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Wildfire Hazards: A Model of Disaster Response

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Wildfire Hazards: A Model of Disaster Response*

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Abstract

Disaster managers allocate resources to accomplish a set of objectives in a highly uncertain environment. The allocation decisions made throughout the course of a disaster inevitably impact final outcomes such as the damage and cost of a disaster. We develop a unique stochastic dynamic model of disaster response in which managers face a temporal tradeoff between disaster containment and the protection of valuable assets, and focus on wildfire response. Comparative dynamics indicate that when the number of threatened assets increases, wildfire managers divert response resources away from wildfire suppression toward asset protection at the expense of fire growth. This leads to a rise in the expected duration, size and cost of a wildfire. Based on our theory, we motivate and develop an econometric model that contributes to the literature on multivariate frailty (hazard) model estimation. We use this model to jointly estimate wildfire duration, size, and cost in a way that exploits the temporal variation in a unique dataset of U.S. wildfires from 2001 to 2008. Our results suggest, among other things, that 100 threatened residential structures leads to an increase in the expected wildfire duration by 8.6%, expected acreage burned by 26%, and the expected response cost by 22.2%. As the wildland urban interface continues to grow, wildfire managers will increasingly face the tradeoff between containment and protection. Consequently federal and state agencies can expect longer, larger, and more expensive wildfires in the future.

JEL Classification: C3, C41, C61, Q2, Q54.

Keywords: Disaster Response, Wildfire Management, Suppression, Protection, Hazard Model, Frailty Model.

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1 Introduction

Disaster managers allocate resources to accomplish a set of objectives in a highly uncertain environment. While the specific objectives of a manager vary depending on the type of disaster, cost minimization and damage mitigation receive significant attention. Managers often face tradeoffs between forms of response effort used to accomplish their objectives. Disaster containment may limit the growth or extent of the disaster while protection may reduce the damage to specific assets. Managers of infectious disease outbreak, invasive species, oil spill, and nuclear contamination all face tradeoffs between containment and protection of threatened assets. The allocation decisions made throughout the course of a disaster inevitably impact final outcomes such as the damage and cost of a disaster.

Wildfire is a destructive and complex form of disaster in the U.S., and the world, with recent management costs exceeding $2 billion annually (Gebert et al., 2008). Management of a wildfire often takes place over the course of several days or even weeks. Decisions are made each day based on new and updated information relating to fire development, including past and present fire behavior, weather changes, differences over space and time in values at risk, and numerous other factors. The dynamic spatiotemporal path of the fire is affected by these environmental factors, and the management decisions, over the course of the fire, until its completion. These environmental factors and resource allocation decisions affect final fire outcomes by affecting spatial outcomes at any point during the response effort.

The wildland urban interface (WUI) has grown significantly over the past several decades, and wildfire managers find themselves devoting more response resources to structure protection than ever before (Mercer and Prestemon, 2005). A growing population and a desire to live near forests have drawn people to build structures in areas inherently susceptible to wildfire. These structures are often high-value homes and cabins. Wildfire managers face significant political pressure to protect these structures, which may come at the expense of other management objectives (Troyer et al., 2003).[1]

[1]Quinn (2005) recounts the management actions taken during one of Oregon's largest wildfires in history,

This paper develops an economic theory of disaster response to characterize the impact of threatened assets on disaster outcomes. A manager chooses containment and protection to minimizes losses subject to a stochastic process representing the disaster. We adapt our theory of disaster response to wildfire management, and formulate three hypotheses: when assets are threatened during the course of a wildfire, the expected 1) duration, 2) final fire size, and 3) total suppression cost increases. We jointly estimate a trivariate hazard model (variously known as duration, time-until-failure, and event history model) of wildfire duration, size, and suppression cost. We utilize daily situation report data on individual wildfires in the United States from 2001 to 2008 to estimate a trivariate hazard model of wildfire duration, size, and cost. The correlation between outcomes is captured through a jointly distributed latent variable.

There exists a rich literature on disaster management that spans several fields of social science. Recent epidemiological-economic models of infectious disease outbreak recognize tradeoffs between treatment of affected populations and prevention through vaccination or distancing (Fenichel et al., 2011; Ludkovski and Niemi, 2010). Other disaster management problems with similar tradeoffs include invasive species, oil spill, and nuclear meltdown (Bassey and Chigbu, 2012). Altay and Green (2006) provide a survey of the operations research literature on disaster management in which they categorize four stages of disaster operations management: preparedness, mitigation, response, and recovery. Gebert et al. (2008) discusses these phases of management as they pertain to wildfire suppression. This paper focuses solely on the response phase of the management process in order to understand the implications of response decisions, throughout the event, on final outcomes.

Very little of the extant empirical economics literature examines the dynamic development of wildfires and their management over time. There have been numerous efforts to estimate the economic relationships and tradeoffs embodied in wildfire management, as

the Biscuit Fire, which ultimately burned over 500,000 acres. After nearly two weeks of unsuccessful containment, response resources were reassigned to protect structures on the fires East side, allowing the fire to grow on the unattended North side.

well as forecasts of suppression costs, fire size, and other outcome measures. Mercer and Prestemon (2005) summarize some of these existing studies, including models of fire ignition rates or ignition risk, individual or aggregate fire extent (e.g. area burned), fire effects models (outcomes of other metrics such as fire intensity or damage), and combinations of these. Some studies develop models based on aggregate level fire and suppression data (Abt, Prestemon, and Gebert, 2008; Prestemon et al., 2008; Cardille, Ventura, and Turner, 2001), and others focus on data at the level of individual fires (Holmes, Huggett, and Westerling, 2008; Butry, Gumpertz, and Genton, 2008; Liang et al., 2008; Gebert, Calkin, and Yoder, 2007). Prestemon et al. (2008) and Butry (2009), among others, focus on the relationship between ex ante wildfire risk mitigation and ex post suppression costs. Another line of research has integrated physical spatiotemporal spread models with suppression (Fried, Gilless, and Spero, 2006; Petrovic and Carlson, 2012; Petrovic, Alderson, and Carlson, 2012). While these models provide useful information on the interaction between resource allocation and fire spread, they do not capture the disaster manager's response to economic trade offs during the response effort.

While disaster response and wildfire outcomes have been analyzed with linear regression and count models, no studies have used hazard models, which are well-suited to characterize the dynamic development of wildfire. Duration models exploit the variation in time-varying covariates over the course of an event. Economists have used hazard models to study factors influencing employment spells, marriage duration, and mortgage default (Meyer, 1990; Bennett, Blanc, and Bloom, 1988; Shumway, 2001). In the medical literature, hazard models are commonly used to compare the efficacy of treatments, and in some studies, multivariate extensions have been developed to account for correlation in the disease treatment of twins (Andersen et al., 1997; Wienke et al., 2005). The most similar empirical model of wildfire to date is Finney, Grenfell, and McHugh (2009), who propose a generalized linear mixed-model (GLMM) of wildfire containment focused on intervals of low- and high-spread. Our trivariate hazard model provides probabilistic information on containment in terms of fire size and cost in addition to duration.

This paper contributes to the literature on disaster and wildfire response as well as empirical hazard models. We have found no published research to date that fully utilizes daily observations over the course of a cross-section of wildfires to estimate a dynamic economic/physical model of response beyond Finney, Grenfell, and McHugh (2009). We derive the components of a hazard model from the theory of disaster response, and estimate a seemingly-unrelated-regression-like system of three equations by maximum simulated likelihood. Our proposed estimation procedure is similar to the seemingly-unrelated-Poisson model of King (1989) and seemingly-unrelated-negative-binomial model of Winkelmann (2000).

We find evidence to support our hypotheses that an increase in the number of threatened assets raises the expected duration, size, and cost of wildfires. In the model, response managers devote resources to asset protection at the expense of overall suppression when the number of threatened assets rises. As resources are diverted from suppression, the fire grows and becomes more difficult to suppress. The resulting fire persists longer than it would have had resources maintained previous suppression efforts. The results of this analysis imply that as the WUI continues to grow, federal and state agencies should expect longer, larger, and more expensive fires.

The paper proceeds as follows. Section 2 develops a stylized model of disaster response management and the connection to empirical hazard models. Section 3 provides a description of the data we use for the analysis. Section 4 provides the results and discussion. Section 5 concludes the paper.

2 A theory of wildfire response

Consider a wildfire a self-perpetuating stock of energy that evolves over time.[2] Like the stock of a renewable resource, the fire persists as long as the stock of energy remains above the

[2]While energy is a flow in the physical sense, we use the term stock to remain consistent with the language of dynamic modeling. The stock of energy should be interpreted as a snapshot of the energy expended by the wildfire at any point in time.

minimum threshold necessary to sustain exothermic reaction. In addition to human management efforts, environmental factors, such as weather, fuel, and geography, may influence the rate of growth (decline) of the energy stock. Eventually the energy stock falls below the physical threshold, and the fire is extinguished.

A management team chooses two forms of management effort at time t: protection effort, which reduces the probability of damage to specific values at risk[3]; and suppression effort, which curtails the overall growth of the fire. Protection can be thought of as intensive asset protection and suppression as extensive asset protection. Fire lines that span large sections of the fire front will stop the growth of the fire along that front. If effective, the lines will limit the spatial extent of the fire.

Modern wildfire management is an organizationally complex endeavor. Response to any single fire may involve the cooperation of numerous federal, state, local, and private organizations. The coordination of resources across many fires during a busy fire season exacerbates the resource allocation problem. We summarize the goals of reducing costs, $c(t)$, and damage to values at risk, $d(t)$, in the management team's loss function[4]

$$\ell(c(t), d(t), t). \tag{1}$$

Losses are increasing in both costs and damage. The general loss function provides flexibility with regard to the weight (marginal losses) of costs and damage, and may be interpreted more generally as a disutility function.[5]

[3]Petrovic, Alderson, and Carlson (2012) develops a simulation-based model of fire spread where suppression reduces the probability that the fire persists in any given cell.

[4]Lowercase symbols are used to represent instantaneous flows at any given point in time whereas capital symbols denote an accumulation of the stream of those flows. The variable definitions are contained in Table 2 of the appendix.

[5]This feature is important to modeling disaster response because, as Troyer et al. (2003) suggests with regard to wildfire, management teams do not always equate a dollars worth of response with a dollars worth of damage. The loss function may also account for a risk averse fire management team; although, risk aversion is not necessary to obtain the results below.

Costs are given by a linear cost equation at time t,

$$c(t) = s_f(t)(w_f + w_f^o(t)) + s_d(t)(w_d + w_d^o(t)) \tag{2}$$

where $s_f(t)$ is suppression effort, $s_d(t)$ is protection effort, w_i is the constant market price of effort $i = \{f, d\}$, and $w_i^o(t)$ is the opportunity cost of effort $i = \{f, d\}$. We assume that the market price of response resources is constant over the fire duration because government agencies often contract resources for the year so the per unit cost is known a priori. The opportunity cost of resources depends on their availability within a geography, which may vary over the course of a fire. These opportunity costs are due to scarcity of quasi-fixed capital during times of high wildfire activity within the region over which response resources are deployed.

Damage is equal to the product of a vector of threatened asset values, the number of threatened assets per acre, and the acres burning at time t,

$$d(t) = \boldsymbol{\nu}(t) \cdot \frac{\mathbf{y}(t)}{s_d(t)} \cdot a(t) \tag{3}$$

where $\boldsymbol{\nu}(t)$ is a $1 \times J$ vector of threatened asset values measured in dollars, $\mathbf{y}(\cdot)$ is a $J \times 1$ vector where each element represents the number of threatened assets per acre corresponding to a particular asset type j, and $a(t)$ is the instantaneous flow of burning area at any point in time t. One may think of $a(t)$ as the contribution to fire size of the fire front as it moves through space. We assume that assets are at risk of destruction at time t, and depending on the level of protection, are destroyed or survive at period $t - \Delta t$.[6] Threatened asset values, $\boldsymbol{\nu}(t)$, may include assets such as endangered species habitat, watersheds, and marketable timber. Protection effort, $s_d(t)$, effectively reduces the concentration per acre of threatened asset values across a given landscape.[7]

[6] This assumption implies that $d(t)$ represents the level of damage that the fire manager will experience in between states t and $t - \Delta t$.

[7] An alternative, but equally valid, interpretation is that $s_d(t)$ reduces the probability that a threatened

The instantaneous growth in fire size at any given point in time t is given by

$$a(t) = a(\mathbf{z}(t), f(t), t). \tag{4}$$

where $\mathbf{z}(t)$ is a vector of exogenous geographic and environmental characteristics such as vegetation and weather.

The energy stock of the fire is $f(t)$. The fire stock evolves according to a stochastic process represented by the distribution function

$$G(f^{'} \mid f(t), s_f(t), \mathbf{z}(t)) = \int_0^{f^{'}} g(q \mid f(t), s_f(t), \mathbf{z}(t), t) \, dq \tag{5}$$

where $f^{'} = \lim_{\Delta t \to 0} f(t + \Delta t)$ is the energy stock at the next moment in time. The distribution is conditional on the current level of energy, $f(t)$, exogenous environmental and geographic characteristics, $\mathbf{z}(t)$, and the amount of suppression effort, $s_f(t)$.[8] By definition, $s_d(t)$ has no impact on $G(\cdot)$ as an approximation.[9] The distribution is lower bounded by zero because the stock of energy must be positive. Suppression effort and exogenous conditions shift the mass of the density over different levels of f depending on whether the variable encourages or discourages growth of the fire stock.[10] We assume that the mass of the density shifts over lower values of f when $s_f(t) \geq 0$ increases. The impact of the elements of the vector $\mathbf{z}$ may affect $G(\cdot)$ differently. For instance, high wind $z_1 \in \mathbf{z}$ and steep terrain $z_2 \in \mathbf{z}$ may shift the mass of $G(\cdot)$ over higher values of f while increased humidity $z_3 \in \mathbf{z}$ may shift the mass of $G(\cdot)$ over lower values of f.[11]

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asset is destroyed at time t. This alternative interpretation is qualitatively similar to the weighted area protection measure developed by Kirsch and Rideout (2005).

[8] Fenichel et al. (2011) and Fraser et al. (2004) use a similar notion of stochastic evolution to model the spread of infectious disease throughout a population.

[9] For example, suppose resources were used to remove vegetation and create a perimeter around a threatened structure. These actions would have a minimal impact on the overall energy content of an established wildfire.

[10] Similarly, Pich, Loch, and De Meyer (2002) formulate a model in which project managers choose actions that impact the probability of event outcomes.

[11] Fenichel et al. (2011) develop a model of infectious disease in which the probability of transmission – growth in the disease stock, analogous to the energy stock – is a function of contact with infected individuals.

The initial fire stock, $f(0) = f_0$, is observed by the management team at the date of discovery and is strictly positive. The wildfire continues to burn until the stock of energy falls below $\bar{f}$ at which point the fire is terminated and the response effort is effectively over.[12] Therefore, a fire begins *iff* $f_0 > \bar{f}$.

The management team's problem is formalized in the recursive Hamilton-Jacobi-Bellman equation

$$\begin{aligned} V(f(0),0) &= \min_{s_d, s_f \geq 0} \left\{ \ell(c(t), d(t), t) + E_t \left\{ V' \right\} \right\} \\ &= \min_{s_d, s_f \geq 0} \left\{ \ell(c(t), d(t), t) + \int_0^\infty V' \; g(f' \mid f(t), s_f(t), \mathbf{z}(t)) \; df' \right\} \end{aligned} \tag{6}$$

where $V(\cdot)$ is the value function and $V' = \lim_{\Delta t \to 0} V(f(t + \Delta t), t + \Delta t)$.[13] The objective function is defined by equation (1) and the components, costs and damage, by equations (2) and (3), respectively. Because wildfire events take place over short time horizons, we assume the discount rate to be approximately zero, and omit it to reduce notational clutter. The probability density function $g(f')$ is defined in equation (5). Suppression and protection effort (s_f and s_d) are lower bounded at zero.[14]

The terminal condition determines the end of the response effort which occurs when the fire's stock of energy falls below an exogenously determined threshold $f(T) = \bar{f}$. Therefore, T is a random variable with the following distribution at any point in time t

$$\begin{aligned} G(\bar{f} \mid s_f(t), \mathbf{z}(t), f(t), t) &= \Pr(f' \leq \bar{f} \mid s_f(t), \mathbf{z}(t), f(t), t) \\ &= \Pr(T \in (t, t + \Delta t] \mid s_f(t), \mathbf{z}(t), f(t), T \geq t). \end{aligned} \tag{7}$$

Equation (7) states that the probability of the fire's energy stock falling below the critical value $\bar{f}$ is equal to the probability that the response effort ends, T, in the next interval of time.

However, individuals choose their level of contact, and thus, affect the probability of disease spread.

[12]Note that $\bar{f}$ is the level of energy above which supports exothermy at any point in $a(t)$.

[13]Management effort is assumed to begin at the date and time of fire discovery $t = 0$.

[14]A degenerative case would entail a fire with no possibility of damage and thus no required effort.

This relationship effectively connects the model of disaster response to an empirical hazard model. Before we develop this connection further, we derive testable hypotheses regarding the impact of threatened assets on expected duration and expected size of wildfires.

2.1 Threatened assets

As the wildland urban interface continues to grow, wildfire management teams face increasingly difficult tradeoffs between suppression and protection when confronted with threatened assets. We are interested in the impact of a sudden[15] increase in the number of threatened assets at any point in time, t, on the final outcomes duration, T, total cost, $C(T)$ and total size $A(T)$, where $C(t) = \int_0^t c(\tau)d\tau$ from equation (2) and $A(t) = \int_0^t a(\tau)d\tau$ from equation (4). We summarize the primary theoretical results of this study in the following propositions.

Proposition 1. *An increase in the number of threatened assets,* $\mathbf{y}(t)$*, at time* t *leads to longer expected wildfire duration when the loss function is separable in costs and damages.*

Proof. To prove this proposition, we demonstrate that an increase in the number of a single type of threatened asset,[16] $y_1 \in \mathbf{y}$ at any time t during the response effort, increases the expected wildfire duration $E_t\{T\}$. Recall that T is directly related to the distribution $G(\cdot)$ through equation (7). By assumption, $dg(f')/ds_f < 0$ and $dg(f')/ds_d = 0$ which imply that only suppression reduces the expected level of the fire's energy stock because protection reduces the density of a given asset without substantially reducing the fire stock. From the system of first-order conditions, suppression is decreasing in the number of threatened assets $ds_f/dy_1 < 0$ and protection is increasing in the number of threatened assets $ds_d/dy_1 > 0$ (see appendix 1 for derivation). Therefore, a sudden increase in the number of threatened properties, $y_1' > y_1$ *ceteris paribus*, causes the management team to shift resources from

[15] We choose this language to make the point that during a response effort, conditions may change such that residential properties become threatened. Our dataset explicitly quantifies the number of threatened properties at various points throughout the response effort.

[16] Let y_1 represent residential property, one of the highest valued assets per unit space that managers protect.

suppression, s_f, to protection, s_d, which reduces the probability that $f' < \bar{f}$ and thus increases $E_t\{T\}$. □

Proposition 2. *An increase in the number of threatened assets,* $\mathbf{y}(t)$*, at time* t *leads to a larger expected wildfire size when the loss function is separable in costs and damages.*

Proof. There are two compounding effects that lead to the result $dE\{A(T)\}/dy_1 > 0$: the longer expected duration of a fire (from proposition 1), and the larger energy stock throughout the remaining duration leads to a larger area burned. The first effect follows from the dependence of final fire size on $E_t\{T\}$. We define cumulative area burned at time t as $A(t) = \int_0^t a(\tau)d\tau$ from equation (4). Therefore, at any point in time t, the expected area can be separated into two parts: the known area burned up until time t, and the area expected to burn during the remaining duration of the fire.

$$E_t\{A(T)\} = \int_0^t a(\mathbf{z}(\tau), f(\tau), \tau)d\tau + E_t\left\{\int_t^T a(\mathbf{z}(\tau), f(\tau), \tau)d\tau\right\} \tag{8}$$

where the fire size at t is known to the management team. Proposition 1 shows that if $y_1(t)$ increases at any point in time, $E_t\{T\}$ increases, which implies that the second portion of equation (8) becomes unambiguously larger, *ceteris paribus*.

The second effect follows from the larger fire on the interval (t, T). By assumption, $da(t)/df(t) > 0$ and $dg(f')/ds_f(t) < 0$, which together with $ds_f(t)/dy_1(t) < 0$, imply that

$$a(\mathbf{z}(t), f(y_1(t), t), t) < E_t\left\{a(\mathbf{z}(\hat{t}), f(y_1(\hat{t}), \hat{t}), \hat{t})\right\} \quad \forall\, \hat{t} \in (t, T). \tag{9}$$

Equation (9) implies that the expected fire size is larger at all points in time after the increase in y_1. In summary, the rise in the number of threatened assets increases the expected area through two channels: the dependence of $A(t)$ on T and the higher expected fire size at all points on the interval (t, T). □

These analytical results are consistent with the simulation results reported in Fried, Gilless, and Spero (2006) who find evidence that larger fires are expected when response resources are diverted to protect structures during initial attack.

While fire size and duration are expected to increase with an increase in the number of threatened assets, the impact on cumulative costs is ambiguous in the model. This result is confounded by the relative prices and magnitudes of change of suppression and protection effort. As resources are shifted from suppression to protection ($ds_f/dy_1 < 0$ and $ds_d/dy_1 > 0$), total cost rises at any point in time if expenditure on protection is greater than on suppression

$$|s_d(t)(w_d + w_d^o(t))| > |s_f(t)(w_f + w_f^o(t))|.$$

Given the explicit costs are constant throughout the response effort, the relative price of the resources depends on the opportunity cost. If the potential marginal damage to threatened assets is large enough, the management team will find it optimal to increase protection such that total costs rise. Additionally, the expected duration of a fire increases (proposition 1), which implies a longer response effort that may or may not require the use of costly resources.

Proposition 3. *Cumulative costs $C(T) = \int_0^T c(t)\,dt$ rise in response to an increase in the number of threatened assets y_1 when the loss function is separable in costs and damages, and if one or both of the following hold:*

1. *increased expenditure on protection is larger than the savings on suppression over (t, T),*
2. *expenditure over the excess duration (proposition 1) exceeds any expenditure reduction due to substitution from suppression toward protection.*

The theory does not provide a definitive prediction of the impact of threatened assets on cost. However, the presence of any valuable asset during the extended duration (proposition 1), would induce a fire manager to apply costly response effort. Therefore, we expect threatened assets to increase the expected cost of a fire, similar to duration and size.

2.2 Hazard model

We now draw the connection between our stochastic dynamic program and empirical hazard models. The following section demonstrates how fire outcomes derived from our model may be empirically estimated as a reduced-form hazard model.

Assume that solutions for $s_f^*(t)$ and $s_d^*(t)$ exist and are functions of the exogenous variables: $\boldsymbol{\nu}(t), \mathbf{y}(t), w_f, w_f^o(t), w_d, w_d^o(t), \mathbf{z}(t)$. After substituting the optimal policy functions into the value function (equation (6)), we can express the fire stock distribution in its reduced form as

$$\begin{aligned} G(\bar{f} \mid s_f^*(t), \mathbf{z}(t), f(t), t) &= G(\bar{f} \mid \boldsymbol{\nu}(t), \mathbf{y}(t), w_f, w_f^o(t), w_d, w_d^o(t), \mathbf{z}(t), f(t), t) \\ &= G(\bar{f} \mid \mathbf{x}(\tau), f(\tau), \tau) \end{aligned} \tag{10}$$

where $\mathbf{x}(t) = [\boldsymbol{\nu}(t), \mathbf{y}(t), w_f, w_f^o(t), w_d, w_d^o(t), \mathbf{z}(t)']'$ represents all exogenous covariates.

Equations (7) and (10) state that the probability of the response effort ending in the next interval of time, conditional on the fire having persisted beyond time t, is equal to the probability that the fire stock falls below the critical value $\bar{f}$. In hazard model terminology, this conditional probability is known as the hazard rate where *hazard* refers to the probability of an event occurring in the next instant of time. In the context of wildfire, the event is the termination of the fire. As the fire progresses over time, the path of hazard rates forms the hazard function denoted

$$\begin{aligned} h(t \mid \mathbf{x}(t)) &= \int_0^t G(\bar{f} \mid \mathbf{x}(\tau), f(\tau), \tau) d\tau \\ &\equiv \lim_{\Delta t \to 0} \frac{\Pr(t \leq T < t + \Delta t \mid \mathbf{x}(t), T \geq t)}{\Delta t} \\ &= \frac{\phi(t \mid \mathbf{x}(t))}{\Psi(t \mid \mathbf{x}(t))}. \end{aligned} \tag{11}$$

The hazard function is equal to the density, $\phi(t \mid \mathbf{x}(t))$, over the survival function, $\Psi(t \mid$

$\mathbf{x}(t)) = 1 - \Phi(t \mid \mathbf{x}(t))$ where $\Phi(t \mid \mathbf{x}(t)) = \int_0^t \phi(\tau \mid \mathbf{x}(\tau))d\tau$.[17] The survival function represents the probability that the fire will persist beyond a given t and is monotonically decreasing. In reference to our model of disaster response, the survival function accounts for the process by which managers accumulate information throughout the disaster.

While hazard models generally study the time until an event occurs, the cost and area of a wildfire are also valid measures of duration (Triplett, 1999; Etzioni et al., 1999; Jain and Strawderman, 2002). Because fire size and costs accumulate over the course of the fire, $A(t) = \int_0^t a(\tau)d\tau$ and $C(t) = \int_0^t c(\tau)d\tau$, we apply the same logic used to derive equation (11) to construct hazard functions of area and cost:[18]

$$h(k \mid \mathbf{x}(k)) = \frac{\phi(k \mid \mathbf{x}(k))}{\Psi(k \mid \mathbf{x}(k))} \qquad \text{where} \quad k = a, c$$

All three outcomes – duration, cost, and size – represent three perspectives on an underlying stochastic process f, which is imperfectly observed by the fire managers, and completely unaccounted for in the data. Due to the joint dependence of duration, size, and cost on the stochastic f, the outcomes are correlated random variables. We exploit this correlation in the development of the trivariate frailty model of wildfire duration, cost, and size.[19]

We employ a parametric proportional hazard model to study the impact of covariates on duration, cost, and fire size. The term *proportional* implies that the covariates shift a parametric baseline hazard function proportionately over the support. The hazard function for each outcome $k = t, a, c$ is

$$h(k \mid \mathbf{x}(k)) = h_0(k)\gamma(\mathbf{x}(k)) \tag{12}$$

[17]While it is instructive to think of the hazard function as a conditional probability, it is not upper bounded by one. See Kalbfleisch and Prentice (1980); Blossfeld, Hamerle, and Mayer (1989); Petersen (1995) for a technical development of the components to hazard analysis.

[18]One may think of cumulative size and cost as alternative measures of the wildfire's progression.

[19]In the hazard literature, the introduction of a latent variable gives rise to a variant of hazard models called frailty models.

where $h_0(k)$ for $k = t, a, c$ are the baseline hazard functions, and the survival functions are

$$\Psi(k \mid \mathbf{x}(k)) = \Psi_0(k)^{\gamma(\mathbf{x}(k))} \tag{13}$$

where $\Psi_0(k)$ for $k = t, a, c$ are the baseline survival functions. Covariates are introduced through a multiplicative function, $\gamma(\mathbf{x}(k)) = \exp\{\mathbf{x}(k)\boldsymbol{\beta}_k\}$ where $k = t, a, c$. The exponential function is commonly used because the hazard function must be non-negative over the support.

Holmes, Huggett, and Westerling (2008) and Strauss, Bednar, and Mees (1989) argue that heavy-tailed distributions most accurately describe the distribution of wildfire size, among other disaster outcomes. In light of their results, we assume each outcome is distributed Weibull, which takes on a heavy tail when the shape parameter is less than one (Embrechts, Klüppelberg, and Mikosch, 1997).

We introduce a jointly distributed random component, $\boldsymbol{\varepsilon}$, correlated across equations, but constant over time, that represents unobserved heterogeneity beyond that captured in the covariates $\mathbf{x}$.[20] The system of hazard and survival functions is

$$h(t|\mathbf{x}, \boldsymbol{\varepsilon}_t) = h_0(t)\exp\{\mathbf{x}\boldsymbol{\beta}_t + \boldsymbol{\varepsilon}_t\} \qquad \Psi(t|\mathbf{x}, \boldsymbol{\varepsilon}_t) = \Psi_0(t)^{\exp\{\mathbf{x}\boldsymbol{\beta}_t + \boldsymbol{\varepsilon}_t\}}$$
$$h(c|\mathbf{x}, \boldsymbol{\varepsilon}_c) = h_0(c)\exp\{\mathbf{x}\boldsymbol{\beta}_c + \boldsymbol{\varepsilon}_c\} \qquad \Psi(c|\mathbf{x}, \boldsymbol{\varepsilon}_c) = \Psi_0(c)^{\exp\{\mathbf{x}\boldsymbol{\beta}_c + \boldsymbol{\varepsilon}_c\}}$$
$$h(a|\mathbf{x}, \boldsymbol{\varepsilon}_a) = h_0(a)\exp\{\mathbf{x}\boldsymbol{\beta}_a + \boldsymbol{\varepsilon}_a\} \qquad \Psi(a|\mathbf{x}, \boldsymbol{\varepsilon}_a) = \Psi_0(a)^{\exp\{\mathbf{x}\boldsymbol{\beta}_a + \boldsymbol{\varepsilon}_a\}}$$

where $\boldsymbol{\varepsilon} = (\boldsymbol{\varepsilon}_t, \boldsymbol{\varepsilon}_c, \boldsymbol{\varepsilon}_a)' \sim N(0, \boldsymbol{\Omega})$ is included in the the function $\gamma(\cdot)$ in equations (12) and (13).[21] The "time-varying" notation on covariates $\mathbf{x}(k)$ is now used only when necessary to distinguish variables in an equation. The log-normal distribution[22] is chosen to describe the unobserved heterogeneity because of the heavy-tail properties as well as its explicit parameterization of the correlation in the covariance matrix $\boldsymbol{\Omega}$.

[20] Wienke et al. (2005) provides a survey of frailty models and their multivariate extensions and discusses the associated estimation methodologies.

[21] The restriction of $\mu = 0$ is inconsequential because any deviation from zero would be subsumed into the scale parameter of the Weibull baseline hazard which may be interpreted as the intercept analog in a linear regression.

[22] Note that $\boldsymbol{\varepsilon}$ appears in the exponential covariate function which implies that the multiplicative effect on the baseline hazard and survival function is log-normal.

The joint conditional density is

$$\phi(t,c,a|\mathbf{x},\boldsymbol{\varepsilon}) = \phi(t|\mathbf{x},\boldsymbol{\varepsilon}_t)\ \phi(c|\mathbf{x},\boldsymbol{\varepsilon}_c)\ \phi(a|\mathbf{x},\boldsymbol{\varepsilon}_a), \tag{14}$$

where $\phi(k|\mathbf{x},\boldsymbol{\varepsilon}_k) = h(k|\mathbf{x},\boldsymbol{\varepsilon}_k)\cdot\Psi(k|\mathbf{x},\boldsymbol{\varepsilon}_k)\ \forall\ k$. The unconditional joint density is obtained by integrating over $\boldsymbol{\varepsilon}$,[23]

$$\phi(t,c,a|\mathbf{x}) = \int_{\boldsymbol{\varepsilon}} \phi(t,c,a|\mathbf{x},\boldsymbol{\varepsilon})\ \exp\{q(\boldsymbol{\varepsilon})\}\ d\boldsymbol{\varepsilon} \tag{15}$$

where

$$q(\boldsymbol{\varepsilon}) = \frac{1}{(2\pi)^{\frac{3}{2}}|\boldsymbol{\Omega}|^{\frac{1}{2}}}\ \exp\left\{-\frac{1}{2}\ \boldsymbol{\varepsilon}'\boldsymbol{\Omega}^{-1}\boldsymbol{\varepsilon}\right\}.$$

The Likelihood function for this trivariate problem over $i = 1\dots n$ wildfire observations is

$$L(\boldsymbol{\theta}|\mathbf{x}) = \prod_{i=1}^{n} \phi(t_i,c_i,a_i|\mathbf{x}_i)^{\delta_i}\Psi(t_i,c_i,a_i|\mathbf{x}_i)^{1-\delta_i} \tag{16}$$

where δ_i is the censoring indicator. Maximization of the full trivariate likelihood function $L(\boldsymbol{\theta}|\mathbf{x})$ in Equation 16, where $\boldsymbol{\theta}$ includes $\boldsymbol{\Omega}$, provides consistent estimates of model parameters. Estimating the duration, cost, and size hazard regressions independently is equivalent to restricting the off-diagonal elements of the covariance matrix $\boldsymbol{\Omega}$ to be zero. Equation 15 requires integration of the CDF over $\boldsymbol{\varepsilon}$. Unfortunately, a closed form solution to this problem does not exist (Wienke et al., 2005). Therefore, we apply a maximum simulated likelihood estimation (MSLE) procedure that approximates the unconditional joint density $\phi(t,c,a|\mathbf{x})$.[24]

We program the likelihood function in Matlab (Mathworks, 2010) and use the constrained optimization interior-point algorithm to maximize the function. The algorithm uses the BFGS method to optimize the nonlinear likelihood function over the parameters $\boldsymbol{\beta}$, $\boldsymbol{\varsigma}$, and $\boldsymbol{\lambda}$

[23]The use of the term "unconditional" here refers to the fact that the joint density is no longer conditional on $\boldsymbol{\varepsilon}_k$. The density, hazard, and survival functions remain conditional on $\mathbf{x}$ throughout the remainder of the analysis.

[24]See Greene (2008) for a brief overview of maximum simulated likelihood estimation.

of the Weibull distribution, and the covariance matrix parameters of the frailty distribution, $\boldsymbol{\Omega}$. The algorithm converges on a solution and terminates once the gradient, the change in the function value, and the change in the norm of the estimated parameter vector reaches tolerance 10^{-4}.

3 Data

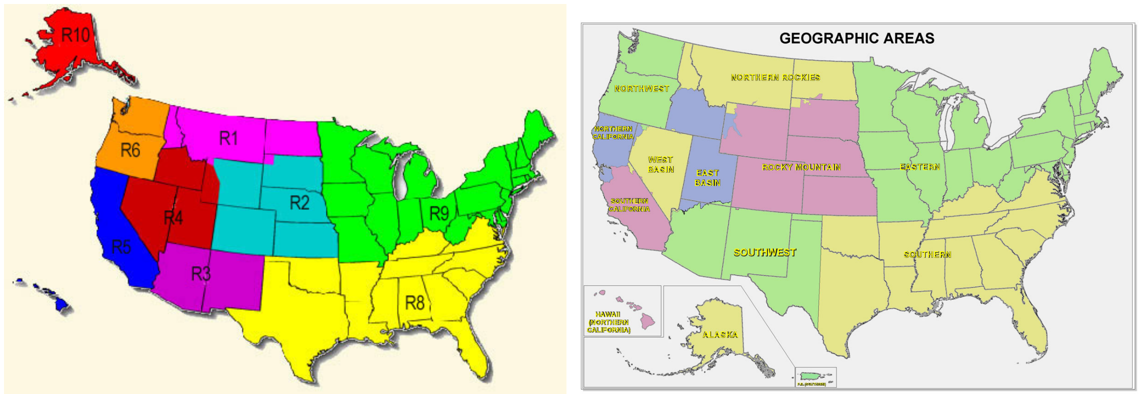
The data used in this analysis comes from the Incident Status Summary (ICS-209) databases, the National Interagency Fire Management Integrated Database (NIFMID), and the 2010 U.S. Census. The National Wildfire Coordination Group maintains a database that contains situational reports, filed intermittently by wildfire incident commanders throughout the suppression effort. These reports begin at the time of discovery and end when the suppression effort is complete. Each report includes data on weather, geographic, and environmental characteristics as well as assets threatened and destroyed by the fire. The wildfire data is supplemented by home value data from the 2010 U.S. Census Bureau (2011) at the Census Designated Place (CDP) level.

Table 3 in the appendix provides variable names, data descriptions, and source information for all variables used in the empirical analysis. The wildfire suppression effort is considered complete when the fire is no longer growing (in terms of our notation, $a_{i\tau-1} = a_{i\tau}$).[25] This definition is consistent with our terminal condition, $f(T) \leq \bar{f}$, in the dynamic program. Because hazard models rely on the accumulation of time, area, and cost, consecutive reports in which the suppression cost or fire size remained constant yield no additional information to the model, and are removed from the dataset.

The structure of the likelihood function suggests that each fire is an independent event. However, we know that fires within a region compete for suppression resources. Therefore, we construct the variable `Resource Scarcity` as an instrument for the availability of resources

[25]Finney, Grenfell, and McHugh (2009) also consider a wildfire contained if the burned area does not increase between reports.

Figure 1: Maps of Forest Service and GACC regions. See Table 4 for FS region names.



(a) Forest Service (b) GACC

during a suppression effort. For each observation, we sum the cumulative growth of all wildfires within a region, within the past five days, and subtract the mean growth of fires in that region during that month across all years. Positive values indicate that fire activity in the region is above average, and resources are likely scarce. Negative values imply that fire activity is less than expected, leaving desired resources available for dispatch. Conditional on `Resource Scarcity` the fires are assumed independent.

Suppression resources are regionally managed by Geographic Area Coordination Centers (GACC). However, our dataset contains the Forest Service region where the wildfire began. Fortunately, the GACC regions correspond to the FS regions with some exceptions as illustrated in Figure 1. The two most important exceptions are the division of California into separate GACC regions and Nevada as its own GACC. We do not believe that these discrepancies significantly affect the results.

There are two general categories of covariate: time-varying covariates, $\mathbf{x}_{i\tau} \in \mathbf{x}$, take on different values over the course of the fire and time-invariant covariates, $\mathbf{x}_i \in \mathbf{x}$, remain constant over the duration of the fire. With the exception of weather covariates, time-varying covariates are lagged one period to avoid endogeneity (Petersen, 1995). For instance, the number of threatened residential homes reported in period $t-1$ is considered predetermined in period t. Recall the conditionality of the hazard function on the survival function.

The survival function effectively accumulates information over the course of the fire such that increases in duration (size, or cost), from one observation to another, are attributable to covariate values during that interval.[26] Therefore, fire growth in period t cannot influence covariate values (e.g., number of threatened residential homes) in period $t-1$. The covariates `Threatened Structures` and `Potential Evacuation` are chosen over their confirmed counterparts (Destroyed Structures and Confirmed Evacuation) because we believe that "threatened" and "potential" reflect the incident commanders assessment of the wildfire's current status. Therefore, any resource allocation decisions made at time t will affect future outcomes.

Table 4 in the appendix presents summary statistics of the covariates used in this analysis. The dataset contains 10,321 observations on 3,829 fires. Means and standard deviations of time-varying covariates $\mathbf{x}_{i\tau}$ are based on all observations (10,321) while the statistics of time-invariant covariates $\mathbf{x}_i$ and the dependent variables duration, cost, and size are based on one record per fire (3,829).

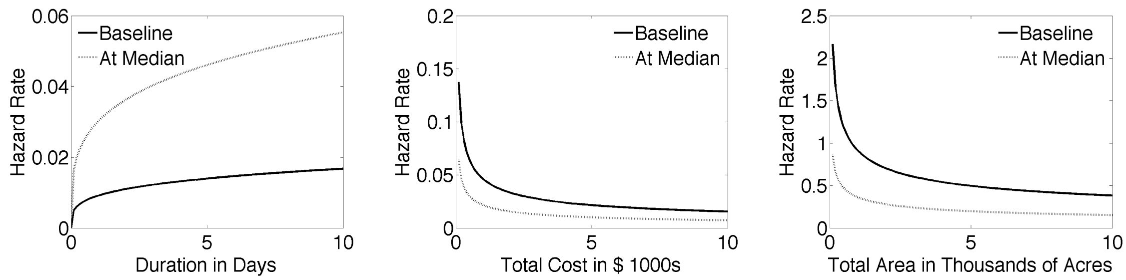
4 Results and discussion

Table 5 in the appendix contains the parameter estimates $\boldsymbol{\beta}_k$ and standard errors for duration, cost, and area, as well as the ancillary Weibull distribution parameters. In order to provide context for the coefficient estimates, we first discuss the ancillary parameters of the Weibull distribution that determine the shape of the baseline hazard function. The baseline is defined as the hazard function where all of the covariates are null. Because some continuous covariates, such as temperature, rarely take a null value, we also present the hazard function with the continuous covariates evaluated at their median value (categorical variables are left as their null values). The environmental and wildfire characteristics represented by the baseline and median scenarios are described in Table 6 in the appendix.

[26] The survival function is qualitatively similar to the number of previous intervals variable included in Finney, Grenfell, and McHugh (2009) GLMM.

The baseline Weibull hazard function $h_0(k) = \lambda_k \varsigma_k \ (\lambda_k k)^{\varsigma_k - 1}$ for $k = t, c, a$ is parameterized by a shape (ς_k) and scale (λ_k) parameter.[27] The baseline hazard functions for duration, cost, and area are presented in Figure 2.[28]

Figure 2: Hazard functions of duration, cost, and area: baseline and at median values of covariates.



(a) Duration (b) Cost (c) Area

The hazard function for duration is increasing over time (ς_t (shape) $= 1.25$), which implies that as the duration of a wildfire grows, the probability of containment (conditional on no containment to date) rises. The decreasing hazard functions of suppression cost ($\varsigma_c = 0.53$) and fire size ($\varsigma_a = 0.63$) imply that as wildfires become more expensive and large in area, the instantaneous probability of containment declines.[29] These results are consistent with those of Holmes, Huggett, and Westerling (2008) and Strauss, Bednar, and Mees (1989) who find that while most fires are contained when the fire is small, some fires grow excessively large and become very difficult to suppress.

The survival function $\Psi(k|\mathbf{x}) = \exp\{-(\lambda_k k)^{\varsigma_k} \cdot \exp\{\mathbf{x}\boldsymbol{\beta}_k\}\}$, as specified in equation (13), represents the probability that a fire persists beyond a given duration (cost, or size). At the point of ignition, the fire is certain to persists until the next instant of time ($\Psi(0) = 1$). As

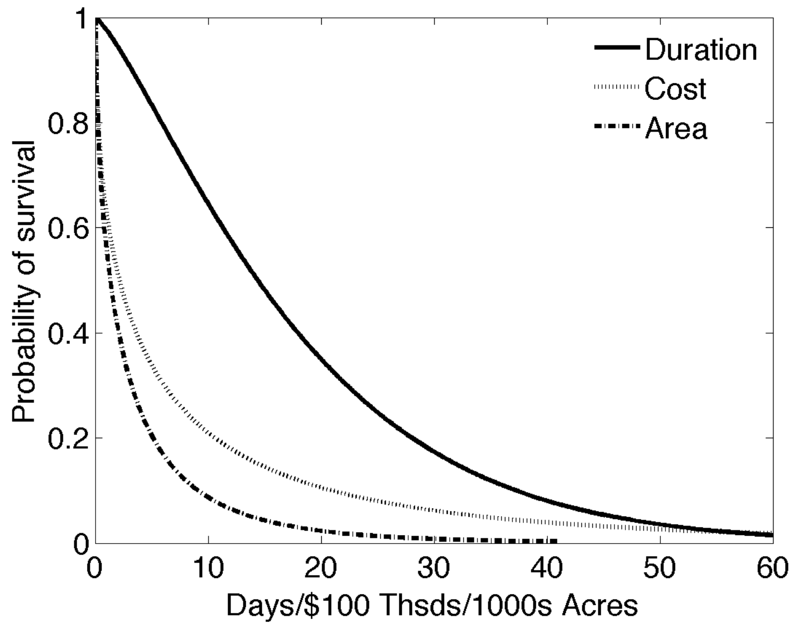
[27] Because the covariates multiplicatively affect the baseline hazard function through the proportionality factor, we can subsume λ_k into the exponential function as $\gamma(\mathbf{x}) = e^{\beta_0 + \mathbf{x}\boldsymbol{\beta}_k}$ where $\beta_0 = \varsigma_k \log(\lambda_k)$. In this form, the scale parameter is analogous to the intercept in a linear regression.

[28] It is important to note that these estimates are not intended to represent the natural, unsuppressed growth of a wildfire. The model should be interpreted as the reduced form of an underlying structural model, with suppression effort implicit in the outcomes.

[29] Note that the cumulative hazard rate is always increasing, even when the instantaneous hazard rate is decreasing over the domain. In other words, the probability of a fire growing larger than 100 thousand acres is less than the probability of a fire growing larger than 10 thousand acres despite the smaller hazard rate at 100 thousand acres.

duration (cost, or size) grows, the probability of persistence, or fire survival, falls. Figure 3 depicts the survival functions of all three models, evaluated at the median covariate values.[30]

Figure 3: Survival functions evaluated at median covariate values



The bottom section of the Table 5 contains the elements of the Cholesky triangle $\mathbf{L}$ which satisfies the equation $\boldsymbol{\Omega} = \mathbf{L}\mathbf{L}'$. The covariance matrix of the normally distributed unobserved heterogeneity, $\boldsymbol{\varepsilon}$, and the associated correlation matrix are

$$\hat{\boldsymbol{\Omega}} = \begin{bmatrix} \hat{\sigma}_{tt} & \hat{\sigma}_{tc} & \hat{\sigma}_{ta} \\ \hat{\sigma}_{tc} & \hat{\sigma}_{cc} & \hat{\sigma}_{ca} \\ \hat{\sigma}_{ta} & \hat{\sigma}_{ca} & \hat{\sigma}_{aa} \end{bmatrix} = \begin{bmatrix} 0.6813 & 0.6521 & 0.7687 \\ 0.6521 & 0.6282 & 0.7409 \\ 0.7687 & 0.7409 & 0.8740 \end{bmatrix}$$

$$\hat{\mathbf{P}} = \begin{bmatrix} 1 & \hat{\rho}_{tc} & \hat{\rho}_{ta} \\ \hat{\rho}_{tc} & 1 & \hat{\rho}_{ca} \\ \hat{\rho}_{ta} & \hat{\rho}_{ca} & 1 \end{bmatrix} = \begin{bmatrix} 1.0000 & 0.9992 & 0.9990 \\ 0.9992 & 1.0000 & 1.0000 \\ 0.9990 & 1.0000 & 1.0000 \end{bmatrix}.$$

While not all of the estimates of $\mathbf{L}$ are statistically different from zero, a likelihood ratio

[30] Recall that the survival function is one minus the probability of containment, $\Psi(t \mid \mathbf{x}(t)) = 1 - \Phi(t \mid \mathbf{x}(t))$. Figure 3 is the complement to Figure 2 in Finney, Grenfell, and McHugh (2009).

test rejects the joint hypothesis that $\sigma_{ij} = 0$ for all $i, j = t, c, a$ with a $LR = 898.32 \sim \chi^2_6$ (p-value=$<$ 0.0001).[31] The off-diagonal elements of the correlation matrix are above 0.99, which indicates that the outcomes, duration, cost, and area covary closely. While such close covariation is expected, this result underscores the importance of jointly modeling duration, cost, and area.

Propositions 1, 2, and 3 are the foundation of three hypotheses: when the number of threatened assets rise, wildfire management teams divert resources from suppression toward protection of threatened assets, which causes 1) longer, 2) larger, and 3) more costly expected wildfires. Table 1 contains a subset of the results that support these hypotheses. Table 5 in the appendix contains the full set of results. The parameter estimates in Table 1 are transformed to show the percent effect of a one unit change in a covariate on the ex ante expected duration, size, and cost of a fire.

Table 1: Selected results. Percent effect of a change in covariate on expected duration, cost, and size

	Duration	Cost	Area
`Threatened Residential` (100s)	8.56**	22.23***	26.02***
`Threatened OutBuildings` (100s)	3.17	16.67**	35.03***
`Potential Evacuation`	120.89***	330.42***	401.83***
`Threatened Commercial` (100s)	-28.95***	-15.99	14.58
`Resource Scarcity` (100k acres)	2.58	3.81*	13.42***

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.001$

Hypothesis tests are based on underlying parameter estimates

Our results indicate that the expected duration, cost and size of a wildfire are affected by an increase in the number of threatened assets. When the number of `Threatened Residential` homes increases by 100, the expected cost and size of a fire rise by 22% and 26%, respectively. These results contrast with those of Donovan, Noordijk, and Radeloff (2004) who find that threatened homes have no statistically significant effect on wildfire

[31] We also conduct a LR test of uncorrelated heterogeneity with a null hypothesis of $\sigma_{ij} = 0$ for all $i \neq j$. $LR = 705.04 \sim \chi^2_3$ (p-value=$<$ 0.0001).

cost.[32] Similarly, an increase of 100 `Threatened Outbuildings` raises the expected cost and size of a fire by 16.6% and 35%, respectively.[33] The impact of threatened resources on the expected wildfire cost suggest that at least one of the conditions in proposition 3 is met. We attribute these results to a reallocation of response resource away from suppression towards protection.

`Potential Evacuation` is a subjective measure of the suppression management team's level of concern regarding the safety of a community. `Potential Evacuation` increases the expected duration by 121%, the expected costs by 330%, and the expected size by 401%. In addition to protecting threatened residences, resources may be used to prepare and manage evacuations, leaving fewer resources available for suppression.

The estimates in the duration and cost equations associated with `Threatened Commercial` are not consistent with hypotheses 1) and 2). An increase of 100 `Threatened Commercial` reduce the expected duration by 29% and reduces the expected costs by 16% (not statistically significant). However, the expected size increases by 15% when the number of `Threatened Commercial` rises by 100. While the duration and cost results do not appear to be consistent with the theory, the management team's perception of insurance may impact resource allocation decisions. Suppose that fire managers believed commercial structures are fully insured against fire damage as opposed to residential structures and outbuildings whose non-market value cannot be fully insured (Yoder, 2010). The management team may treat commercial structures as a low value asset and continue suppression rather than shifting suppression effort toward protecting specific assets.

The ability of suppression management teams to optimally allocate resources to a specific fire is contingent on the availability of the desired resources, which depends on fire activity in the region. The cost function in the theoretical model given by equation (2) includes an opportunity cost w_i^o for $i = f, d$ meant to capture the availability of resources. We include

[32]Note that Donovan, Noordijk, and Radeloff (2004) treat wildfire size as an exogenous variable in a linear regression of total suppression costs, whereas our model considers suppression costs an endogenous outcome.

[33]The magnitude of the estimates for `Threatened Residential` and `Threatened Outbuildings` are not statistically different from each other.

the variable `Resource Scarcity` as an instrument for w_i^o. We find that when fire activity in the region increases by 100,000 acres within a span of five days: the expected duration increases by 2.5% (not statistically significant), the expected cost increases by 4%, and the expected size increases by 13%. As the opportunity cost of suppression resources rises, the management team may not apply sufficient suppression effort leading to larger and more costly expected fire outcomes.

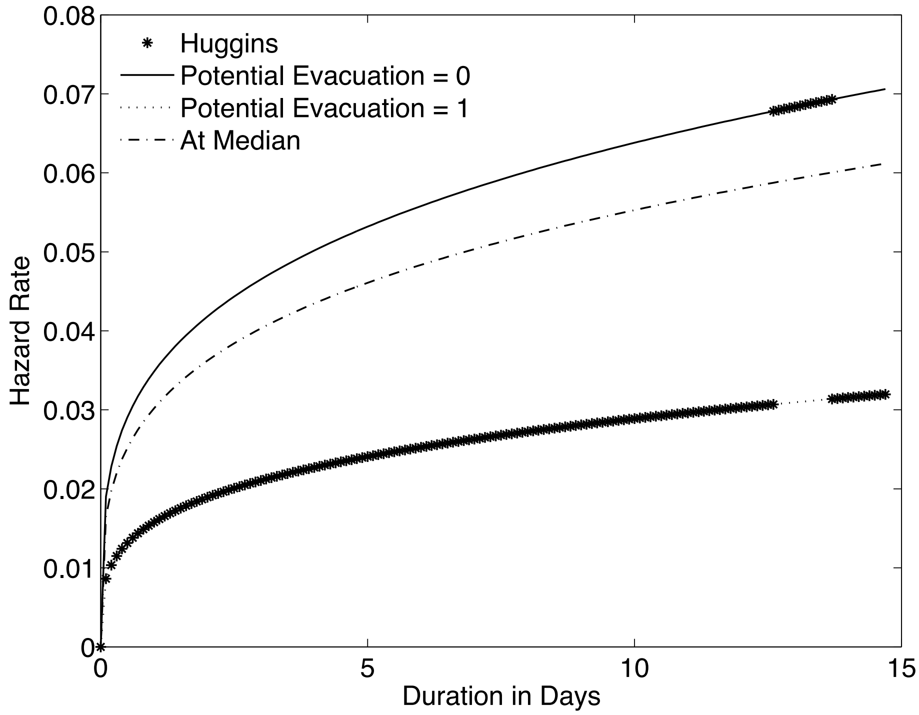
The covariate coefficients provide information about the impact of a covariate at any given point in time. However, conditions change over the course of the wildfire. These changes cause the hazard function to "jump" from the hazard rate reflecting the previous covariate state to that of the new state. Therefore, the hazard function is not continuous over the course of a particular wildfire. The survival function, on the other hand, is continuous and provides an intuitive representation of the wildfire's progression over time.

Figure 4 contains the hazard and survival functions of the Huggins fire, a wildfire in Curry County, Oregon that started in July 2005. In order to illustrate the effect of a time-varying covariate on the hazard rate and survival function, we fix all covariates, except for `potential evacuation`, to their within-fire median values. The Huggins wildfire lasted 40 days, however, the transition between covariate values is clearer when the duration axis is truncated.

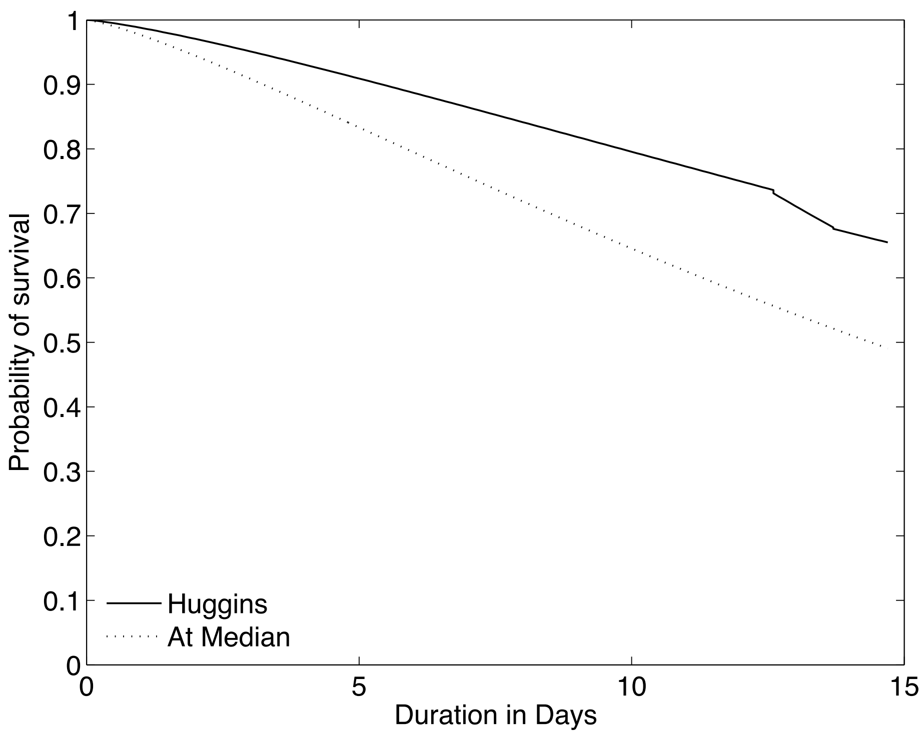
Figure 4(a) displays four hazard functions: 1) at the population median covariate levels, 2) at `Potential Evacuation` = 0 (with other covariate fixed to their within-fire median), 3) at `Potential Evacuation` = 1, and 4) the hazard function based on the Huggins fire data. The initial duration hazard rate corresponds to `Potential Evacuation` = 1 that persists for 12.5 days.[34] Given that `potential evacuation` reduces the hazard of containment ($\beta_{\text{pot evac}} = -0.792$ from table 5), the Huggins fire hazard function lies below the median population hazard function. The jump in the Huggins hazard function represents the instan-

[34]Note that the dataset contains numerous reports within the 12 day interval in which other time-varying covariates change. However, we fix all other covariate to their median level to illustrate the marginal effects of a change in a time-varying covariate during a fire.

Figure 4: Hazard and survival function of Oregon wildfire experiencing potential evacuation.



(a) Hazard function



(b) Survival function

taneous transition to a new state corresponding to a new ICS-209 report. During the interval 12.5 to 13.7 days, the potential for evacuation is removed, and the hazard of containment jumps to the hazard function associated with `Potential Evacuation` = 0. After day 13, the potential for evacuation was reinstated, and the hazard of containment fell once again. We attribute these large shifts in the probability of containment to shifts between suppression and protection strategies in response to threatened assets.

Figure 4(b) displays the population median survival function, and the Huggins fire survival function, which represents the cumulative effects of changing `potential evacuation` over the course of the fire. During the first 12.5 days, the probability of the fire persisting another day gradually declines due to the existing potential for evacuation. Midway through day 12, the potential for evacuation is removed and the slope of the survival function decreases. The kink in the survival function reflects the instantaneous transition to the higher hazard rate depicted in figure 4(a). Once the potential for evacuation is reinstated, the slope of the survival function rises to a level consistent with that during the interval from 0 to 12.5 days. However, the curve has shifted down, due to the interval of increased containment probability, reflecting the cumulative nature of the survival function. We consider the cumulative aspect of the survival function an important feature of this hazard model approach because it approximates the wildfire manager's accumulation of information over the course of the fire.

5 Conclusion

We have proposed a model of disaster response applied to wildfire in which managers face tradeoffs between suppression and protection while attempting to minimize costs and damage. We hypothesize that an increase in the number of threatened assets causes managers to shift resources from suppression toward asset protection. The model predicts that the diversion of resources causes the expected duration, cost, and size of the fire to rise.

The model of disaster response, and the focus on disaster outcomes, lends itself to an empirical duration model. We derive the foundations of a duration model from our dynamic model of disaster response and estimate a trivariate hazard model. Our results support the predictions of the theory: an increase in the number of threatened assets increases the expected duration, cost, and size of the fire. In addition, we find that the trivariate hazard model with correlated unobserved heterogeneity outperforms the model with independent unobserved heterogeneity, which outperforms the model without unobserved heterogeneity. This result underscores the importance of jointly estimating disaster outcomes; specifically, modeling wildfire size and cost as jointly determined outcomes.

The results of this analysis imply that the growing wildland urban interface has, and will continue to influence wildfire size and cost. As more structures are built in zones with high risk of fire, our results imply that fires are expected to last longer, grow larger, and cost more as structures are threatened during wildfire.

The modeling framework proposed in this analysis provides ex-ante and ex-post information to fire-fighting agencies at all levels. The Huggins fire example demonstrates how the results of this model may be used by wildfire managers to understand the probabilistic behavior of wildfire given the limited information they accrue throughout the response effort. Results from the hazard model may be integrated with existing management tools such as the Wildland Fire Decision Support System, used to project fire behavior and future resource needs (Service, 2011). Furthermore, analyzing the survival function after the response effort is complete may provide information that would help agencies such as the U.S. Forest Service and Bureau of Land Management assess their management and resource prepositioning strategies.

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Appendix

1 Comparative dynamic results

In this section, we derive the result $ds_f/d\nu_1 < 0$ and $ds_d/d\nu_1 > 0$ from the system of first order conditions of the recursive Bellman equation (6). Because we have data on threatened properties, which represent a subset of all threatened assets, we present this result in the context of an increase in $\nu_1 \in \boldsymbol{\nu}$. We begin by restating equation (6) and dropping the time notation for ease of exposition

$$V(f) = \min_{s_d, s_f \geq 0} \left\{ \ell \left[c(s_f, s_d), d(s_d) \right] + \int_0^\infty V^{'} \ g(f^{'} \mid f, s_f, \mathbf{x}) \ df^{'} \right\}$$

where $c(\cdot)$ and $d(\cdot)$ are defined by (2) and (3), respectively. Assuming that $g(\cdot)$ is a continuous, but not necessarily stationary, distribution, the optimal feedback rules must satisfy the following first-order conditions,

$$\frac{\partial \ell \left[c(s_f, s_d), d(s_d) \right]}{\partial c}(w_f + w_f^o) + \int_0^\infty V^{'} \frac{\partial g(f^{'} \mid f, s_f, \mathbf{x})}{\partial s_f} df^{'} = 0$$

$$\frac{\partial \ell \left[c(s_f, s_d), d(s_d) \right]}{\partial c}(w_d + w_d^o) - \frac{\partial \ell \left[c(s_f, s_d), d(s_d) \right]}{\partial d} \frac{y_1}{s_d^2} \nu_1 a(\mathbf{x}, f) = 0$$

We can then totally differentiate the FOCs and rearranging them into the form $Ax = b$. For simplicity, we assume that the loss function is separable in costs and damages (i.e., $\partial^2 \ell / \partial c \partial d = 0$). This assumption is equivalent to equal weights on costs and damage in the loss function. Alternative assumptions complicate the derivation but do not change the results.

$$\begin{bmatrix} \frac{\partial^2 \ell}{\partial c^2}\left(w_f + w_f^o\right)^2 + \int_0^\infty V' \frac{\partial^2 g}{\partial s_f^2} & \frac{\partial^2 \ell}{\partial c^2}\left(w_d + w_d^o\right)\left(w_f + w_f^o\right) \\ \frac{\partial^2 \ell}{\partial c^2}\left(w_d + w_d^o\right)\left(w_f + w_f^o\right) & \frac{\partial^2 \ell}{\partial c^2}\left(w_d + w_d^o\right)^2 + \frac{\partial^2 \ell}{\partial d^2}\left(\frac{y_1 \nu_1 a}{s_d^2}\right)^2 + \frac{\partial \ell}{\partial d}\frac{y_1}{s_d^3}\nu_1 a \end{bmatrix} \begin{bmatrix} ds_a \\ ds_d \end{bmatrix}$$
$$= - \begin{bmatrix} 0 \\ -\frac{\partial^2 \ell}{\partial d^2}\frac{y_1}{s_d^3}(\nu_1 a)^2 - \frac{\partial \ell}{\partial d}\frac{1}{s_d^2}\nu_1 a \end{bmatrix} dy_1$$

We now solve for ds_f/dy_1 and ds_d/dy_1. We assume the following properties

- The loss function, $\ell\left[c(s_f, s_d), d(s_d)\right]$, is increasing in both arguments at an increasing rate
- The expectation of energy stock, $E_t\{\partial V'/\partial s_f\} = \int_0^\infty V' \frac{\partial g(f' \mid f, s_f, \mathbf{x})}{\partial s_f}\, df'$, conditional on suppression, is decreasing in suppression activity at a decreasing rate.

These properties are summarized in the following table.

$$\begin{array}{lll} \frac{\partial \ell(c,d)}{\partial i} > 0 & \frac{\partial^2 \ell(c,d)}{\partial i^2} > 0 & i = c, d \\ \frac{\partial g\left(f' \mid s_f\right)}{\partial s_f} < 0 & \frac{\partial g\left(f' \mid s_f\right)}{\partial s_f} \geq 0 & \end{array}$$

Given that we are minimizing an objective function, the determinant of A in the equation $Ax = b$ is positive definite, which implies

$$|A| = \left(\overbrace{\frac{\partial^2 \ell}{\partial c^2}\left(w_f + w_f^o\right)^2}^{+} + \overbrace{\int_0^\infty V' \frac{\partial^2 g}{\partial s_f^2} df'}^{+}\right)\left(\overbrace{\left(\frac{\partial^2 \ell}{\partial c^2}\left(w_d + w_d^o\right)^2\right)}^{+} + \overbrace{\frac{\partial^2 \ell}{\partial d^2}\left(\frac{y_1 \nu a}{s_d}\right)^2}^{+} + \overbrace{\frac{\partial \ell}{\partial d}\frac{y_1 \nu a}{s_d^3}}^{+}\right)$$
$$- \overbrace{\left(\frac{\partial^2 \ell}{\partial c^2}\left(w_d + w_d^o\right)\left(w_f + w_f^o\right)\right)^2}^{+} > 0$$

Then by Cramer's rule, we have

$$\frac{ds_f}{dy_1} = \frac{\left(-\frac{\partial^2 \ell}{\partial d^2}\frac{y_1}{s_d^3}(\nu_1 a)^2 - \frac{\partial \ell}{\partial d}\frac{y_1}{s_d^2}\nu_1 a\right)\frac{\partial^2 \ell}{\partial c^2}\left(w_d + w_d^o\right)\left(w_f + w_f^o\right)}{|A|} < 0$$

$$\frac{ds_d}{dy_1} = \frac{\left(\frac{\partial^2 \ell}{\partial c^2}\left(w_f + w_f^o\right)^2 + \int_0^\infty V'\frac{\partial^2 g}{\partial s_f^2}\right)\left(-\left(-\frac{\partial^2 \ell}{\partial d^2}\frac{y_1}{s_d^3}(\nu_1 a)^2 - \frac{\partial \ell}{\partial d}\frac{y_1}{s_d^2}\nu_1 a\right)\right)}{|A|} > 0$$

whereby each derivative is signed by the assumed properties of the comprising functions. We have shown that under fairly general assumptions, the sudden increase of threatened assets causes the wildfire management to increase protection while decreasing suppression effort. This result is due to the temporal effect of suppression. The moment an asset (e.g., home) becomes threatened, the management finds protection the most effective technique for reducing losses.

2 Tables

Table 2: Description of variables used throughout the paper.

Variable	Description
$s_d(t)$	protection effort at time t
$s_f(t)$	suppression effort at time t
$l(c(t), d(t), t)$	loss function
$a(t)$	instantaneous flow of area burning at time t
$A(t)$	cumulative burned area at time t
$c(t)$	instantaneous flow of management expenditure
$C(t)$	cumulative costs accrued until time t
$d(t)$	instantaneous flow of losses at time t
w_f	wage per unit of suppression fixed over duration of fire
$w_f^o(t)$	opportunity cost of suppression resources
w_d	wage per unit of protection fixed over duration of fire
$w_d^o(t)$	opportunity cost of protection resources
$\mathbf{y}(t)$	vector of asset densities
$\mathbf{z}(t)$	a vector of exogenous environmental and geographic characteristics at time t
$\boldsymbol{\nu}(t)$	a vector of threatened asset values at time t
$f(t)$	the fire's energy stock at time t measured in kilowatts
$g(f')$	probability density function of the fire stock in the next instant of time where $f' = \lim_{\Delta t \to 0} f(t + \Delta t)$
$G(f')$	cumulative distribution function (transition function) of the fire stock in the next instant of time
k	index of fire outcomes duration (t), cost (c), size (a)
$\phi_k(k \mid \mathbf{x}(k))$	PDF describing the probability of the fire termination in the next unit of duration, cost, or size
$\Phi_k(k \mid \mathbf{x}(k))$	CDF along dimension k
$\Psi_k(k \mid \mathbf{x}(k))$	survival function along dimension k ($\Psi_k(k \mid \mathbf{x}(k)) = 1 - \Phi_k(k \mid \mathbf{x}(t))$)
$h_k(k \mid \mathbf{x}(k))$	hazard function along dimension k ($= \phi_k/\Psi_k$)
$\phi(t, c, a \mid \mathbf{x}(k))$	unconditional joint density
$\gamma(\mathbf{x}(k))$	exponential proportionality factor
ε_k	unobserved heterogeneity in equation k
$\boldsymbol{\varepsilon}$	jointly distributed unobserved heterogeneity
$\boldsymbol{\Omega}$	covariance matrix of unobserved heterogeneity $\boldsymbol{\varepsilon}$
$\boldsymbol{\beta}_k$	vector of covariate coefficients in equation k
δ	censoring indicator
ς_k	shape parameter in regression equation k
λ_k	scale parameter in regression equation k (subsumed into $\gamma(\cdot)$ for estimation)

Table 3: Variable labels used in this analysis, descriptions, and source information

Variable name	Brief description and source
`Duration` (t)	Duration is calculated as the difference measured in days between the report date:time (ICS box 1,2) and the discovery date:time (NIFMID) or start date:time (ICS box 7).
`Cost` (c)	Cost is the suppression cost to date (ICS box 19) in thousands of dollars.
`Area` (a)	Fire size is the total area burned to date (ICS box 15) in thousands of acres.
`Threatened X`$_\tau$	Thousands of structures threatened lagged one period for `X`=(`Residential`, `Commercial`, `Outbuildings`)(ICS box 24).
`Injuries`$_\tau$	Number of reported injuries lagged one period (ICS box 22).
`Fatalities`$_\tau$	Number of reported fatalities lagged one period (ICS box 23).
`Potential Evacuation`$_\tau$	Binary; equals 1 if evacuations were reported imminent (ICS box 25) and 0 if no evacuation necessary. The variable is lagged one period.
`Wind`$_\tau$	Wind speed, mph/100 (ICS box 27).
`Temperature`$_\tau$	Temperature, degrees Fahrenheit/100 (ICS box 27).
`Relative Humidity`$_\tau$	Relative humidity on scale of $0-1$ (ICS box 27).
`Resource Scarcity`$_\tau$	The sum of the growth of all other wildfires within a Forest Service region within five days of the report less the monthly average growth of fires in the region. Fire growth is calculated as the difference in acres burned between any two reports of a given fire ($a_t - a_{t-1}$); in 100 thousands of acres
`Latitude`	Latitude of fire start location, degrees/100 (ICS box 13).
`Day of Year`	Calculated by converting the report date into radians and applying sin and cos transformations.
`Cause Lightning`	Binary; equals 1 if cause of the wildfire is lightning (baseline=human) (ICS box 8).
`Cause Unknown`	Binary; equals 1 if cause is unknown or under investigation (baseline=human) (ICS box 8).
`Year`	Binary; equals 1 for fires that began in year $i = 2001, \ldots, 2008$ (baseline is 2001)
`FS Region` i `(name)`	Binary; equals 1 for fires that began in Forest Service region $i = 1, \ldots, 10$, respectively (baseline is region 8 (south))
`Distance`	The natural logarithm of the distance in miles between the ignition of a fire and the centroid of the nearest Census Designated Place (CDP); calculation based on the latitude and longitude from ICS box 13 and the 2010 Census.
`Value20`	The total housing value of any CDP with a centroid located 20 miles or less from the fire's point of ignition; in billions of dollars.

Table 4: Summary statistics of variables used in hazard model.

	Obs.	Mean	Std. Dev.	Min	Max
Duration	3,829	15.19	15.88	0.25	178.68
Cost	3,829	4,408.20	10,574.00	0.00	152,660.00
Area	3,829	23.81	77.17	0.00	1,322.90
Threatened Commercial	10,321	0.01	0.05	0.00	3.00
Threatened OutBuildings	10,321	0.04	0.24	0.00	5.00
Threatened Residential	10,321	0.11	0.57	0.00	24.00
Injuries	10,321	0.17	0.65	0.00	18.00
Fatalities	10,321	0.03	0.41	0.00	14.00
Potential Evacuation	10,321	0.31	0.46	0.00	1.00
Wind	10,321	0.07	0.06	0.00	0.90
Temperature	10,321	0.76	0.14	0.00	1.26
Relative Humidity	10,321	0.30	0.19	0.00	1.00
Resource Scarcity	10,321	0.00	1.22	-5.88	21.07
Latitude	3,829	0.41	0.07	0.25	0.67
Day of Year (Sin)	3,829	-0.20	0.52	-1.00	1.00
Day of Year (Cos)	3,829	-0.73	0.39	-1.00	1.00
Cause Lightning	3,829	0.65	0.48	0.00	1.00
Cause Unknown	3,829	0.16	0.37	0.00	1.00
Year	3,829	4.14	2.16	0.00	7.00
FS Region 6 (Pacific Northwest)	3,829	0.13	0.34	0.00	1.00
FS Region 5 (Pacific Southwest)	3,829	0.25	0.43	0.00	1.00
FS Region 1 (North)	3,829	0.18	0.38	0.00	1.00
FS Region 2 (Rocky Mountain)	3,829	0.04	0.20	0.00	1.00
FS Region 4 (Intermountain)	3,829	0.13	0.34	0.00	1.00
FS Region 9 (Eastern)	3,829	0.03	0.16	0.00	1.00
FS Region 3 (Southwest)	3,829	0.11	0.31	0.00	1.00
FS Region 10 (Alaska)	3,829	0.03	0.18	0.00	1.00
ln(Distance)	3,829	2.39	0.85	-1.75	4.33
Value20	3,829	3.31	17.83	0.00	283.98

Interpretation of Hazard Model Parameter Estimates in Table 5

The coefficient estimates in a proportional hazard model represent a proportional shift of the baseline hazard function over the domain of the function (duration, cost, or area). The wildfire characterized by the baseline hazard function is described in Table 6. A negative sign on a covariate coefficient indicates that an increase in the associated variable leads to a downward shift in the hazard function. A downward shift in the hazard function implies a lower conditional probability of fire termination at any point in time (cost, or area). A lower probability of event occurrence leads to a longer expected fire duration (higher final costs, or larger final fire size). For example, an increase in the distance between a fire's point of ignition and the nearest town (`ln(Distance)`) leads to a reduction (-0.135) in the baseline area hazard function. This lower hazard function, representing the probability that the fire is contained, implies that the expected fire size is larger.

Table 5: Parameter estimates of jointly estimated trivariate hazard model.

	Duration		Cost		Area	
	$\boldsymbol{\beta}_t$	S.E.	$\boldsymbol{\beta}_c$	S.E.	$\boldsymbol{\beta}_a$	S.E.
`Threatened Commercial`	3.418***	(0.979)	1.743	(1.703)	-1.361	(3.966)
`Threatened OutBuildings`	-0.312	(0.541)	-1.542**	(0.761)	-3.003***	(0.997)
`Threatened Residential`	-0.822**	(0.319)	-2.007***	(0.385)	-2.312***	(0.460)
`Injuries`	0.098***	(0.038)	-0.137***	(0.051)	-0.034	(0.045)
`Fatalities`	-0.237***	(0.068)	-0.298***	(0.069)	-0.263***	(0.076)
`Potential Evacuation`	-0.792***	(0.079)	-1.460***	(0.071)	-1.613***	(0.087)
`Wind`	0.294	(0.290)	-0.115	(0.345)	-1.739***	(0.369)
`Temperature`	2.065***	(0.234)	0.837***	(0.161)	0.106	(0.334)
`Relative Humidity`	0.806***	(0.113)	1.151***	(0.121)	1.343***	(0.160)
`Resource Scarcity`	-0.255	(0.359)	-0.374*	(0.208)	-1.259***	(0.331)
`Latitude`	1.043	(0.904)	-2.187	(1.457)	-0.932*	(0.492)
`Day of Year (Sin)`	0.073*	(0.039)	0.138***	(0.048)	-0.018	(0.039)
`Day of Year (Cos)`	0.440***	(0.068)	0.481***	(0.058)	0.044	(0.064)
`Cause Lightning`	-0.740***	(0.055)	-0.206***	(0.057)	-0.496***	(0.053)
`Cause Unknown`	0.063	(0.055)	-0.025	(0.059)	0.034	(0.059)
`Year`	-0.042***	(0.010)	-0.028***	(0.009)	-0.024**	(0.010)
`FS Region 6 (Pacific Northwest)`	0.114	(0.138)	-0.741***	(0.153)	-0.155	(0.117)
`FS Region 5 (Pacific Southwest)`	0.691***	(0.089)	-0.824***	(0.090)	0.093	(0.092)
`FS Region 1 (North)`	-0.004	(0.139)	-0.431***	(0.155)	-0.113	(0.111)
`FS Region 2 (Rocky Mountain)`	0.522***	(0.134)	-0.449***	(0.123)	-0.058	(0.128)
`FS Region 4 (Intermountain)`	0.559***	(0.104)	-0.236**	(0.106)	-0.183*	(0.099)
`FS Region 9 (Eastern)`	0.449***	(0.118)	0.044	(0.148)	0.224**	(0.112)
`FS Region 3 (Southwest)`	0.400***	(0.090)	-0.461***	(0.094)	-0.317***	(0.097)
`FS Region 10 (Alaska)`	-0.024	(0.244)	-0.395	(0.528)	-1.238***	(0.241)
`ln(Distance)`	-0.100***	(0.026)	0.038	(0.024)	-0.135***	(0.025)
`Value20`	0.000	(0.001)	-0.001	(0.001)	0.001	(0.001)
Ancillary Parameters						
ς `(shape)`	1.263***	(0.017)	0.527***	(0.007)	0.624***	(0.008)
$\beta_0 = \varsigma_k \log(\lambda_k)$	-4.925***	(0.428)	-2.433***	(0.574)	0.378	(0.446)
Elements of the Cholesky Triangle						
Duration	0.825***	(0.038)	0		0	
Cost	0.790***	(0.048)	-0.063	(0.081)	0	
Area	0.931***	(0.049)	-0.082	(0.093)	0.009	(0.058)

Observations = 10,321; Number of Wildfires (n) = 3,829; Completed Fires = 3,398
$\ln L = -43,546$; Likelihood Ratio Statistic = 5,214 $\sim \chi^2_{84}$

* $p < 0.1$, **$p < 0.05$, ***$p < 0.001$

Table 6: Baseline and Median covariate values.

Continuous Covariates	Baseline	Median
Threatened Residential	0	0
Threatened Commercial	0	0
Threatened Outbuildings	0	0
Injuries	0	0
Fatalities	0	0
Wind (mph)	0	5
Temperature (°F)	0	79
Relative Humidity	0	0
Resource Scarcity	0	0
Latitude (0 = equator)	0	41°
Day of Year (Sin)	0	-.26
Day of Year (Cos)	0	-.89
Year	0 (2001)	5 (2006)
Distance	0	2.48
Value20	0	< 0.0001
Categorical Covariates		
Cause	Human	
Forest Service Region	8 (South)	