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Entry Deterrence in the Commons with Multiple Incumbents

By

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Abstract

We examine an entry-deterrence model with multiple incumbents who strategically increase their individual appropriation in order to prevent entry. We find that, as the number of incumbents increases entry deterrence can only be supported if the resource is abundant. Additionally, we show that entry deterrence yields a welfare improvement, relative to contexts of unthreatened entry, if few firms exploit a moderately abundant resource. When several firms compete, however, welfare losses can arise. Consequently, the presence of several incumbents recommends the implementation of policies that hinder entry under larger conditions than when a single incumbent exploits the commons.

KEYWORDS: Entry Deterrence; Multiple Incumbents; Common Pool Resources.
JEL CLASSIFICATION: L12, Q20, D62.

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1 Introduction

Common pool resources (CPRs) are often exploited by several firms. Examples include fishing grounds, forests and oil and gas reservoirs. In the forest industry, for instance, Associated Oregon Loggers reports that 65 logging companies currently operate in the Willamina/Tillamook region.[1] In the case of fishing grounds, 300 boats have been fishing for tuna in the nearby Indian islands of Agatti, Suheli, Minicoy, Bitra and Androth for the past 15 years.[2] Oil drilling also exhibits examples of several companies exploiting the same area. For instance, the U.S. Energy Information Administration[3] reports that 26 companies operate in the Houston and San Antonio areas in Texas. These firms usually face entry threats from potential entrants who also seek to benefit from the exploitation of the resource. Mason and Polasky (1994) examine entry deterrence in contexts where a single incumbent initially operates in the CPR. Most commons are, however, exploited by multiple firms and, as this paper demonstrates, the presence of several incumbents entails different equilibrium results and policy implications compared with CPRs with a single incumbent.

This paper considers a two period entry-deterrence game in which several incumbents independently select their initial appropriation of the resource in order to prevent entry. Our model assumes that second-period costs increase in aggregate first-period appropriation (reflecting that first-period appropriation makes future exploitation more difficult), and that the commons partially regenerate over time. We first show that three different equilibria can be sustained in which: (1) entry is blockaded when the CPR is scarce, and incumbents behave as if entry threats were absent; (2) incumbents deter entry by increasing their first-period appropriation (relative to no entry threats) if the commons are moderately abundant; and (3) firms allow entry when the CPR is significantly abundant. Our results demonstrate that, as the number of incumbents increases, the entry-deterring equilibrium can only be supported if the commons become very abundant. We furthermore identify a critical number of firms above which blockaded entry is the only equilibrium prediction under all stock levels. Summarizing, a larger number of incumbents expands the region of parameter values under which blockaded entry exists, thus shrinking the area in which entry deterrence can be sustained. In contrast, a faster regeneration of the resource shrinks the set of parameter conditions for which blockaded entry can be supported.

In order to evaluate the welfare consequences of the incumbents' entry-deterring behavior, we first examine whether their increase in first-period appropriation yields an overexploitation of the resource relative to the social optimum. We show that the CPR is underexploited when the resource is relatively scarce and few incumbents operate.[4] Firms' appropriation levels are, nonetheless,

[1] Similarly, more than 80 sawmill companies exploit forests mainly in central Sweden and some portions of the Norrland region, as reported by the Swedish Forest Industries Association (Skogsindustrierna).

[2] See TNAU Agricultural University, http://agritech.tnau.ac.in/fishery/fish_fishing_methods.html.

[3] Likewise, BP and ConocoPhillips exploit the Alaskan Prudhoe Bay oil field, with their subsidiaries BP Exploration Alaska Inc. and ConocoPhillips Alaska Inc. In addition, ExxonMobil and Chevron operate in the Alaskan North Pole oil field (with their subsidiaries Flint Hills Resources Alaska LLC and Petro Star Inc, respectively); as documented by the Resource Development Council for Alaska, Inc. (http://www.akrdc.org/issues/oilgas/overview.html.)

[4] This underexploitation is observed in different fishing grounds. In particular, as reported by the United Nations Food and Agriculture Organization (2005), the incumbent fleet exploiting the silver hake in the North Atlantic (a

higher than when entry threats are absent, thus entailing a welfare improvement. In contrast, the commons is overexploited when the resource is abundant and several incumbents operate.[5] Such overexploitation, however, is not necessarily welfare reducing. In particular, we identify a critical level of the stock below which overexploitation is relatively small and, hence, social welfare improves. If, in contrast, the stock is above such a critical level and the number of firms is large, overexploitation is significant, thus becoming welfare reducing. Hence, relative to settings in which a single incumbent operates, the introduction of several firms that practice entry deterrence entails an unambiguous welfare loss. Our results are further emphasized when appropriation generates biodiversity losses, i.e., environmental damage, since in these settings overexploitation is more likely to become welfare reducing.

Different policy implications can be drawn from our results, depending on the number of incumbents exploiting the commons and the abundance of the resource. Specifically, if few firms operate in a relatively scarce CPR, regulators should promote entry threats (e.g., by lowering entry costs) since firms' entry-deterring practices can yield welfare gains under relatively large parameter conditions. By contrast, when several firms exploit a very abundant commons, incumbents' entry-deterring behavior yields a significant overexploitation, thus generating a welfare loss. In this setting, regulators should hinder entry threats by increasing fixed entry costs, e.g., more costly fishing permits, higher administrative costs, etc.

Our equilibrium results also suggest a policy implication that may go against common belief. In particular, policies that protect the CPR from further entry are unnecessary when the resource is scarce and few incumbents exploit the commons, since in this context their entry-deterring practices can be welfare improving. In contrast, our results imply that the regulator should be especially protective with the resource (e.g., by raising entry costs) when the CPR is abundant, since in these settings the incumbents' entry-deterring behavior yields a significant overexploitation of the commons, inducing welfare losses under large parameter conditions.

Related literature. Since the seminal work of Hardin (1968), several studies have analyzed the overexploitation of the commons in contexts in which a given set of firms are unthreatened by potential entry; for a detailed review of the literature, see Ostrom (1990), Ostrom et al. (1994) and Faysee (2005).[6] Under the threat of entry, such overexploitation can be emphasized given the incumbent's entry-deterring behavior, as shown by Mason and Polasky (1994) for the case of a single incumbent exploiting the CPR. Our paper contributes to this line of literature by examining equilibrium appropriation levels in commons where multiple incumbents initially operate. If few

relatively scarce resource) has consistently underexploited it below its annual sustainable catch since the late 1990s. This organization also found the underexploitation of other CPRs, such as the Argentine anchovy in the Southern Atlantic and the yellowfin sole in the Pacific Northwest.

[5] Even in fishing grounds that are particularly abundant, overexploitation is commonly observed. For instance, the United Nations Food and Agriculture Organization (2005) identifies such overexploitation for the blue whiting in the Northeast Atlantic, the Chilean jack mackerel in the Southeast Pacific and the Peruvian anchovy in the Southeast Pacific.

[6] Other studies, such as Cornes et al. (1986), Mason et al. (1988) and Mason and Polasky (1997), analyze overexploitation, and examine the optimal number of firms under different settings. Our model, in contrast, allows incumbents to strategically increase appropriation in order to deter entry.

incumbents compete, we demonstrate that the overexploitation result identified by Mason and Polasky (1994) can be sustained if the resource is moderately abundant. In contrast, when several firms operate in the CPR, such overexploitation can only be supported if the resource becomes very abundant. Otherwise, the presence of several firms blockades entry.

Our model also connects to entry-deterrence games with multiple incumbents, as in Gilbert and Vives (1986). Unlike this paper, however, we consider a CPR which partially regenerates over time, and hence, second-period costs are negatively affected by first-period output decisions.[7] Finally, a recent line of studies investigates entry-deterring practices under incomplete information settings (Polasky and Bin, 2001; Espinola-Arredondo and Munoz-Garcia, 2011), whereby potential entrants are uninformed about the level of the stock while the single incumbent is informed.[8] Nonetheless, the presence of several firms helps disseminate information about the stock and, hence, CPRs with multiple incumbents can be more appropriately analyzed under contexts of complete information, as in our paper.

The following section describes the model. Sections three and four examine equilibrium appropriation in the second and first-period, respectively. Section five analyzes under which conditions the commons are overexploited, while section six identifies when such an overexploitation entails welfare gains. Finally, section seven concludes.

2 Model

Consider a common pool resource (CPR) initially exploited by a set of N incumbents. A potential entrant decides whether or not to exploit the CPR given an entry cost, $F > 0$, which represents the investment required to start exploiting the commons. The initial stock of the CPR is represented by $\theta \in [0, 1]$, which is perfectly observed by all players in the game. In particular, we study a two-stage complete-information game with the following time structure:

1. In the first stage, every incumbent independently chooses its appropriation level $x_i > 0$.
2. Given the first-period aggregate appropriation $X \equiv \sum_{i=1}^{N} x_i$, the potential entrant chooses whether or not to join the CPR.
   (a) If entry does not occur, the N incumbents independently and simultaneously choose their second-period appropriation levels $q_1, ..., q_N$, while the entrant's profits from staying out are normalized to zero.

[7] Importantly, Gilbert and Vives (1986) considers that incumbent firms commit to a given production level during the first period of the game, which is sold in the second period once the potential entrant has decided whether or not to enter. The potential entrant hence might be deterred if the incumbents' aggregate production is relatively large, leaving an insufficient market share to the entrant. Unlike incumbent firms in our paper who produce in both time periods at different costs, Gilbert and Vives (1986) assume that incumbents make a single production decision during the first period.

[8] Other authors have theoretically and experimentally analyzed uncertainty regarding the profitability of the CPR; see Suleiman and Rapoport (1988), Suleiman et al. (1996) and Apesteguia (2006). Unlike Polasky and Bin (2001) and Espinola-Arredondo and Munoz-Garcia (2011), this literature does not allow for informational asymmetries among players.

(b) If entry ensues, agents compete for the CPR, selecting their appropriation levels $q_1, ..., q_N, q_{N+1}$, for the N incumbents and the entrant.

In the first stage of the game, incumbent $i \in N$ appropriates x_i, with an associated total cost of $c(x_i, \theta) = (1-\theta)x_i$, which is increasing in first-period appropriation, x_i, and decreasing in the available stock, θ. Intuitively, a more abundant stock facilitates the exploitation of the resource. The inverse demand curve is $p(X) = 1 - X$.

In the second-period, appropriation costs are affected by first period appropriation as well as the regeneration rate of the CPR. In particular, firm i's second-period costs are given by $c(q_i, X, \theta, \beta) = [1 - (\theta - (1-\beta) X)] q_i$, which increase in firm i's second-period appropriation level, q_i, and in aggregate first-period appropriation, X. In contrast, costs decrease in the initial stock, θ, and in the regeneration rate of the CPR, $\beta \in (0,1)$. Specifically, a larger appropriation during previous periods, X, depletes the resource, thus increasing second-period costs. Furthermore, such an increase becomes more severe when the resource does not regenerate across periods, i.e., $\beta \to 0$. However, if the CPR completely regenerates $\beta \to 1$ (i.e., biological regeneration offsets first-period appropriation), then X does not affect second-period costs, i.e., costs converge to $(1-\theta)q_i$. Finally, consider that, similar to the first-period game, firms face inverse demand curve $p(Q) = 1 - Q$, where Q denotes aggregate second-period appropriation. We next solve the game operating by backward induction.

3 Second period

No entry. During the second period, if entry does not occur, the same set of N incumbents exploit the resource, each of them appropriating $q_i > 0$. Therefore, each firm solves

$$\max_{q_i > 0} \pi_i(q_i, Q_{-i}) \equiv (1 - q_i - Q_{-i})q_i - [1 - (\theta - (1-\beta) X)] q_i, \tag{1}$$

where $Q_{-i} = \sum_{j \neq i} q_j$, by selecting best response function $q_i(Q_{-i}) = \frac{\theta-(1-\beta)X}{2} - \frac{Q_{-i}}{2}$, entailing an individual equilibrium appropriation of $q_i^{NE}(X) = \frac{\theta-(1-\beta)X}{N+1}$, where NE denotes no entry. The aggregate equilibrium appropriation is thus $Q^{NE}(X) = Nq_i^{NE}(X)$, which is increasing in the level of the stock, θ, and the regeneration rate, β, but decreasing in first-period appropriation, X. Let $\overline{\pi}_i^{NE}(X)$ denote incumbent i's second-period equilibrium profits, where $\overline{\pi}_i^{NE}(X) = \left(\frac{\theta-(1-\beta)X}{N+1}\right)^2$.

Entry. If entry occurs in the second period, $N+1$ firms (N incumbents and the entrant) compete for the common resource. Every individual firm selects the level of q_i that solves (1) by choosing $q_i^E(X) = \frac{\theta-(1-\beta)X}{N+2}$, implying an aggregate appropriation of $Q^E(X) = (N+1)q_i^E(X)$, where superscript E denotes entry.[9] Equilibrium profits are therefore $\overline{\pi}_i^E(X) = \left(\frac{\theta-(1-\beta)X}{N+2}\right)^2$, which lie below those when entry does not occur, $\overline{\pi}_i^{NE}(X)$, for all X. Note that profits $\pi_{ent}^E(X)$ are

[9] Note that the incumbent considers that Q_{-i} captures the second-period appropriation from the other $N-1$ incumbents and that of the entrant.

decreasing in first-period appropriation, X, thus making entry unprofitable if X is sufficiently high; as figure 1 depicts. Intuitively, the resource becomes so depleted that the potential entrant's profits from exploiting the CPR do not offset its entry costs, F. Specifically, the entrant's second-period profits satisfy $\overline{\pi}^{E}_{ent}(X) \leq F$ for all $X \geq X_{ED}$, where $X_{ED} \equiv \frac{\theta - \sqrt{F}(N+2)}{1-\beta}$. For compactness, we refer to X_{ED} as the "entry-deterring appropriation level."

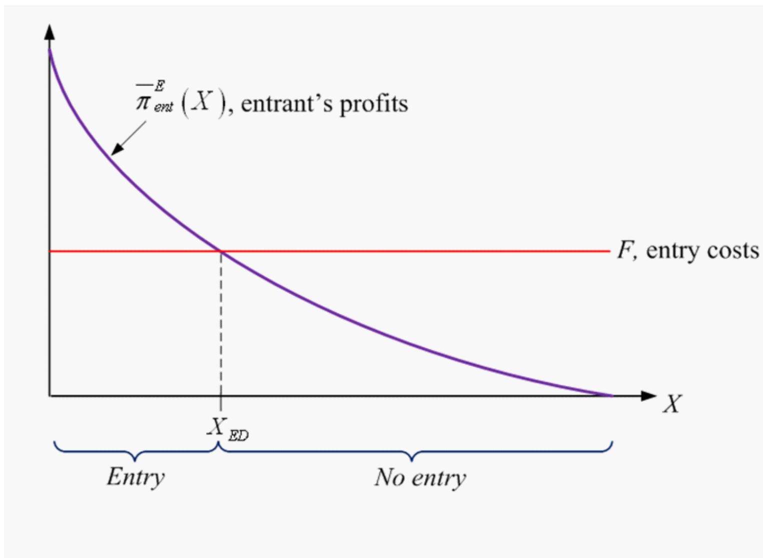


Fig 1. Entry-deterring appropriation, X_{ED}.

4 First period

In the first stage of the game, given an aggregate appropriation from all firms $j \neq i$, X_{-i}, firm i must select its individual appropriation x_i taking into account that its choice might deter entry. In particular, three cases emerge depending on X_{-i}: (1) if all other incumbents' appropriation satisfies $X_{-i} > X_{ED}$, then entry is blockaded; (2) if $X_{-i} \leq X_{ED}$ and firm i's exploitation of the resource reaches the entry-deterring appropriation level, i.e., $x_i = X_{ED} - X_{-i}$, then entry is successfully deterred; and (3) if $X_{-i} \leq X_{ED}$ but firm i's exploitation is insufficient to deter entry, i.e., $x_i < X_{ED} - X_{-i}$, entry is allowed. We next separately analyze each of these cases.

Blockaded entry. When other incumbents' appropriation satisfies $X_{-i} > X_{ED}$ entry is blockaded, and every firm i selects x_i in order to maximize its overall discounted profits

$$\max_{x_i > 0} \pi_i^{NE}(x_i; X_{-i}) = (1 - x_i - X_{-i})x_i - (1-\theta)x_i + \delta\overline{\pi}_i^{NE}(X), \tag{2}$$

where $X \equiv x_i + X_{-i}$, $\overline{\pi}_i^{NE}(X)$ represents incumbent i's second-period profits, and $\delta \in [0, 1]$ denotes its discount factor. The above maximization problem hence yields the best response function

$$x_i^{NE}(X_{-i}) = \frac{\left[(1+N)^2 - 2(1-\beta)\delta\right]\theta}{2(1+N)^2 - 2(\beta-1)^2\delta} - \frac{\left[(1+N)^2 - 2(\beta-1)^2\delta\right]}{2(1+N)^2 - 2(\beta-1)^2\delta} X_{-i} \tag{3}$$

entailing overall profits of $\widehat{\pi}_i^{BE}(X_{-i}) \equiv \pi_i^{NE}(x_i^{NE}(X_{-i}); X_{-i})$, where BE reflects blockaded entry.[10] Similarly, when $X_{-i} \leq X_{ED}$ and firm i produces according to $x_i^{NE}(X_{-i})$ such that $X_{-i} + x_i^{NE}(X_{-i}) \geq X_{ED}$, then entry is blockaded and firm i produces ignoring the threat of entry. In particular, let $\overline{X}_{-i}(X_{ED})$ denote the appropriation level from all firms $j \neq i$, X_{-i}, that solves $X_{-i} + x_i^{NE}(X_{-i}) = X_{ED}$.

Deterred entry. When $X_{-i} \leq X_{ED}$ and firm i deters entry by reaching the entry-deterring appropriation level, i.e., $x_i = X_{ED} - X_{-i}$, its overall profits are $\widehat{\pi}_i^{ED}(X_{-i}) \equiv \pi_i^{NE}(X_{ED} - X_{-i}; X_{-i})$.

Allowed entry. Finally, when other incumbents' appropriation is still relatively low, $X_{-i} \leq X_{ED}$, but firm i's exploitation of the CPR allows entry, i.e., $x_i < X_{ED} - X_{-i}$, its maximization problem becomes

$$\max_{x_i > 0} \pi_i^E(x_i; X_{-i}) = (1 - x_i - X_{-i})x_i - (1-\theta)x_i + \delta\overline{\pi}_i^E(X), \tag{4}$$

which yields the best response function

$$x_i^{E}(X_{-i}) = \frac{\left[(2+N)^2 - 2(1-\beta)\delta\right]\theta}{2(2+N)^2 - 2(\beta-1)^2\delta} - \frac{\left[(2+N)^2 - 2(\beta-1)^2\delta\right]}{2(2+N)^2 - 2(\beta-1)^2\delta} X_{-i} \tag{5}$$

ultimately entailing overall profits of $\widehat{\pi}_i^{AE}(X_{-i}) \equiv \pi_i^{E}(x_i^{E}(X_{-i}); X_{-i})$, where AE represents that entry is allowed.[11] In addition, let $\underline{X}_{-i}(X_{ED})$ solve $\widehat{\pi}_i^{ED}(X_{-i}) = \widehat{\pi}_i^{AE}(X_{-i})$, i.e., the level of X_{-i} that makes firm i indifferent between deterring and allowing entry. The following lemma identifies the precise value of cutoffs $\underline{X}_{-i}(X_{ED})$ and $\overline{X}_{-i}(X_{ED})$.

Lemma 1. *When the commons are initially exploited by N incumbents, every firm $i \in N$ is indifferent between deterring and allowing entry when $X_{-i} = \underline{X}_{-i}(X_{ED})$. In addition, for any $X_{-i} \geq \overline{X}_{-i}(X_{ED})$ incumbents produce according to $x_i^{NE}(X_{-i})$, and entry is blockaded, where*

$$\overline{X}_{-i}(X_{ED}) = \frac{2\left[(1+N)^2 - (\beta-1)^2\delta\right]}{(1+N)^2} X_{ED} - \frac{\theta\left[(1+N)^2 + 2(\beta-1)^2\delta\right]}{(1+N)^2}, \text{ and}$$

$$\underline{X}_{-i}(X_{ED}) = \frac{2AX_{ED} + 2\sqrt{B}[(\beta-1)X_{ED} + \theta]^2 - (1+N)^2\left[(2+N)^2 + 2(\beta-1)\delta\right]\theta}{(2+3N+N^2)^2},$$

where $A \equiv (2+3N+N^2)^2 - (\beta-1)^2(N-1)^2\delta$ *and* $B \equiv (1+N)^2(3+2N)\delta[(2+N)^2 - (\beta-1)^2\delta]$.

[10] Note that the slope of firm i's best response function is negative but larger than -1 for all parameter values.

[11] It is straightforward to show that the slope of best response function $x_i^E(X_{-i})$ is larger in absolute value than that of $x_i^{NE}(X_{-i})$, indicating that a given increase in X_{-i} leads firm i to more significantly reduce its first-period appropriation when entry ensues than when it does not.

Figure 2 summarizes our results about firm i's strategic appropriation decision, by comparing its profits from deterring entry, $\widehat{\pi}_i^{ED}(X_{-i})$, with those from allowing entry, $\widehat{\pi}_i^{AE}(X_{-i})$. In particular, the figure identifies three regions for firm i's production: Region I, where other firms' appropriation, X_{-i}, is so low that firm i would need to significantly increase its first-period exploitation in order to deter entry. Such increase in production, however, is too costly for firm i since $\widehat{\pi}^{ED}(X_{-i}) < \widehat{\pi}^{AE}(X_{-i})$, leading this firm to produce according to best response function $x_i^E(X_{-i})$ for all $X_{-i} < \underline{X}_{-i}(X_{ED})$, and thus allow entry. In Region II, X_{-i} is relatively higher, inducing firm i to deter entry by appropriating $x_i = X_{ED} - X_{-i}$, since profits satisfy $\widehat{\pi}^{ED}(X_{-i}) \geq \widehat{\pi}^{AE}(X_{-i})$ for all $\overline{X}_{-i}(X_{ED}) > X_{-i} \geq \underline{X}_{-i}(X_{ED})$. Finally, in Region III, X_{-i} is sufficiently large to allow firm i to behave as if entry threats were absent, i.e., incumbent i uses best response function $x_i^{NE}(X_{-i})$ for all $X_{-i} \geq \overline{X}_{-i}(X_{ED})$ and entry is blockaded.

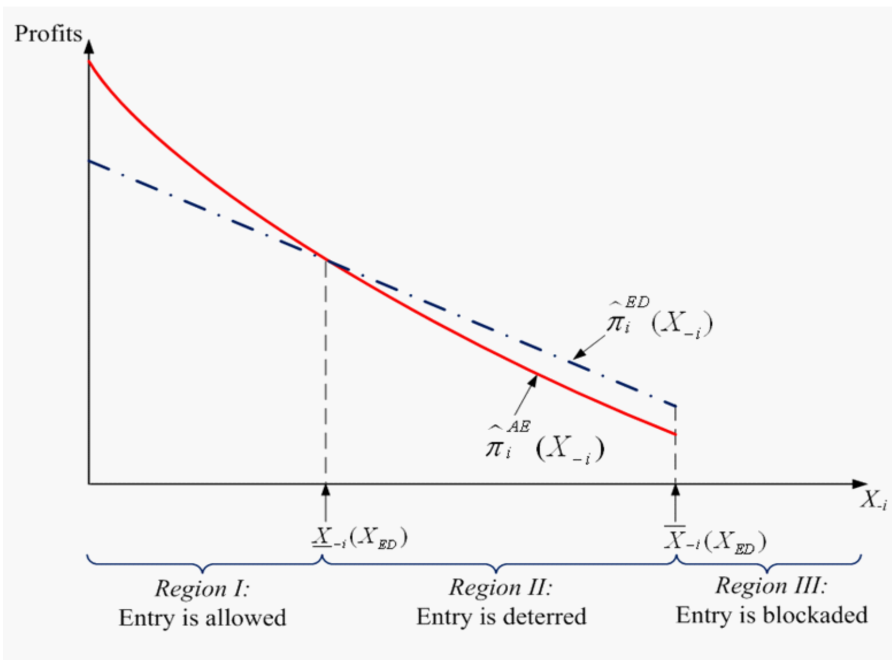


Fig 2. Incumbent i's first-period appropriation decision.

Given the above best response function for every incumbent $i \in N$, the following proposition describes the subgame perfect equilibrium (SPE) of the entry game. For compactness, let x_i^{NE} denote the equilibrium appropriation level emerging from all N incumbents behaving according to best response function $x_i^{NE}(X_{-i})$, i.e., $x_i^{NE} = \frac{\left[(1+N)+2(\beta-1)^2\delta\right]\theta}{(1+N)^3-2(\beta-1)^2 N\delta}$, entailing an aggregate output $X^{NE} \equiv \sum_{i=1}^{N} x_i^{NE}$. Similarly, let x_i^E represent first-period appropriation when incumbents use best response function $x_i^E(X_{-i})$, i.e., $x_i^E = \frac{\left[(2+N)^2+2(\beta-1)^2\delta\right]\theta}{(1+N)(2+N)^2-2(\beta-1)^2 N\delta}$, which yields an aggregate output of $X^E \equiv \sum_{i=1}^{N} x_i^E$.

Proposition 1. *In the entry game where N incumbents exploit the commons, there exists a unique SPE, depending on the precise value of the entry-deterring output, X_{ED}:*

1. *If $X_{ED} \leq X^{NE}$, each incumbent produces x_i^{NE} (entailing an aggregate output of X^{NE}), and entry is blockaded;*
2. *If $X^{NE} < X_{ED} \leq \overline{X}_{ED}$, any output profile $x \in \mathbb{R}_+^N$ that yields an aggregate output equal to the entry-deterring output, i.e., $\sum_{i=1}^N x_i = X_{ED}$, where every individual production level x_i satisfies $X_{ED} - \underline{X}_{-i}(X_{ED}) \geq x_i \geq x_i^{NE}(X_{-i})$, is an equilibrium in which entry is deterred;*
3. *If $X_{ED} \geq \overline{X}_{ED}$, each incumbent produces x_i^E (entailing an aggregate output of X^E), and entry is allowed;*

where $\overline{X}_{ED}$ solves $\underline{X}_{-i}(X_{ED}) = \frac{(N-1)X_{ED}}{N}$, i.e., $\overline{X}_{ED}$ denotes the largest value of X_{ED} for which deterring entry is an equilibrium.

Intuitively, the above proposition identifies equilibrium behavior in three different regions, depending on the value of the appropriation level that all N incumbents must reach in order to prevent entry, X_{ED}. First, when X_{ED} is relatively low, i.e., $X_{ED} \leq X^{NE}$, firms can blockade entry by just selecting their output levels as if entry threats were absent. Second, if X_{ED} lies between X^{NE} and $\overline{X}_{ED}$, then entry is successfully deterred. Specifically, in this entry-deterring equilibrium every individual incumbent must produce more than when entry is blockaded, i.e., $x_i \geq x_i^{NE}(X_{-i})$, but does not produce more than what is strictly necessary to reach the entry-deterring output X_{ED}. Finally, when X_{ED} is sufficiently large, practicing entry deterrence becomes too costly, and incumbents allow entry. The following subsection analyzes how the regions supporting these three equilibrium outcomes are affected by changes in the number of incumbents, N, the regeneration rate, β, and entry costs, F.

4.1 Comparative statics

Number of incumbents. Let us first examine how entry is affected by the introduction of more incumbents exploiting the CPR. The next corollary shows that the equilibrium where entry is allowed shrinks as we increase the number of incumbents.

Corollary 1. *The region of parameter values sustaining blockaded entry (allowed entry) expands (shrinks, respectively) as the number of incumbents increases.*

For comparison purposes, the next figure illustrates the case in which a single incumbent exploits the commons; as in Mason and Polasky (1994).[12] When the level of the stock, θ, is relatively scarce,

[12] To facilitate the comparisons and graphical representation of our results, we consider $\beta = 0.8$ and no discounting in future payoffs. In addition, this combination of parameters implies that the entrant's profits become $\overline{\pi}_{ent}^E(X) = \frac{\theta - 0.2X}{49}$, thus entailing that $\overline{\pi}_{ent}^E(X)$ originates at $\overline{\pi}_{ent}^E(0) = \frac{\theta}{49}$ and monotonically decreases in X (see figure 1). Hence, entry deterrence is only possible if entry costs satisfy $F < \overline{\pi}_{ent}^E(0)$, e.g., $F = 0.0204$. Different parameter values yield similar results, and can be provided by the authors upon request.

i.e., $\theta \leq \theta_1$ in figure 3, X_{ED} satisfies $X_{ED} \leq X^{NE}$ and entry is blockaded. However, when the stock becomes more abundant, i.e., $\theta_1 < \theta \leq \theta_3$, the unique incumbent increases its exploitation level to reach X_{ED}, and successfully deters entry. Finally, when the stock is relatively high, i.e., $\theta > \theta_3$, X_{ED} lies above $\overline{X}_{ED}$. In this setting, the incumbent would need to significantly raise its first-period appropriation level in order to prevent entry, which is not profitable, and thus acquiesces entry.

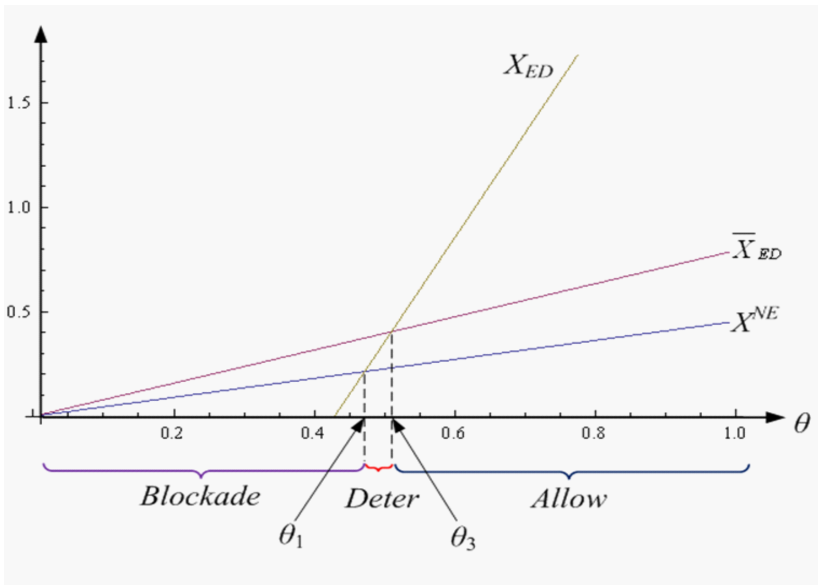


Fig 3. Equilibrium with $N = 1$ incumbent.

When we allow for $N = 2$ incumbents (figure 4a), the region where entry is blockaded expands. As a consequence, for entry deterrence to be profitable the stock must be more abundant when $N = 2$ than when $N = 1$. In addition, the set of parameter values under which entry is allowed shrinks. Intuitively, when the stock is relatively scarce and two firms exploit the resource, entry becomes unprofitable and incumbents do not need to overexploit the commons in order to deter entry (blockaded entry expands in figure 4a). This tendency is further emphasized when more incumbents exploit the commons. As figure 4b depicts, when four firms exploit the CPR, X_{ED} lies below X^{NE} (and entry is thus blockaded) for all levels of the stock. Hence, there exists a minimal number of firms N_{ED} for which the entry-deterring equilibrium cannot be supported where, in particular, N solves $\theta_1 = 1$, and hence blockaded entry becomes the unique equilibrium prediction for all $N > N_{ED}$.

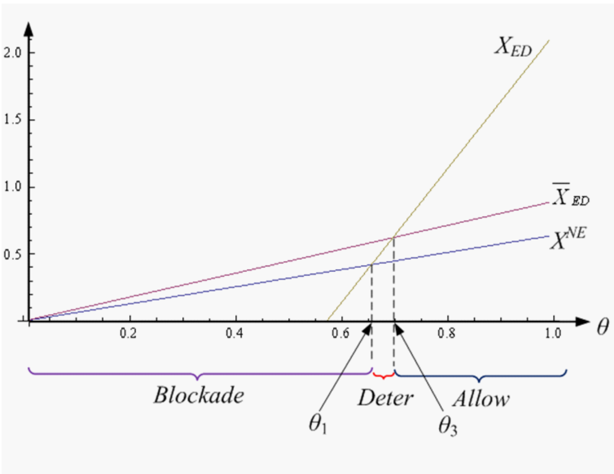


Fig 4a. $N = 2$ incumbents.

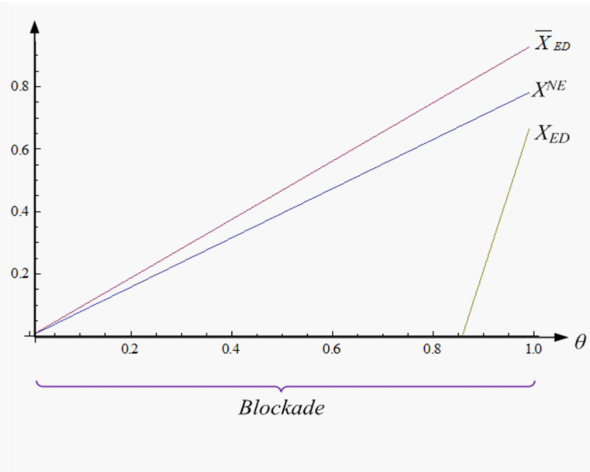


Fig 4b. $N = 4$ incumbents.

Our results hence provide a direct comparison with those in Mason and Polasky (1994). In particular, the introduction of additional incumbents implies that the overexploitation of the resource needed to deter entry can only be sustained for more stringent parameter conditions. Indeed, when the number of incumbents is sufficiently large, the entry-deterring outcome reported in Mason and Polasky (1994) cannot be sustained under any parameter values. As a consequence, ignoring the number of incumbents operating in the commons entails completely different equilibrium predictions.

Regeneration rate. When the stock fully regenerates across periods, $\beta \to 1$, the incumbents' costs remain constant over time, and hence their production decisions resemble those of firms producing goods whose current costs are not affected by their production history; similarly as in the entry deterrence game examined by Gilbert and Vives (1986). In contrast, if the regeneration rate decreases, the CPR becomes less attractive, and equilibria where entry is blockaded or deterred emerge under larger parameter conditions. The next figure depicts the case in which β decreases from $\beta = 0.8$ (as in figure 4a) to $\beta = 0.5$, thus expanding the region where entry is blockaded.

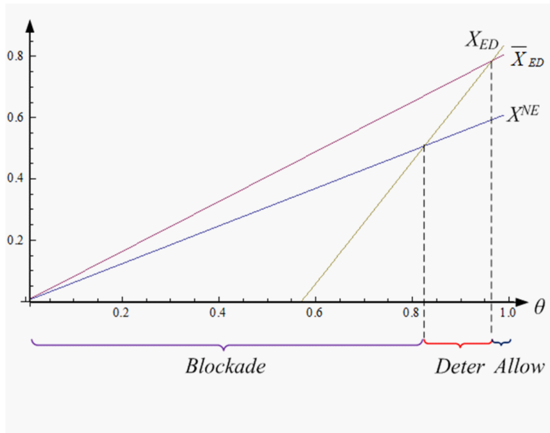
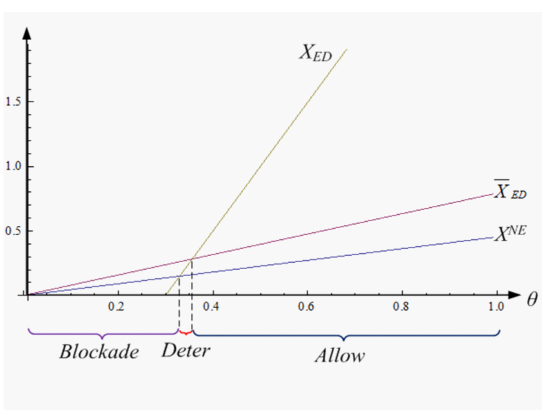


Fig 5. $N = 2$ incumbents when β decreases.

Entry costs. Finally, in order to examine the effect of a decrease in entry costs, figure 6a represents equilibrium outcomes when $F = 0.01$, as opposed to figure 3 where $F = 0.0204$. A comparison of figures 3 and 6a shows that a reduction in entry costs makes entry more attractive, implying that the region of stocks where entry is allowed (blockaded) expands (shrinks, respectively). When more incumbents exploit the commons, the region where entry is allowed narrows, but it is still present (figure 6b), unlike the case where entry costs are high (figure 4b). Nonetheless, it can be shown that if the number of incumbents is sufficiently high, in particular if $N \geq N_{ED} = 7$, only blockaded entry can be supported.[13]



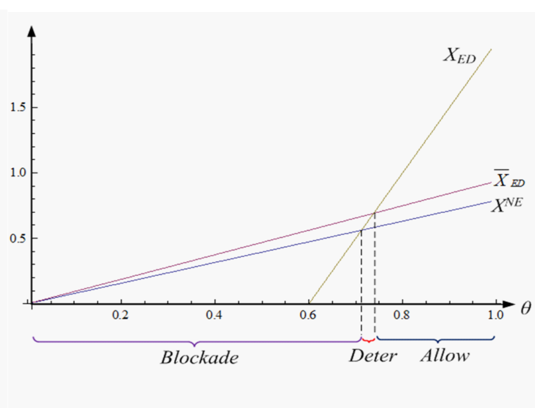


Fig 6a. $N = 1$ incumbent when F decreases. Fig 6b. $N = 4$ incumbents when F decreases.

[13] Note that if agents assign a low value to future payoffs, e.g., $\delta \to 0$, then incumbents are not willing to practice entry deterrence under any value of θ, and firms behave as when entry threats are absent. (In particular, cutoffs θ_1 and θ_3 in the proof of Proposition 1 coincide when $\delta \to 0$, thus nullifying the region of θ that supports entry deterrence).

Let us finally generalize our previous findings by analyzing under which conditions blockaded entry can be supported as the unique equilibrium of the game. In particular, define θ_1 as the lower bound of the entry-deterring equilibrium, i.e., the level of the stock that solves $X_{ED} = X^{NE}$.

Corollary 2. *Cutoff θ_1 increases in the number of incumbents, N. In addition, it experiences an upward (downward) shift in entry costs, F (in the regeneration rate, β).*

Therefore, since entry is blockaded for the range of θ in which $\theta \leq \theta_1$, an increase in the number of incumbents enlarges such range; as the shaded area in figure 7a illustrates. In addition, recall that N_{ED} solves $\theta_1 = 1$. Hence, for all $N \geq N_{ED}$ cutoff $\theta_1 \geq 1$, and the only equilibrium outcome that can be sustained is that where entry is blockaded. Moreover, this minimal number of firms N_{ED} decreases in F, since entry becomes less attractive for potential entrants, thus expanding the shaded region where entry is blockaded in equilibrium (see figure 7b). In contrast, an increase in β shifts cutoff N_{ED} rightward, thus shrinking the shaded region of blockaded entry. Intuitively, as the resource is more replenished in the second-period game, entry becomes more attractive, and hence blockaded entry can only be sustained under more restrictive conditions.

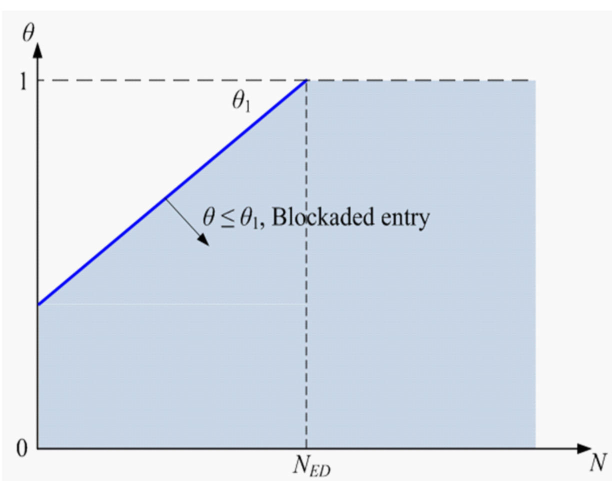


Fig 7a. Region of blockaded entry.

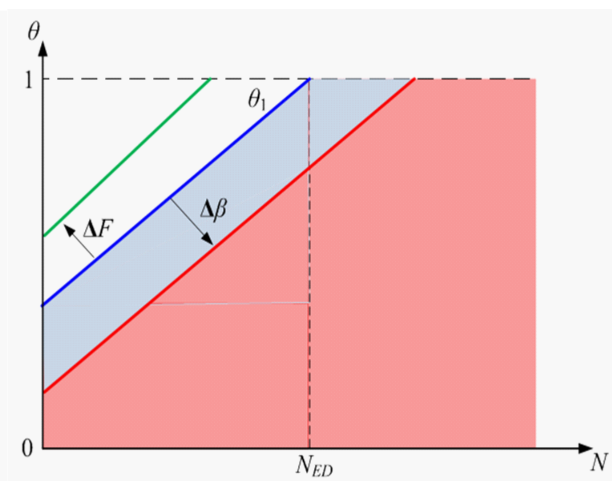


Fig 7b. Comparative statics of θ_1.

5 Overexploitation of the commons

In order to develop welfare comparisons, let us first examine appropriation levels with and without entry threats. Hence, in this section we confine our analysis to the entry-deterring equilibrium, supported in the region $\theta \in (\theta_1, \theta_3]$.

Lemma 2. *Aggregate first-period (second-period) appropriation is larger (smaller, respectively) when entry threats are present than when they are absent. In addition, aggregate first-period appropriation under entry threats exceeds socially optimal appropriation if and only if* $\theta > \widetilde{\theta}$, *where* $\widetilde{\theta} \equiv \frac{(2+N)\sqrt{F}\left[1-(\beta-1)^2\delta\right]}{\beta}$.

Figure 8 depicts first- and second-period aggregate appropriation. Firms produce X_{UI} when they are unthreatened by entry, where subscript UI denotes unthreatened incumbents (X_{UI} thus coincides with X_{BE}), while firms increase aggregate first-period appropriation to X_{ED} when they face entry threats. Such output, however, lies above the socially optimal output, X_{SO}, for stock levels $\theta > \widetilde{\theta}$. In the second-period game, the resource is more depleted and, therefore, aggregate appropriation under entry threats lies below that when incumbents are unthreatened, i.e., $Q_{ED} < Q_{UI}$. Importantly, both Q_{ED} and Q_{UI} are lower than the socially optimal appropriation level, and hence, we hereafter focus on first-period exploitation.[14]

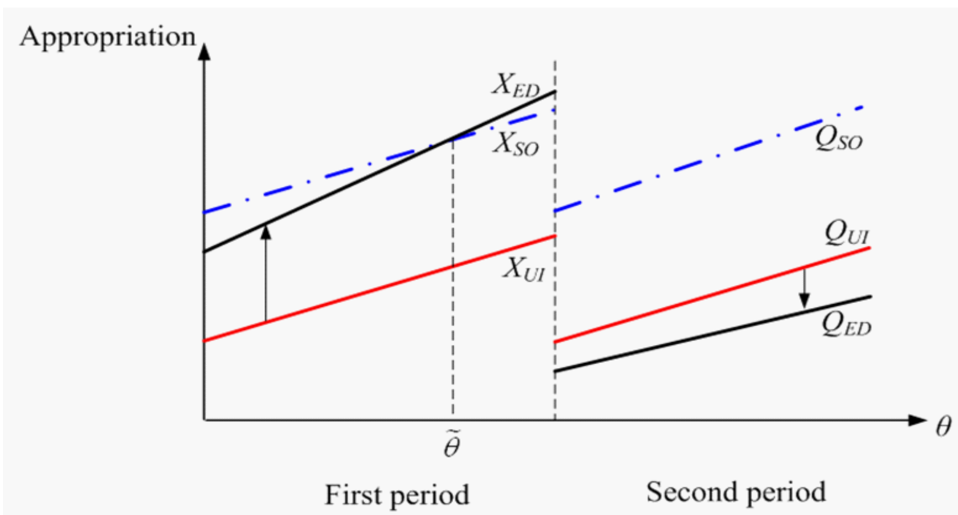


Fig 8. Effects of entry deterrence on appropriation.

The following corollary examines how the region of overexploitation in the entry-deterring equilibrium, i.e., $\theta_3 \geq \theta > \widetilde{\theta}$, is affected by the number of incumbents. Figure 9 illustrates our findings by breaking the area of stock levels where the entry-deterring equilibrium is sustained, $\theta_1 < \theta \leq \theta_3$, into two subareas: one in which entry deterrence yields an overexploitation of the resource (the shaded region where $\theta > \widetilde{\theta}$, as described in Lemma 2), and another in which entry deterrence entails an underexploitation of the CPR, i.e., $\theta \leq \widetilde{\theta}$.

[14] For completeness, the proof of Lemma 2 identifies first- and second-period equilibrium appropriation levels, both with and without entry threats, as well as the socially optimal appropriation levels. In addition, note that a one unit increase in first-period appropriation, X, produces a less-than-proportional reduction in second-period appropriation. Specifically, as described in section 3, an increase in X produces a reduction of $\frac{N}{N+1}(1-\beta)$ in $Q^{NE}(X)$, which is smaller than one for all parameter values.

Corollary 3. *Let N^* be the number of firms that solves $\widetilde{\theta} = \theta_3$. Then, for all $N < N^*$, underexploitation is supported under all θ. Similarly, let N^{**} be the number of firms that solves $\widetilde{\theta} = \theta_1$. Then, for all $N > N^{**}$, overexploitation is sustained under all θ.*

As depicted in figure 9, corollary 3 divides the number of incumbents into three regions, where: (1) underexploitation holds for all parameter values,[15] $N < N^*$; (2) under- and overexploitation can be sustained, depending on the stock's abundance, $N^* \leq N \leq N^{**}$; and (3) overexploitation is supported for all parameter conditions, $N > N^{**}$.

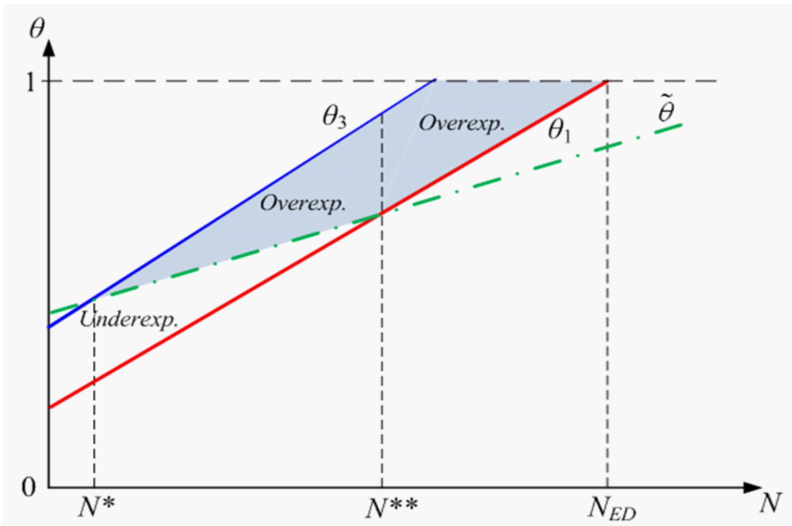


Fig 9. Under- and overxploitation in the ED equilibrium

For instance, for parameter values $\beta = 0.8$, $F = 0.005$ and no discounting, these cutoffs become $N^* = 1$, $N^{**} = 6$, and $N_{ED} = 10$.[16] That is, if a single incumbent operates in the commons, entry-deterring practices only lead to an underexploitation of the CPR for all θ's. Otherwise, from $N = 1$ to $N = 6$ we observe that entry deterrence yields overexploitation when the level of the stock is sufficiently high, i.e., $\theta > \widetilde{\theta}$, where $\widetilde{\theta} = \frac{6+3N}{25\sqrt{2}}$. Finally, if $6 < N \leq 10$, the commons is overexploited for all θ's.[17]

[15] Note that $\widetilde{\theta}$ originates above θ_1 for all parameter values, and it also originates above θ_3 (as depicted in figure 9) when the regeneration rate is sufficiently strong. In particular, under no discounting, $\widetilde{\theta}$ starts above θ_3 if and only if $\beta > 2/3$. Otherwise, $\widetilde{\theta}$ originates between θ_1 and θ_3, and the region where underexploitation is sustained for all θ's does not exist. Finally, cutoffs $\widetilde{\theta}$ and θ_3 are both increasing in N for the relevant range of N, i.e., $N \in [1, N_{ED}]$ for all admissible parameter values.

[16] Note that, for each of these critical number of firms, N^*, N^{**} and N_{ED}, we report the next largest integer by using the ceiling function $\lceil N \rceil$.

[17] Note that if the regeneration rate of the CPR, β, decreases, cutoffs N^*, N^{**} and N_{ED} experience a leftward shift, approaching them to the origin. For instance, in our previous parametric example, if β decreases from 0.8 to 0.3, N^{**} is reduced from 6 to 2, while N_{ED} decreases from 10 to 5. (Regarding cutoff N^*, it decreases from 0.9 to 0.7, implying that the largest integer for which overexploitation can be sustained under all values of θ is still $N^* = 1$). Intuitively, the commons become less attractive for the potential entrant, implying that incumbents' entry-deterring practices only arise in equilibrium when $N < 5$.

6 Welfare comparisons

In this section we examine whether the increase in appropriation due to entry deterrence yields welfare gains or losses. In particular, let us define social welfare in a given period t as the sum of consumer and producer surplus,[18] and overall welfare as its discounted stream across both periods, i.e., $SW = SW_1 + \delta SW_2$.

Proposition 2. *Social welfare under entry threats exceeds that under no threats when the stock level is sufficiently scarce, i.e., $\theta < \overline{\theta}$, where*

$$\overline{\theta} \equiv \frac{\sqrt{F}T\left[V^5 - D^2NV^2\left(4+N\left(3+N\right)\right)\delta + 2D^4N^2T\delta^2\right]}{V^4\left(BT-1\right)+D^2NV^2\left(T+\beta\left(NT-4\right)\right)\delta}$$

where $T=(2+N)$, $D=(\beta-1)$ and $V=(1+N)$.

As described in lemma 2, when incumbents are unthreatened, aggregate appropriation lies below the social optimum. Under entry threats, entry-deterring practices also yield an underexploitation of the resource if $\theta \leq \widetilde{\theta}$ but, since output approaches the social optimum, welfare unambiguously increases (Region A in figure 10).[19]

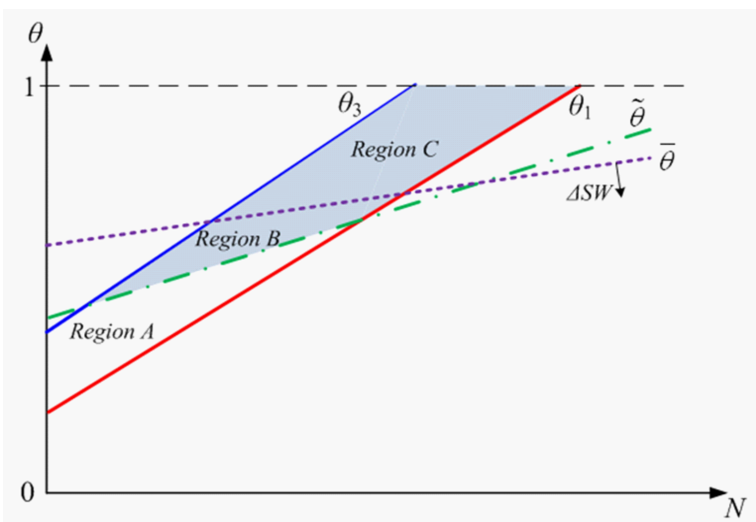


Fig 10. Social welfare in the ED equilibrium

[18] This social welfare function has been considered in similar studies analyzing exploitation of the commons, such as Mason and Polasky (1994), and hence allows for more direct comparisons. Nonetheless, section 6.2 below extends our findings to social welfare functions which include the environmental damage from firms' appropriation, arising for instance when firms' exploitation reduces the biological diversity of the CPR, and shows that our qualitative results are unaffected.

[19] Note that $\overline{\theta}$ originates above $\widetilde{\theta}$ (as depicted in figure 10) when the regeneration rate of the CPR is relatively strong. Specifically, under no discounting, $\overline{\theta}$ originates above $\widetilde{\theta}$ under relatively large parameter conditions, i.e., for all $\beta \geq 0.2$.

However, if entry deterrence generates overexploitation, $\theta > \widetilde{\theta}$, such welfare result becomes dependent on the stock abundance: specifically, if $\theta < \overline{\theta}$ overexploitation is small, and thus social welfare with entry threats is larger than without threats (region B), while if $\theta \geq \overline{\theta}$ overexploitation becomes larger, and welfare is lower when incumbents are threatened by potential entrants than otherwise (region C).

6.1 Discussion

Our results suggest that, when a relatively small number of incumbents exploit the CPR, their entry-deterring practices can entail welfare improvements, both when they under- and overexploit the commons relative to the social optimum; as discussed in Mason and Polasky (1994) for a single incumbent. However, when the number of firms increases, the threat of entry yields an overexploitation of the resource that entails a welfare loss for stock levels above $\overline{\theta}$. In addition, if the number of firms is further augmented, entry deterrence yields an unambiguous welfare loss under all parameter values.[20]

Our equilibrium predictions hence lead to different policy implications depending on the number of incumbents exploiting the commons and the abundance of the resource. Specifically, if few incumbents operate in a relatively scarce CPR (regions A and B), regulators should promote entry threats (e.g., by lowering entry costs) since firms' entry-deterring practices can yield welfare gains under relatively large parameter conditions. By contrast, when several firms exploit a very abundant commons (region C), incumbents' entry-deterring behavior yields a significant overexploitation, thus generating a welfare loss. In this setting, regulators should hinder entry threats by increasing the fixed entry cost, e.g., more costly fishing permits, higher administrative costs, etc.

6.2 Introducing environmental damage

Let us now consider the case in which firms' exploitation produces a loss in the biodiversity of the commons. Specifically, we include environmental damage $EnvD \equiv d \times \left(X^2 + \delta Q^2\right)$ into the social welfare function, implying that welfare becomes $SW(d) = SW_1 + \delta SW_2 - EnvD$, where $d \in [0, 1]$. In this context, firms' equilibrium behavior coincides with that in previous sections (and so do the bounds of the entry-deterring equilibrium, θ_1 and θ_3), but such output entails a different equilibrium welfare level. The following proposition examines how our equilibrium results are affected by the introduction of environmental damages.

Proposition 3. *Cutoffs θ_1 and θ_3 are constant in the loss of environmental quality, d. However, cutoffs $\widetilde{\theta}$ and $\overline{\theta}$ are both decreasing in d.*

Since $\widetilde{\theta}$ decreases in d, the region of stock levels for which entry-deterring practices yield an overexploitation of the resource expands, i.e., the shaded area in figure 9 enlarges. Intuitively, society

[20] For instance, for the parameter values considered in our previous example, $\overline{\theta}$ crosses θ_1 at $N = 5.26$, and hence welfare gains arise for all $N \leq 6$. Otherwise, overexploitation entails welfare losses under all stock levels, θ.

assigns a larger value to the biodiversity of the resource, and thus the incumbents' appropriation exceeds the socially optimal level under larger parameter conditions than when environmental damage is absent from the regulator's objective function. In addition, cutoff $\overline{\theta}$ also decreases in d, implying that the region of overexploitation that entails a welfare loss (region C in figure 10) expands. Hence, incumbents' entry-deterring appropriation becomes more harmful under larger parameter values. Our results thus imply that the threat of entry yields a social improvement under a smaller number of incumbents when environmental damage is considered than otherwise, i.e., region A in figure 10 shrinks; whereas an increase in the number of firms leads to welfare losses under larger parameter conditions.

7 Conclusions

Our paper analyzes an entry-deterrence model with multiple incumbents who strategically increase appropriation in order to prevent entry. We find that, when the number of firms is relatively small, entry deterrence can be sustained if the resource is moderately abundant. As the number of incumbents grows, such behavior can only be supported if the resource becomes more abundant. In addition, when few firms operate in a CPR, we demonstrate that entry-deterring appropriation levels are higher than those in contexts where firms are unthreatened by entry (but still lie below the social optimum, i.e., underexploitation), thus yielding a welfare improvement. When several firms compete for the resource, entry deterrence entails an overexploitation of the commons, which generates welfare gains (losses) if the CPR is moderately abundant (very abundant, respectively).

Our model can be extended in several directions. First, we introduce environmental damages by assuming that, for simplicity, appropriation imposes a flow externality —whose negative effects fully dissipate at the end of the first period— rather than a stock externality, whose impacts remain across time. An extension of the paper could consider that a proportion of the damage caused by first-period appropriation persists in future periods. In such setting, firms' entry-deterring practices would entail an overexploitation of the resource under larger parameter conditions, thus yielding a welfare reduction in more circumstances. Second, we consider that potential entrants are perfectly informed about the stock's abundance. However, as pointed out by Polasky and Bin (2001) and Espinola-Arredondo and Munoz-Garcia (2011), it might be especially difficult for potential entrants to accurately assess the profitability of a CPR that has been exploited by a small number of firms during long periods of time. Under such information structure, potential entrants would use incumbents' aggregate appropriation levels (or market prices) as a signal to infer the stock level; similarly as in Harrington (1987), who uses a limit-pricing game to analyze how incumbent duopolists independently choose their output levels in order to signal their common cost structure to an uninformed entrant. Unlike our model, his setting does not consider CPRs since neither the costs of incumbent firms nor those of the entrant are affected by previous production decisions.

8 Appendix

8.1 *Proof of Lemma 1*

Let us first find cutoff $\overline{X}_{-i}(X_{ED})$, which denotes the appropriation level from all firms $j \neq i$, X_{-i}, that solves $X_{-i} + x_i^{NE}(X_{-i}) = X_{ED}$. Using expression (3) of the best response function $x_i^{NE}(X_{-i})$, we have

$$X_{-i} + \left[\frac{\left[(1+N)^2 - 2(1-\beta)\delta\right]\theta}{2(1+N)^2 - 2(\beta-1)^2\delta} - \frac{\left[(1+N)^2 - 2(\beta-1)^2\delta\right]}{2(1+N)^2 - 2(\beta-1)^2\delta} X_{-i} \right] = X_{ED}$$

and solving for X_{-i} yields $\overline{X}_{-i}(X_{ED}) = \frac{2\left[(1+N)^2-(\beta-1)^2\delta\right]}{(1+N)^2} X_{ED} - \frac{\theta\left[(1+N)^2+2(\beta-1)^2\delta\right]}{(1+N)^2}$ for a general value of X_{ED}. Plugging the value of $X_{ED} \equiv \frac{\theta-\sqrt{F}(N+2)}{1-\beta}$ we obtain

$$\overline{X}_{-i}(X_{ED}) = \frac{2\sqrt{F}\left[(1+N)^2 - (\beta-1)^2\delta\right] - \theta(1+\beta)(1+N)^2}{(\beta-1)(1+N)^2}.$$

Let us now find cutoff $\underline{X}_{-i}(X_{ED})$, which solves $\widehat{\pi}_i^{ED}(X_{-i}) = \widehat{\pi}_i^{AE}(X_{-i})$. When entry is deterred, firm i selects $x_i = X_{ED} - X_{-i}$ implying that its profits from deterring entry

$$\pi_i^{NE}(x_i; X_{-i}) = (1 - x_i - X_{-i})x_i - (1-\theta)x_i + \delta\left(\frac{\theta - (1-\beta)(x_i - X_{-i})}{N+1}\right)^2$$

become

$$\widehat{\pi}_i^{ED}(X_{-i}) = (1 - X_{ED})(X_{ED} - X_{-i}) - (1-\theta)(X_{ED} - X_{-i}) + \delta\left(\frac{\theta - (1-\beta)X_{ED}}{N+1}\right)^2.$$

Similarly, when entry is allowed, firm i uses best response function $x_i^E(X_{-i})$, implying that its profits become

$$\widehat{\pi}_i^{AE}(X_{-i}) = (1 - x_i^E(X_{-i}) - X_{-i})x_i^E(X_{-i}) - (1-\theta)x_i^E(X_{-i}) + \delta\left(\frac{\theta - (1-\beta)\left[x_i^E(X_{-i}) + X_{-i}\right]}{N+2}\right)^2$$

and using expression (5) of the best response function $x_i^E(X_{-i})$, the above profits from allowing entry can be more compactly expressed as

$$\widehat{\pi}_i^{AE}(X_{-i}) = \frac{[(2+N)^2 + 4\beta\delta]\theta^2 - 2\left[(2+N)^2 - 2(\beta-1)\beta\delta\right]\theta X_{-i} + (2+N)^2 X_{-i}^2}{4(2+N)^2 - 4(\beta-1)^2\delta}$$

We can now find the value of X_{-i} that solves $\widehat{\pi}_i^{ED}(X_{-i}) = \widehat{\pi}_i^{AE}(X_{-i})$,

$$\underline{X}_{-i}(X_{ED}) = \frac{2\left[AX_{ED} + \sqrt{B}[(\beta-1)X_{ED} + \theta]\right] - (1+N)^2\left[(2+N)^2 + 2(\beta-1)\delta\right]\theta}{(2+3N+N^2)^2},$$

where $A \equiv (2+3N+N^2)^2 - (\beta-1)^2(N-1)^2\delta$ and $B \equiv (1+N)^2(3+2N)\delta[(2+N)^2 - (\beta-1)^2\delta]$.
■

8.2 *Proof of Proposition 1*

Let $\underline{X}_{ED}$ indicate the entry-deterring appropriation level that solves $\underline{X}_{-i}(X_{ED}) = (N-1)x_i^E$. Intuitively, when all other incumbents produce x_i^E, firm i is indifferent between allowing and preventing entry when the entry-deterring appropriation level that firm i must reach is exactly $X_{ED} = \underline{X}_{ED}$. In addition, let $\overline{X}_{ED}$ denote the largest value of X_{ED} for which deterring entry is an equilibrium. That is, $\overline{X}_{ED}$ solves $\underline{X}_{-i}(X_{ED}) = \frac{(N-1)X_{ED}}{N}$, where entry is deterred by having all $j \neq i$ incumbents equally sharing the burden of reaching the entry-deterring appropriation X_{ED}.

In addition, cutoff $\overline{X}_{ED} > \underline{X}_{ED}$ since function $\underline{X}_{-i}(X_{ED})$ is linear and increasing in X_{ED}, and $\frac{(N-1)X_{ED}}{N} > (N-1)x_i^E$, as figure A1 depicts. [In the special case of $N = 1$, both $\overline{X}_{ED}$ and $\underline{X}_{ED}$ solve $\underline{X}_{-i}(X_{ED}) = 0$, and hence $\overline{X}_{ED} = \underline{X}_{ED}$.]

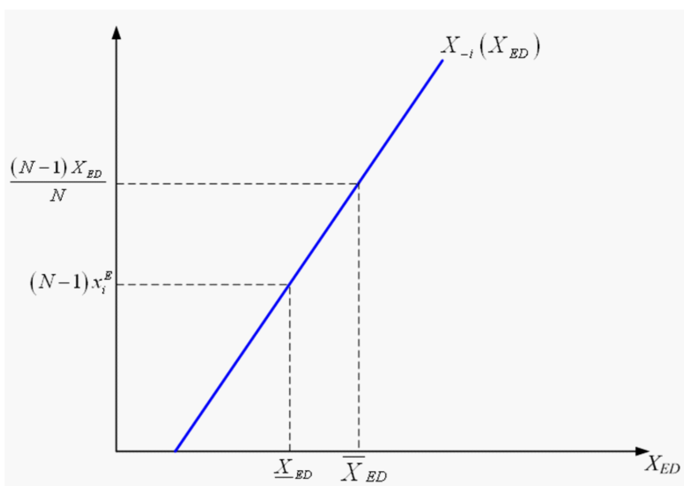


Fig A1. $\underline{X}_{-i}(X_{ED})$

Therefore, three equilibria can be identified depending on the value of X_{ED}: (1) one where entry is blockaded for all $X_{ED} \leq X^{NE}$ (where X^{NE} denotes the aggregate output under no entry); (2) one where entry is prevented when $X^{NE} < X_{ED} \leq \overline{X}_{ED}$; and (3) one where entry is allowed for all $X_{ED} > \overline{X}_{ED}$. As a consequence, in the region where $\underline{X}_{ED} < X_{ED} \leq \overline{X}_{ED}$ two types of equilibrium outcomes coexist (entry deterrence and allowing entry).

These three equilibria can be alternatively expressed in terms of θ. First, note that X_{ED} can be rewritten as a linear function of θ as follows

$$X_{ED} = \frac{-\sqrt{F}(N+2)}{1-\beta} + \frac{1}{1-\beta}\theta$$

and similarly X^{NE} can be expressed as

$$X^{NE} = \frac{N\left[(1+N)^2 + 2(\beta-1)\delta\right]}{(1+N)^3 - 2(\beta-1)^2 N\delta}\theta$$

Let θ_1 denote the value of θ at which X_{ED} crosses X^{NE}. This crossing point is unique, since both X_{ED} and X^{NE} are linear in θ, but X_{ED} originates in the negative quadrant, $\frac{-\sqrt{F}(N+2)}{1-\beta}$, while X^{NE} originates at zero (see figure A2 below). Hence, entry is blockaded for all $\theta \leq \theta_1$ where

$$\theta_1 \equiv \frac{\sqrt{F}(N+2)\left[(1+N)^2 - 2(\beta-1)^2 N\delta\right]}{(1+N)^2(1+\beta N)}$$

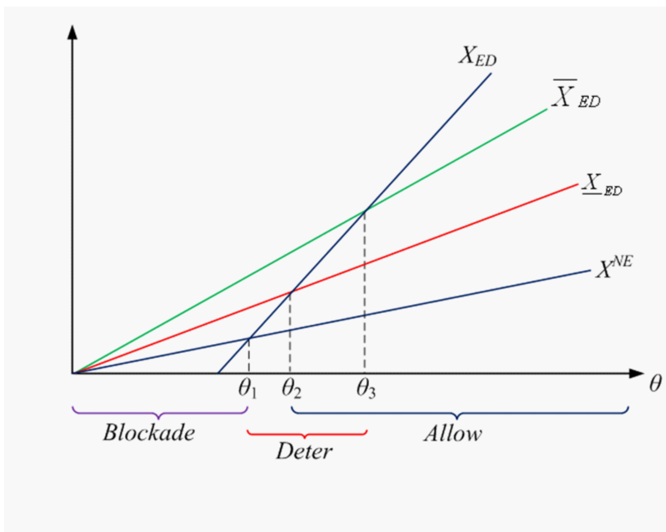


Fig A2

Similarly, let θ_2 represent the value of θ at which X_{ED} crosses $\underline{X}_{ED}$. Cutoff $\underline{X}_{ED}$ also originates at zero, and is linear in θ. Hence, entry is deterred for all $\theta_1 < \theta \leq \theta_2$, where

$$\theta_2 \equiv \frac{\sqrt{F}(N+2)}{(1-\beta)\left[\frac{1}{1-\beta} - D + E\right]}$$

and

$$D \equiv \frac{(1+N)(1+\beta N)[(2+N)^2 - (\beta-1)^2\delta]\sqrt{(3+2N)\delta[(2+N)^2 - (\beta-1)^2\delta]}}{[(1+N)(2+N)^2 - (\beta-1)^2 N\delta]\left[((1+N)^2 - (\beta-1)^2\delta)\left((2+N)^2 - (\beta-1)^2\delta\right)\right]}$$

and $E \equiv \frac{-N(1+N)^2(2+N)^2 + (\beta-1)(1+N)(N-3(-5+\beta+(-3+\beta)N))\delta + 2(\beta-1)^3 N\delta^2}{(1+N)^3(2+N)^2 - (\beta-1)^2(1+N)(4+3N(2+N))\delta + 2(\beta-1)^4 N\delta^2}$. Finally, let θ_3 represent the value of θ at which X_{ED} crosses $\overline{X}_{ED}$. Cutoff $\overline{X}_{ED}$ also originates at zero, and is linear in θ.

Hence, entry is allowed for all $\theta > \theta_3$, where

$$\theta_3 \equiv \frac{\sqrt{F}(N+2)}{(1-\beta)\left[\frac{1}{1-\beta} + \frac{G_1+G_2+G_3}{G_4}\right]}$$

where $G_1 \equiv -2\sqrt{N^2(1+N)^2(3+2N)(1+\beta N)^2\delta\left[(2+N)^2-(\beta-1)^2\delta\right]}$, $G_2 \equiv N(-(1+N)^3(2+N)^2 + 2(\beta-1)(-1+N(\beta(1+N)^2-(2+N)(5+2N)))\delta$, $G_3 \equiv 4(\beta-1)^3N\delta^2$, and $G_4 \equiv (1+N)^4(2+N)^2 - 4(\beta-1)^2N(1+N(2+N)(3+N))\delta + 4(\beta-1)^4N^2\delta^2$. In addition, $\theta_1 > \theta_2 > \theta_3$, since $\overline{X}_{ED}$, $\underline{X}_{ED}$ and X^{NE} originate at zero, and $\overline{X}_{ED} > \underline{X}_{ED} > X^{NE}$, while X_{ED} originates at $\frac{-\sqrt{F}(N+2)}{1-\beta} < 0$.

Finally, let us show that, within the coexistence region, only the equilibrium in which entry is prevented is undominated in terms of firms' profits.

Lemma A. *In the coexistence region, $\theta \in [\theta_2, \theta_3]$, every firm i's profits in the equilibrium where entry is prevented are larger than in that where entry is allowed.*

Proof of Lemma A. In the equilibrium where entry is allowed, all firms use the best response function $x_i^E(X_{-i})$, and by symmetry $x_i^E = \frac{\left[(2+N)^2+2(\beta-1)^2\delta\right]\theta}{(1+N)(2+N)^2-2(\beta-1)^2N\delta}$. Therefore, firm i's equilibrium profits become

$$\pi^{AE} \equiv \frac{[T^4 + T^2(-1+B(2+(4+\beta(-2+N))N))\delta - 4D^2\beta N\delta^2]\theta^2}{[VT^2 - 2D^2N\delta]^2}$$

where $T = (2+N)$, $D = (\beta-1)$ and $V = (1+N)$. A continuum of equilibria exist in which entry is deterred, whereby firms' aggregate appropriation is X_{ED} and x_i satisfies $X_{ED} - \underline{X}_{-i}(X_{ED}) \geq x_i \geq x_i^{NE}(X_{-i})$. For simplicity, we focus on the symmetric entry-deterring equilibrium where every firm selects $x_i = \frac{X_{ED}}{N}$. In this setting, firm i's equilibrium profits from deterring entry become

$$\pi^{DE} \equiv \frac{FT^2(D^2N\delta - V^2) + V^2\theta\left[(1+\beta)\sqrt{F}T - \beta\theta\right]}{D^2NV^2}$$

In addition, $\pi^{AE} < \pi^{DE}$ for all $\theta > \widehat{\theta}_2$, where

$$\widehat{\theta}_2 \equiv \frac{L + \sqrt{\begin{array}{c} D^2FV^2(VT^2-2D^2N\delta)^2(T^4(-1+N^2)^2 - 4NT^2(-N(3+2N) - \beta(4+NT(1+N^2)) + \\ \beta^2(1+N(-3-4N+N^3)))\delta + 4D^2N^2(-3-2N+\beta(1+NT(-3+N^2))))\delta^2) \end{array}}}{2V^2(T^3(\beta+N)(1+\beta N) + D^2NT(-1+\beta(-2+\beta(-2+N)N))\delta)}$$

and $L = (1+\beta)\sqrt{F}((2+3N+N^2)^2 - 2D^2NV\delta)^2$.

Furthermore, cutoff $\widehat{\theta}_2$ satisfies $\widehat{\theta}_2 < \theta_2$. Hence, for all θ in the coexistence region $\theta \in (\theta_2, \theta_3]$, profits satisfy $\pi^{AE} < \pi^{DE}$. We can therefore identify a unique equilibrium prediction for each value of X_{ED}: blockaded entry when $X_{ED} \leq X^{NE}$ (i.e., $\theta \leq \theta_1$), entry deterrence when $X^{NE} < X_{ED} \leq \overline{X}_{ED}$ (i.e., $\theta_1 < \theta \leq \theta_3$), and allowing entry when $X_{ED} > \overline{X}_{ED}$ (i.e., $\theta > \theta_3$). ■

8.3 *Proof of Corollary 1*

As shown in the proof of Proposition 1, cutoffs $\overline{X}_{ED}$, $\underline{X}_{ED}$ and X^{NE} originate at zero, and $\overline{X}_{ED} > \underline{X}_{ED} > X^{NE}$, while X_{ED} originates at $\frac{-\sqrt{F}(N+2)}{1-\beta} < 0$. Hence, $\theta_1 > \theta_2 > \theta_3$. In addition, since θ_1 increases in N, the region of parameter values supporting the equilibrium where entry is allowed (blockaded) shrinks (expands, respectively). ■

8.4 *Proof of Corollary 2*

From the proof of Proposition 1, cutoff $\theta_1(N)$ is $\theta_1(N) \equiv \frac{\sqrt{F}T\left[V^2-2D^2N\delta\right]}{V^2(1+\beta N)}$, where $T = (2+N)$, $D = (\beta - 1)$ and $V = (1 + N)$. Cutoff θ_1 increases in N since

$$\frac{\partial\theta_1}{\partial N} = \frac{\sqrt{F}\left[V^3\left(3+2N+\beta(N^2-2)\right)+2D^2(\beta N^2(3+N)-2)\delta\right]}{V^3(1+\beta N)^2}$$

which is strictly positive for all admissible values of $\beta, \delta \in (0,1)$ and $N \geq 1$. In addition, θ_1 increases in F given that

$$\frac{\partial\theta_1}{\partial F} = \frac{T\left[V^3-2D^2N\delta\right]}{2\sqrt{F}V^2(1+\beta N)}$$

which is strictly positive for all admissible values of $\beta, \delta \in (0,1)$ and $N \geq 1$. Finally, θ_1 decreases in β since

$$\frac{\partial\theta_1}{\partial\beta} = -\frac{\sqrt{F}NT\left[V^3+2D(2+N+\beta N)\delta\right]}{V^2(1+\beta N)^2}$$

which is strictly negative for all admissible parameter values. ■

8.5 *Proof of Lemma 2*

First-period output. From the proof of Proposition 1, we know that aggregate first-period appropriation X_{ED} (in the entry-deterring equilibrium when entry threats are present) is strictly larger than X^{NE} (i.e., the aggregate first-period appropriation when entry threats are absent) for all $\theta > \theta_1$. Since the entry-deterring equilibrium can only be supported if θ satisfies $\theta_1 < \theta \leq \theta_3$, we can then conclude that $X_{ED} > X^{NE}$ holds.

Second-period output. Evaluating the second-period appropriation $Q^{NE}(X)$ at the entry-deterring output X_{ED} yields an equilibrium appropriation level of $Q^{NE}(X_{ED}) = \frac{\sqrt{F}N(2+N)}{1+N}$ when entry threats are present. Similarly, when entry threats are absent, we evaluate $Q^{NE}(X)$ at X^{NE} to obtain an equilibrium appropriation level of $Q^{NE}(X^{NE}) = \frac{N(1+N)(1+\beta N)\theta}{(1+N)^3-2(\beta-1)^2N\delta}$. Comparing $Q^{NE}(X_{ED})$ and $Q^{NE}(X^{NE})$ yields that $Q^{NE}(X_{ED}) < Q^{NE}(X^{NE})$ for all $\theta > \theta_1$. Since $\theta_1 < \theta \leq \theta_3$ holds in the entry-deterring equilibrium, $Q^{NE}(X_{ED}) < Q^{NE}(X^{NE})$ must be satisfied.

Socially-optimal output. In order to compare first-period appropriation in the entry-deterring equilibrium, X_{ED}, with the socially optimal output, X_{SO}, let us first find the value of X_{SO}. Starting in the second period, it is straightforward to show that, for a given first-period appropriation, X,

$Q_{SO}(X) = \theta + (\beta - 1)X$ maximizes second-period welfare

$$\frac{Q^2}{2} + (1 - Q)Q - [1 - (\theta - (1 - \beta) X)] Q,$$

entailing a second-period welfare of $SW_2 = \frac{[\theta+(\beta-1)X]^2}{2}$. Given this second-period welfare, the social planner selects the first-period appropriation level that maximizes overall social welfare across both periods

$$\max_{X \geq 0} \frac{X^2}{2} + (1 - X)X - (1 - \theta) X + \delta \frac{[\theta + (\beta - 1)X]^2}{2},$$

which is maximized at $X_{SO} = \frac{(1-\delta+\beta\delta)\theta}{1-(\beta-1)^2\delta}$. Hence, evaluating the second-period appropriation at X_{SO}, we find that the equilibrium second-period appropriation is $Q_{SO}(X_{SO}) = \frac{\theta\beta}{1-(\beta-1)^2\delta}$. Comparing $X_{ED} \equiv \frac{\theta-\sqrt{F}(N+2)}{1-\beta}$ and X_{SO}, and solving for θ, we obtain that $X_{ED} > X_{SO}$ for all $\theta > \widetilde{\theta}$, where $\widetilde{\theta} \equiv \frac{(2+N)\sqrt{F}\left[1-(\beta-1)^2\delta\right]}{\beta}$. ■

8.6 *Proof of Proposition 2*

In order to analyze the welfare effect of entry threats, we need to compare social welfare in the entry deterring equilibrium (arising when θ satisfies $\theta_1 < \theta \leq \theta_3$) with that under no entry threats (i.e., blockaded entry) using the same range of θ. Under entry deterrence, equilibrium social welfare is

$$SW^{ED} = \frac{FT^2 \left(NT \left(D^2\delta - 1\right) - 1\right) + V^2\theta \left(2\beta\sqrt{F}T + (1 - 2\beta)\theta\right)}{2D^2V^2}$$

whereas under no entry threats (blockaded entry), social welfare is

$$SW^{BE} = \frac{N \left[V^4T + V^2 \left(-2 - 3N + \beta \left(4 + N \left(2 \left(6 + N\right) + \beta \left(-4 + NT\right)\right)\right)\right) \delta - 4D^4 \left(2\beta - 1\right) N\delta^2\right] \theta^2}{2 \left(V^3 - 2D^2N\delta\right)^2}$$

Hence, the difference $SW^{ED} - SW^{BE}$ for $\theta \in [\theta_1, \theta_3]$ is positive for all $\theta \leq \overline{\theta}$, where

$$\overline{\theta} \equiv \frac{\sqrt{F}T \left[V^5 - D^2NV^2 \left(4 + N \left(3 + N\right)\right) \delta + 2D^4N^2T\delta^2\right]}{V^4 \left(BT - 1\right) + D^2NV^2 \left(T + \beta \left(NT - 4\right)\right) \delta}$$

where $T = (2 + N)$, $D = (\beta - 1)$ and $V = (1 + N)$. ■

8.7 *Proof of Proposition 3*

Output comparisons. In order to compare first-period appropriation in the entry-deterring equilibrium, X_{ED}, with the socially optimal output considering environmental damage, $X_{SO}(d)$, let us first find the value of $X_{SO}(d)$. Starting in the second period, it is straightforward to show that, for a given first-period appropriation, X, $Q_{SO}(X) = \frac{\theta+(\beta-1)X}{1+2d}$ maximizes second-period welfare

$$\frac{Q^2}{2} + (1-Q)Q - [1 - (\theta - (1-\beta)X)]\, Q - dQ^2,$$

entailing a second-period welfare of $SW_2 = \frac{[\theta+(\beta-1)X]^2}{2+4d}$. Given this second-period welfare, the social planner selects the first-period appropriation level that maximizes overall social welfare across both periods

$$\max_{X\geq 0} \frac{X^2}{2} + (1-X)X - (1-\theta)\, X - dX^2 + \delta\frac{[\theta + (\beta-1)X]^2}{2+4d},$$

which is maximized at $X_{SO}(d) = \frac{(1+2d+(\beta-1)\delta)\theta}{(1+2d)^2-(\beta-1)^2\delta}$. Hence, evaluating the second-period appropriation at $X_{SO}(d)$, we find that the equilibrium second-period appropriation is $Q_{SO}(X_{SO}) = \frac{\theta(\beta+2d)}{1+4d(1+d)-(\beta-1)^2\delta}$. Comparing $X_{ED} \equiv \frac{\theta-\sqrt{F}(N+2)}{1-\beta}$ and $X_{SO}(d)$, and solving for θ, we obtain that $X_{ED} > X_{SO}$ for all $\theta > \widetilde{\theta}(d)$, where $\widetilde{\theta}(d) \equiv \frac{(2+N)\sqrt{F}\left[(1+2d)^2-(\beta-1)^2\delta\right]}{(1+2d)(\beta+2d)}$. Finally, note that $\frac{\partial\widetilde{\theta}(d)}{\partial d} < 0$ for all admissible parameter values.

Welfare comparisons. Comparing social welfare in entry-deterring equilibrium, $SW^{ED}(d)$, with that under no entry threats (i.e., blockaded entry), $SW^{BE}(d)$, we obtain that $SW^{ED}(d) > SW^{BE}(d)$ for all $\theta < \overline{\theta}(d)$, where

$$\overline{\theta}(d) \equiv \frac{\sqrt{F}T\left(2D^2N\delta - V^3\right)\left((1+2d)\,V^2 + D^2N(-T+2dN)\delta\right)}{V^2(V(1-2d(1+2N)+\beta(-T+2dN)) + D^2N(-T+2d(4+N)+\beta(4+N(-T+2dN)))\delta)}$$

where $T = (2+N)$, $D = (\beta-1)$ and $V = (1+N)$. Finally, note that $\frac{\partial\overline{\theta}(d)}{\partial d} < 0$ for all admissible parameter values. ■

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