School of Economic Sciences

Working Paper Series
WP 2013-12

# Product Quality, Advertising Intensity and Market Size

By

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September 2013

WASHINGTON STATE UNIVERSITY
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# Product Quality, Advertising Intensity and Market Size

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Abstract:
We develop a model of product differentiation in which firms strategically compete in product quality and advertising intensity. Consumers face a trade-off between higher quality goods and price. Increased competition may lead to higher or lower quality products. Consumers always benefit from more competition as a reduction in price offsets reduced quality.

Keywords: advertising intensity, consumer surplus, market size, product quality
JEL: L110; L150; L220

# 1. Introduction

Most markets exhibit product differentiation and often, products vary by quality. The objective of this research is to analyze the impacts of an increase in the number of firms on product quality and the associated welfare changes. We consider a random, non-localized utility model, where each firm competes simultaneously in quality, price and advertising intensity.

Product quality is difficult for consumers to detect initially for experience goods, where quality cannot be ascertained until consumption (Nelson, 1970). Orosel and Zauner (2011) examine product quality differentiation in an experience goods market with costly entry. They use a fixed distribution of consumers' willingness to pay for the most popular product, which does not reflect the impact of quality on preferences. We examine preferences for experience goods also, but introduce a stochastic relationship between utility and product quality.

Advertising plays an important role in markets for experience goods. Schmalensee (1978) and Milgrom and Roberts (1986) present models of product quality based on signaling games, where goods have different qualities and firms advertise as a signal of qualities. Christou and Vettas (2008) show firms may increase their profits by charging higher prices when using informative advertising. Hamilton (2009) and Brouhle and Khanna (2007) examine welfare implications of informative advertising in oligopoly markets with differentiated products. They illustrate that increased product quality may result in a decrease of social welfare due to too much product differentiation. We find an increase in quality may decrease consumer surplus also, but focus on the interaction between price and quality.

In our model, a number of firms compete with quality differentiated products. Producers inform some consumers about the price and quality of products through advertising. Firms choose quality and advertising intensities simultaneously. Consumers choose to purchase a unit of the product that generates the highest utility given their information. We analyze the existence of a symmetric equilibrium and the related comparative static properties, in order to identify the impacts of the number of firms in the market. In addition, we discuss the trade-off between price and product quality for consumers surplus.

This study contributes to the existing literature by providing a more general method to more accurately capture consumer preferences. The utility distribution allows consumers' product preferences to vary with the quality. In addition, we carefully define the market to capture the relationships between total product demand, price, quality, market size, and consumer surplus.

## 2. Model

Consider a market with *n* firms ( $n \geq 2$ ) indexed by $i$. Each firm sells a single product, which can be differentiated by quality. Consumers are aware of the product only if they receive a costly advertisement. Consumers are equally likely to receive any advertisement.

Each firm has two choice variables, $q_i$, product quality and $\phi_i$, advertising intensity, which is the percentage of consumers that receive an advertisement. Following Grossman and Shapiro (1984), we assume no fixed production cost and per unit production cost, $C(q_i)$, increases with quality. Firms face an advertising cost, $A(\phi_i)$, such that it is increasingly more expensive to reach more consumers.

The population of consumers is normalized to 1, and each consumer has unit demand. A given consumer $\kappa$ who consumes one unit of the product of firm $i$ has value $v_i^\kappa + \underline{u}$. The value of $v_i^\kappa$, for any firm $i$ and any consumer $\kappa$, are randomly drawn from distribution $\Omega$, with density $\omega$. The reserve utility $\underline{u}$, associated with purchase from the class of heterogeneous goods is the same across all consumers and assumed to equal zero. Unlike the popular assumption of a uniform distribution, we assume that $\Omega$ is a normal distribution, $N(q_i,1)$. Thus, $E(v_i^\kappa) = q_i$.The expected utility of firm $i$'s product equals $q_i + \underline{u}$ and is increasing in quality.

The profit function of firm $i$ is

$$\pi_i = p_i D_i(q_i,\phi_i) - C(q_i) D_i(q_i,\phi_i) - A(\phi_i), \tag{1}$$

where $p_i$ is the price and $D_i$ is the demand for firm $i$. Firms enjoy a degree of market power due to product differentiation. Inspired by the Cournot-Bertand model in Bowley (1924), Dixit(1979), Singh and Vives(1984) and Naimzada and Tramontana(2012), we define the inverse demand function as:

$$p_i = a - D_i - \sum_{j=1, j\neq i}^{n} r_j D_j \;. \tag{2}$$

Here, $a > 0$, and $D_j$ is firm $j$'s demand. We represent an index of product differentiation with $r_j$ where $r_j \in [0,1]$. Products are homogeneous when $r_j = 1$, and highly differentiated when $r_j = 0$. Specifically, we set $r_j = 0.5^{|q_i - q_j|}$, which allows products to serve as substitutes, but does not require them to be perfect substitutes.

Similar to Christou and Vettas (2008), we describe the firm's demand as,

$$D_i(q_i,\phi_i)=\phi_i\sum_{k=0}^{n-1}\frac{(n-1)!}{k!(n-k-1)!}(1-\phi_j)^{n-k-1}\phi_j{}^k\ \Pr(q_i,q_j,k). \tag{3}$$

We assume the advertising intensity of firm $j$ ( $j\neq i$ ) is $\phi_j=\phi$, and the product quality is $q_j=q$. The demand function of firm $i$ can be thought of as $(n-1)!/(k!(n-1-k)!)$ different combinations with which a consumer may receive the advertisements of $k$ firms other than firm $i$. Each combination occurs with probability $(1-\phi)^{n-k-1}\phi^k$. The probability a consumer receives the $k$ advertisements of firm $i$ and $j$ other firms and chooses to purchase from firm $i$ is defined as $\Pr(q_i,q_j,k)$. Unlike previous work, we use price to capture the probability of purchasing product $i$, $\Pr(q_i,q_j,k)=\Pr(q_i,q_j)^k=P(v_i>v_j)^k$, where $P(v_i>v_j)$ is the probability the utility from firm $i$ is higher than that received from firm $j$.

## 3. Equilibrium

To solve for the symmetric equilibrium we being with the first-order conditions of the profit function of firm $i$, in equation (1)

$$\frac{\partial\pi_i(q_i,q_j,\phi_i,\phi_j)}{\partial q_i}=0$$
$$\frac{\partial\pi_j(q_i,q_j,\phi_i,\phi_j)}{\partial\phi_i}=0\ . \tag{4}$$

We assume that each firm $j$ chooses the same quality $q$ and advertising intensity $\phi$ to solve the first order condition with respect to quality. Setting all qualities other than $q_i$ equal to $q$ and all advertising intensities equal to $\phi$, we obtain

$$\frac{\partial\pi_i(q_i,q,\phi,\phi)}{\partial q_i}=(\frac{\partial p_i(q_i,q,\phi,\phi)}{\partial q_i}-\frac{\partial C(q_i)}{\partial q_i})D_i(q_i,q,\phi,\phi)+(p_i(q_{i,}q,\phi,\phi)-C(q_i))\frac{\partial D_i(q_i,q,\phi,\phi)}{\partial q_i} \tag{5}$$

where $\frac{\partial D_i(q_i,q,\phi,\phi)}{\partial q_i}=\phi\sum_{k=0}^{n-1}\frac{(n-1)!}{k!(n-1-k)!}(1-\phi)^{n-1-k}\phi^k\,\frac{\partial\Pr(q_i,q,k)}{\partial q_i}$. (6)

At a symmetric equilibrium, firms maximize their profits at $q_i=q$, which yields

$$\frac{\partial D_i(q,q,\phi,\phi)}{\partial q_i} = \phi \frac{1}{\sqrt{2\pi}}(1-(1-\phi)^{n-1}). \tag{7}$$

All firms share the market equally when $q_i = q$ and $\phi_i = \phi$. The total market demand equals the probability that a consumer receives at least one advertisement, $1-(1-\phi)^n$,

$$D_i(q,q,\phi,\phi) = \frac{1-(1-\phi)^n}{n}. \tag{8}$$

Next, we consider the first order condition with respect to advertising intensity. We set all quality equal to $q$ and all advertising intensities other than $\phi_i$ equal to $\phi$. The first order condition can be written as

$$\frac{\partial \pi_i(q,q,\phi_i,\phi)}{\partial \phi_i} = (p(q,q,\phi_i,\phi) - C(q_i))\frac{\partial D_i(q,q,\phi_i,\phi)}{\partial \phi_i} + \frac{\partial p(q,q,\phi_i,\phi)}{\partial \phi_i} D - \frac{\partial A(\phi_i)}{\partial \phi_i} \tag{9}$$

and $D_i(q,q,\phi_i,\phi) = \phi_i \dfrac{1-(1-\phi)^n}{n\phi}$. (10)

To analytically solve the model, we assume a standard quadratic cost function of advertising $A(\phi) = \phi^2$ and a linear cost function of product quality $C(q) = q$, which are specific functions consistent with the general model assumptions. Using these functions and plugging expression (10) into expression (5) yields,

$$(-2n\phi \frac{1}{\sqrt{2\pi}}(1-(1-\phi)^{n-1}) - 1)D + (a - 2nD - q)\phi \frac{1}{\sqrt{2\pi}}(1-(1-\phi)^{n-1}) = 0. \tag{11}$$

We solve the equilibrium quality level,

$$q = a - 2(1-(1-\phi)^n) - \frac{\sqrt{2\pi}(1-(1-\phi)^n)}{n\phi(1-(1-\phi)^{n-1})}. \tag{12}$$

Differentiating expression (10) with respect to $\phi_i$ yields

$$\frac{\partial D_i(q,q,\phi_i,\phi)}{\partial \phi_i} = \frac{1-(1-\phi)^n}{n\phi}. \tag{13}$$

Plugging expressions (11) and (13) into expression (12), we conclude that any equilibrium values of $(n,\phi)$ will satisfy the condition,

$$\frac{\sqrt{2\pi}}{n^2\phi^2}\frac{(1-(1-\phi)^n)^2}{(1-(1-\phi)^{n-1})} = \frac{\partial A(\phi)}{\partial \phi} = 2\phi. \tag{14}$$

For any $n$ greater than 1, there exists a corresponding advertising intensity $\phi$ in equilibrium.

Although we assume firms select the same quality and advertising intensity in order to solve the symmetric equilibrium, product differentiation enters through the consumer decision. Consumers' values from consuming the same product can be characterized with the i.i.d. distribution of $\Omega$. In other words, consumers' utilities may vary when they consume the same goods.

Now, we consider an endogenous market size. Firms enter the market and pay an entry cost, $M$. When $n$ is endogenous, per-firm's profit under symmetric equilibrium can be presented as,

$$\pi(n) = (p(\phi,n) - C(a,\phi,n))D(\phi,n) - A(\phi) - M \,. \tag{15}$$

**Lemma 1.** As the number of firms in the market increases, each firm advertises less.

**Proof.** Equation (14) determines the equilibrium level of advertising intensity as a function of $n$. Define $L = \frac{\sqrt{2\pi}}{n^2\phi^2}\frac{(1-(1-\phi)^n)^2}{(1-(1-\phi)^{n-1})} = 2\phi$. Using a first order Taylor expansion, $L \approx \frac{\sqrt{2\pi}}{(n-1)\phi}$. We can approximately obtain $\frac{d\phi}{dn} = -\frac{\partial L/\partial n}{\partial L/\partial\phi - 2} = -\frac{-1/(n-1)^2\phi}{-1/(n-1)\phi^2 - 2} < 0$. Figure 1 illustrates this negative relationship between $\phi$ and $n$ clearly.

Intuitively, informative advertising fosters quality competition instead of advertising competition. For any given advertising intensity, increasing the number of firms will increase the expected consumers who receive the advertisement. This will induce firms to increase quality competition. Consequently, the marginal profit of advertising will decrease. As a result, firms invest in product quality instead of advertising.

**Lemma 2.** Product quality initially increases and then decreases as the number of firms in the market increases.

**Proof.** We provide the proof in the Appendix. To understand the intuition behind this result, we compare the marginal profits from quality and advertising. When $n$ is small, the marginal profit from product quality is larger than the marginal profit from advertising. Firms do not need to invest in advertising to attract more consumers. Instead, they invest in product quality with little increase in price and profits increase.

However, when $n$ is large, the marginal profit from product quality is less than the marginal profit from advertising. Here, when firms increase product quality, they increase price also. When their market power is relatively small due to the large number of firms, this will result in a decrease in profits. Therefore, firms advertise more in order to capture more consumers, and reduce product quality.

Figure 1 depicts that increasing the number of firms in the market leads to increased product quality followed by decreased quality. Advertising intensity is always decreasing with market size. Product quality is increasing in market size when market size is small and is decreasing when market size is larger. When $n \le 5$, product quality and advertising intensity are strategic substitutes as reducing advertising intensity or raising product quality can help firms make profits. However when $n > 5$, product quality and advertising intensity become strategic complements as firms choose to lower product quality as well as advertising intensity to increase profits.

**Lemma 3.** The profit of each firm is strictly decreasing in the number of firms in the market.

**Proof.** It suffices to show that $\frac{\partial \pi}{\partial n} < 0$. Specifically, we have

$$\begin{aligned}
\frac{d\pi(n)}{dn} &= \frac{d}{dn}(D(p-C)) - \frac{dA}{d\phi}\frac{d\phi}{dn} \\
&= \phi\frac{d}{dn}\left(\frac{1-(1-\phi)^n}{n\phi}(p-C)\right) + \left(\frac{1-(1-\phi)^n}{n\phi}(p-C)\right)\frac{d\phi}{dn} - \frac{dA}{d\phi}\frac{d\phi}{dn} \\
&= \phi\frac{d}{dn}\left(\frac{dA}{dn}\right) + \frac{dA}{d\phi}\frac{d\phi}{dn} - \frac{dA}{d\phi}\frac{d\phi}{dn} \\
&= \phi\frac{d^2A}{d\phi^2}\frac{d\phi}{dn} < 0
\end{aligned}$$

with $\frac{d\phi}{dn} < 0$ and $\frac{d^2A}{d\phi^2} > 0$.

In the symmetric equilibrium, the more firms operating, the smaller market share they receive. Consequently, per-firm profit will decrease.

To examine consumer surplus we plug the equilibrium product quality and advertising intensity in to the inverse demand function. We derive the expected consumer surplus as

$$E(CS) = (E(v_i^{\kappa} + \underline{u}) - P)\cdot TD\,, \tag{16}$$

where $TD = n \cdot D$.

**Lemma 4.** The expected consumer surplus, $E(CS)$, is increasing with market size.

**Proof.** Under the symmetric equilibrium, the expected consumer surplus can be rewrite as $E(CS) = ((\underline{u} + q) - P) \cdot n \cdot D$. We cannot analytically solve for this result. However, Figure 1 illustrates the numerical results depicting the relationship between expected consumer surplus and market size.

Although the expected consumer surplus is always increase with market size, there is a trade-off between price and quality. Product quality is increasing with market size when $n < n^{\text{Hi Qual}}$ and decreases when $n > n^{\text{Hi Qual}}$, depicted in Figure 1. Associated price is decreasing with market size with $n < n^{\text{Hi Qual}}$ and is increasing with $n > n^{\text{Hi Qual}}$. Accordingly, the quality and price impacts expected consumer surplus in two stages. In the first stage, when $n < n^{\text{Hi Qual}}$, an increase in quality leads to an increase in expected consumer surplus and expected consumer surplus decreases with the increase in price. The quality effect dominates as the expected consumer surplus is increasing the market size. Similarly, in the second stage, $n > n^{\text{Hi Qual}}$, quality decrease causes a drop in the expected consumer surplus and price decease results in an increase in the expected consumer surplus. In this period, the price effect dominates.

## 4. Conclusion

Our model represents a straightforward way to study the relationship between competition and product quality. Firms compete strategically in product quality and informative advertising. In equilibrium, a higher level of advertising tends to lower profits. As more firms enter the market, product quality initially increases, then decreases. With few firms, product quality and advertising intensity are strategic substitutes, but become strategic complements with more firms. More firms increase consumer surplus due to the trade-off between quality and price.

# Appendix

We illustrate the existence of a market size which generates the highest product quality. We need to show the first order derivative of product quality with respect to market size is positive when market size is small and becomes negative when market size is large.

We use the per-firm profit maximization problem and construct the first order conditions with respect to $\phi$ and $q$, respectively.

$$\begin{cases} G: \partial\pi(q,q,\phi,\phi)/\partial q = 0 \\ H: \partial\pi(q,q,\phi,\phi)/\partial\phi = 0 \end{cases} \tag{A.1}$$

Then, we obtain the differential equation system in terms of $dq/dn$ and $d\phi/dn$.

$$\begin{cases} \frac{\partial G}{\partial\phi}\cdot\frac{d\phi}{dn}+\frac{\partial G}{\partial q}\cdot\frac{dq}{dn}+\frac{\partial G}{\partial n}=0 \\ \frac{\partial H}{\partial\phi}\cdot\frac{d\phi}{dn}+\frac{\partial H}{\partial q}\cdot\frac{dq}{dn}+\frac{\partial H}{\partial n}=0 \end{cases}. \tag{A.2}$$

Consequently,

$$\frac{dq}{dn}=\begin{vmatrix} \frac{\partial G}{\partial\phi} & -\frac{\partial G}{\partial n} \\ \frac{\partial H}{\partial\phi} & -\frac{\partial H}{\partial n} \end{vmatrix} / \begin{vmatrix} \frac{\partial G}{\partial\phi} & \frac{\partial G}{\partial q} \\ \frac{\partial H}{\partial\phi} & \frac{\partial H}{\partial q} \end{vmatrix} = \frac{\frac{\partial^2\pi}{\partial q\partial\phi}*\frac{\partial^2\pi}{\partial\phi\partial n}-\frac{\partial^2\pi}{\partial\phi^2}*\frac{\partial^2\pi}{\partial q\partial n}}{\frac{\partial^2\pi}{\partial\phi^2}*\frac{\partial^2\pi}{\partial q^2}-\frac{\partial^2\pi}{\partial q\partial\phi}*\frac{\partial^2\pi}{\partial\phi\partial q}}. \tag{A.3}$$

$$\frac{\partial^2\pi}{\partial\phi^2}=p_{\phi,\phi}D+p_\phi D_\phi+p_\phi D_\phi+pD_{\phi,\phi}-qD_{\phi,\phi}-A_{\phi,\phi} \tag{A.4}$$

$$\frac{\partial^2\pi}{\partial q^2}=0 \tag{A.5}$$

$$\frac{\partial^2\pi}{\partial q\partial\phi}=-D_\phi \tag{A.6}$$

$$\frac{\partial^2\pi}{\partial\phi\partial n}=p_{\phi,n}D+p_\phi D_n+p_n D_\phi+pD_{\phi,n}-pD_{\phi,n} \tag{A.7}$$

$$\frac{\partial^2\pi}{\partial q\partial n}=-D_n \tag{A.8}$$

First, we use the above results to define the sign of denominator of expression (A.3).

$$\frac{\partial^2\pi}{\partial\phi^2}*\frac{\partial^2\pi}{\partial q^2}-\frac{\partial^2\pi}{\partial q\partial\phi}*\frac{\partial^2\pi}{\partial\phi\partial q}$$

$$= 0 * (p_{\phi,\phi} D + p_\phi D_\phi + p_\phi D_\phi + pD_{\phi,\phi} - qD_{\phi,\phi} - A_{\phi,\phi}) - (-D_\phi)^2 < 0 . \qquad (A.9)$$

Next, we use the Taylor expansion to detect the sign of numerator of expression (A.3.).

Apply second order Taylor expansion to equilibrium demand. We can get $D \approx \frac{1 - n\phi + \frac{1}{2}n(n-1)\phi^2}{n}$ , $p \approx a - 2(1 - n\phi + \frac{1}{2}n(n-1)\phi^2)$ , $q \approx a - 2(1 - n\phi + \frac{1}{2}n(n-1)\phi^2) - \frac{D}{D_q}$

and $D_q \approx \frac{1}{\sqrt{2\pi}}\phi((n-1)\phi + \frac{1}{2}(n-1)(n-2)\phi^2)$ .

Plugging this approximation to the numerator of expression (A.3), we get

$$\frac{\partial^2 \pi}{\partial q \partial \phi} * \frac{\partial^2 \pi}{\partial \phi \partial n} - \frac{\partial^2 \pi}{\partial \phi^2} * \frac{\partial^2 \pi}{\partial q \partial n}$$

$$= \frac{1 - n\phi + \frac{1}{2}n(n-1)\phi^2}{n}(1 - (n-1)\phi)(2 - 2\phi(2n-1) + \frac{\sqrt{2\pi}}{(n-1)\phi(1 - \frac{1}{2}(n-2)\phi)})$$

$$-6n((n-1)\phi - 1)^2 (\frac{1}{n^2} - \frac{1}{2}\phi^2) - 2\phi n(1 - (2n-1)\phi)(1 - (n-1)\phi)^2$$

$$-(\frac{1}{n^2} - \frac{1}{2}\phi^2)(1 - n\phi + \frac{1}{2}n(n-1)\phi^2)(-2(n-1) + \frac{\sqrt{2\pi}}{(n-1)\phi(1 - \frac{1}{2}(n-2)\phi)}) - 2(\frac{1}{n^2} - \frac{1}{2}\phi^2) \cdots\cdots (A.10)$$

The highest order in expression (A.10) in term of $n$ is 4 and the coefficient is $8\phi^4$, and the lowest order in expression (A.10) is negative 4 and the coefficient is $-\frac{2\sqrt{2\pi}}{\phi^3}$. Thus, we have

$$\lim_{n \to \infty} \frac{\partial^2 \pi}{\partial q \partial \phi} * \frac{\partial^2 \pi}{\partial \phi \partial n} - \frac{\partial^2 \pi}{\partial \phi^2} * \frac{\partial^2 \pi}{\partial q \partial n} \to \infty > 0 \quad \text{and} \quad \lim_{n \to 0} \frac{\partial^2 \pi}{\partial q \partial \phi} * \frac{\partial^2 \pi}{\partial \phi \partial n} - \frac{\partial^2 \pi}{\partial \phi^2} * \frac{\partial^2 \pi}{\partial q \partial n} \to -\infty < 0 .$$

Combining with the negative denominator, the first order derivative is positive when $n$ goes to 0 and is negative when $n$ goes to infinite. It is implicit to conclude product quality $q$ is increasing with market size $n$ and then decreasing with market size $n$.

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