, and Kenneth Lehn (1985) "The structure of corporate ownership: Causes and consequences." Journal of Political Economy 93, no. 6: 1155-1177.

[11] Dietzenbacher, Erik, Bert Smid, and Bjørn Volkerink (2000) "Horizontal integration in the Dutch financial sector." International Journal of Industrial Organization 18, no. 8: 1223-1242.

[12] Ellis, Christopher J. (2001) "Common pool equities: An arbitrage based non-cooperative solution to the common pool resource problem." Journal of Environmental Economics and Management 42, no. 2: 140-155.

[13] Ellis, Christopher J., and Anne van Den Nouweland (2006) "A mechanism for inducing cooperation in noncooperative environments: theory and applications." Journal of Public Economic Theory 8, no. 1: 23-47.

[14] Fanti, Luciano (2015) "Partial Cross-Ownership, Cost Asymmetries, and Welfare." Economics Research International, Article ID 324507.

[15] Farrell, Joseph, and Carl Shapiro (1990) "Asset ownership and market structure in oligopoly." The RAND Journal of Economics: 275-292.

[16] Faysse, Nicolas (2005) "Coping with the tragedy of the commons: Game structure and design of rules." Journal of Economic Surveys 19, no. 2: 239-261.

[17] Gardner, Roy, Elinor Ostrom, and James M. Walker (1990) "The nature of common-pool resource problems." Rationality and Society 2, no. 3: 335-358.

[18] Gilo, David, Yossi Moshe, and Yossi Spiegel (2006) "Partial cross ownership and tacit collusion." The RAND Journal of Economics37, no. 1: 81-99.

[19] Grafton, R. Quentin (1996) "Individual transferable quotas: theory and practice." Reviews in Fish Biology and Fisheries 6, no. 1: 5-20.

[20] Hackett, Steven, Edella Schlager, and James Walker (1994) "The role of communication in resolving commons dilemmas: experimental evidence with heterogeneous appropriators." Journal of Environmental Economics and Management 27, no. 2: 99-126.

[21] Hardin, Garrett. "The Tragedy of the Commons' (1968) 162." Science, 1243.

[22] Jones, Madeleine M. (1995) "Fishing debris in the Australian marine environment." Marine Pollution Bulletin 30, no. 1: 25-33.

[23] Kirkley, James E., Dale Squires, Mohammad Ferdous Alam, and Haji Omar Ishak (2003) "Excess capacity and asymmetric information in developing country fisheries: the Malaysian purse seine fishery." American Journal of Agricultural Economics 85, no. 3: 647-662.

[24] Kitts, Andrew W., and Steven F. Edwards (2003) "Cooperatives in U.S. fisheries: realizing the potential of the fishermen's collective marketing act." Marine Policy 27, no. 5: 357-366.

[25] Kitts, Andrew, Patricia Pinto da Silva, and Barbara Rountree (2007) "The evolution of collaborative management in the Northeast USA tilefish fishery." Marine Policy 31, no. 2: 192-200.

[26] La Porta, Rafael, Florencio Lopez-de-Silanes, and Andrei Shleifer (1999) "Corporate ownership around the world." The Journal of Finance 54, no. 2: 471-517.

[27] Levy, Marc, and Ariane Szafarz (2017) "Cross-Ownership: A Device for Management Entrenchment?." Review of Finance 21, no. 4: 1675–1699.

[28] Malueg, David A (1992) "Collusive behavior and partial ownership of rivals." International Journal of Industrial Organization 10, no. 1: 27-34.

[29] Milazzo, Matteo (1998) "Subsidies in world fisheries: a reexamination." Vol. 23. World Bank Publications.

[30] MRAG (2007) "Assessment of alternative approaches to implementing Individual Transferable Quotas (ITQs) in the Australian Northern Prawn Fishery (NPF) and identification of the impacts on the fishery of those approaches", September.

[31] Ono, Hiroshi, Takuya Nakazato, Colin Davis, and Alley Wilson (2004) "Partial ownership arrangements in the Japanese automobile industry; 1990-2000." Journal of Applied Economics 7, no. 2: 355-367.

[32] Ostrom, Elinor, Joanna Burger, Christopher B. Field, Richard B. Norgaard, and David Policansky (1999) "Revisiting the commons: local lessons, global challenges." Science 284, no. 5412: 278-282.

[33] Ostrom, Elinor, Roy Gardner, and James Walker (1994) *Rules, Games, and Common-pool Resources.* University of Michigan Press.

[34] Parker, Philip M., and Lars-Hendrik Röller (1997) "Collusive conduct in duopolies: multimarket contact and cross-ownership in the mobile telephone industry." The RAND Journal of Economics: 304-322.

[35] Pérez, Ivone, María Manuela González, and Juan Luis Jiménez (2017) "Size matters? Evaluating the drivers of waste from ships at ports in Europe." Transportation Research Part D: Transport and Environment 57: 403-412.

[36] Qin, Cheng-Zhong, Shengping Zhang, and Dandan Zhu (2017) "A Model of Endogenous Cross-Holdings in Oligopoly." Frontiers of Economics in China, no. 12, 3: 465-479.

[37] Reynolds, Robert J., and Bruce R. Snapp (1986) "The competitive effects of partial equity interests and joint ventures." International Journal of Industrial Organization 4, no. 2: 141-153.

[38] Rogers, Peter P. (2009) "Co-management strategies for WA State Managed Fisheries using the Exmouth Gulf Prawn (Trawl) Fishery as a case study." Murdoch University, Centre for Fish and Fisheries Research.

[39] Suleiman, Ramzi, and Amnon Rapoport (1988) "Environmental and social uncertainty in single-trial resource dilemmas." Acta Psychologica 68, no. 1: 99-112.

[40] Sullivan, Joseph M (2001) "Harvesting cooperatives and U.S. antitrust law: recent developments and implications." Proceedings of the 10th IIFET conference, July10–14, 2000. Corvallis, Oregon, U.S.A; 2001.

[41] Topping, Paul, David Morantz, and Glen Lang (1997) "Waste disposal practices of fishing vessels: Canada's east coast, 1990–1991." In Marine Debris, pp. 253-262. Springer, New York.

[42] Yang, Yuwen, Ross Cullen, Edward Hearnshaw, and Ian A. Macdonald (2014) "An evaluation of self-governance in the New Zealand Bluff oyster fishery–the indicator system approach." Marine Policy, 43: 273-282.