

Working Paper Series  
**WP 2011-5**

**Unobserved Capacity Constraints  
and Entry Deterrence**

By

**Felix Munoz-Garcia and Gulnara  
Zaynutdinova**

**May 2011**

# Unobserved Capacity Constraints and Entry Deterrence\*

Felix Munoz-Garcia<sup>†</sup>

School of Economic Sciences  
Washington State University

Gulnara Zaynutdinova<sup>‡</sup>

Department of Finance  
and Management Science  
Washington State University

May 2011

## Abstract

This paper examines entry deterrence and signaling when an incumbent firm experiences a capacity constraint, arising from either her productive efficiency or the high market demand she faces. In both cases, we demonstrate that separating and pooling equilibria can be sustained. Our results show that if the costs that constrained and unconstrained incumbents face when expanding their facilities are substantially different, the separating equilibrium can be supported under large parameter values. In this case, information is perfectly transmitted to the entrant. If, in contrast, both types of incumbent face similar expansion costs, a policy reducing expansion costs can help move the industry from a pooling equilibrium to the separating equilibrium with associated efficient entry. Nonetheless, our results show that if this policy is overemphasized entry patterns remain unaffected, suggesting a potential disadvantage of policies that significantly reduce firms' expansion costs.

KEYWORDS: Business expansion; Signaling; Entry deterrence.

JEL CLASSIFICATION: L12, D82.

---

\* We would like to especially thank Ana Espinola-Arredondo, Jia Yan, Jill McCluskey and Levan Elbakidze for their insightful comments and suggestions.

<sup>†</sup> 103G Hulbert Hall, School of Economic Sciences, Washington State University, Pullman, WA, 99164-6210. Tel. 509-335-8402. Fax 509-335-1173. E-mail: [fmunoz@wsu.edu](mailto:fmunoz@wsu.edu).

<sup>‡</sup> 481 Todd Hall, Department of Finance and Management Science, Washington State University, Pullman, WA, 99164-4750. E-mail: [gzaynutdinova@wsu.edu](mailto:gzaynutdinova@wsu.edu).

# 1. Introduction

Capacity constraints constitute a limiting factor for industries experiencing a sudden increase in demand. Indeed, some firms may only be able to satisfy additional demand by producing at higher marginal costs, while others may find it extremely costly to produce above their current capacity, thus making them unable to satisfy unexpected increases in demand. An expansion of the current facility might be an attractive option, since it alleviates such capacity constraint. In the absence of entry threats, firms expand if their direct benefits from expanding are positive, i.e., if the increase in future profits associated to the expansion offset expansion costs. Under entry threats, however, firms must consider not only this direct benefit but also the indirect effects that such expansion might entail. In particular, expansion might signal a high demand and attract potential entrants to the industry, thus suggesting that firms suffering a capacity constraint might face a tradeoff when considering whether or not to expand their facility. Such a tradeoff is specifically relevant in periods of economic recovery, where several firms start experiencing larger customer traffic and sales, making them more likely to experience capacity constraints.<sup>1</sup>

In this paper we examine this tradeoff by studying entry deterrence in a context where the incumbent is privately informed about her capacity constraint. Specifically, the incumbent is constrained if she cannot produce her profit-maximizing output because she faces a limited plant capacity. This occurs, for instance, when her technological efficiency or the market demand she faces are relatively high. In contrast, an unconstrained incumbent can produce her profit-maximizing output. Our model considers that, first, the incumbent chooses whether to expand her facility where the fixed costs from such expansion may differ between the constrained and unconstrained type of incumbent. The potential entrant does not observe whether the incumbent suffers a capacity constraint, and must therefore base her entry decision on the information he infers from the incumbent's expansion.

We first show that both separating and pooling equilibria can be sustained, where information is either perfectly conveyed to the potential entrant or concealed from him, respectively.<sup>2</sup> In the separating equilibrium, such information allows the entrant to base his entry decision on more accurate information about his post-entry competition. In contrast, in the pooling equilibrium the entrant cannot accurately assess the profitability of the market, and thus may enter a market that is actually unprofitable. Hence, the separating equilibrium supports entry in similar contexts as under complete information. Instead, the

---

<sup>1</sup> Industries that have recently reported relatively severe capacity constraints include, among others, the oil industry (as documented by the U.S. Energy Information Administration), the biopharmaceutical manufacturing industry (as reported in a survey conducted among European and U.S. firms by BioPlan Associates, Inc.), and the freight-transportation industry (according to the University of Denver's Intermodal Transportation Institute).

<sup>2</sup> In addition, both the separating and pooling equilibria survive standard equilibrium refinements in signaling games, i.e., the Cho and Kreps' (1987) Intuitive Criterion, under relatively general parameter conditions.

pooling equilibrium predicts possible entry in industries that the entrant would have avoided under complete information, entailing negative net profits for the entrant and subsequent exit.

The fully informative separating equilibrium can be sustained, specifically, when the constrained incumbent faces relatively lower expansion costs than the unconstrained firm. This difference in expansion costs might arise if, for instance, financial institutions discriminate constrained and unconstrained incumbents, charging different financial costs to each type. In this case, only the separating equilibrium arises where, as suggested above, entry patterns coincide with those under complete information, and no policy intervention is needed. In other contexts, both types of firm might face relatively similar expansion costs, illustrating situations where financial markets are not capable of differentiating constrained and unconstrained incumbents, thus charging both types of incumbent similar financial costs. In this setting, our paper shows that government intervention might be welfare improving under certain conditions, even when the regulator is uninformed about the incumbent's cost structure. In particular, we demonstrate that a policy reducing the financial costs associated with expansion induces a change in the equilibrium outcome from a pooling to a separating equilibrium.<sup>3</sup> We also predict that, despite the potential benefits from lowering financial costs—inducing similar entry patterns as under complete information—such policy can be easily *overdone*, which occurs when expansion costs are reduced beyond certain levels. In particular, under extremely low expansion costs, both types of incumbent expand their facilities, changing the equilibrium prediction, from a pooling equilibrium where no type of incumbent expands to one where both types expand. However, entry patterns coincide in the pooling equilibrium with and without expansion.

Our results are especially useful for predicting the potential effects of federal and state policies reducing firms' expansion costs. After the economic crisis, several firms have started to experience larger customer traffic and sales.<sup>4</sup> In this context, policies reducing expansion costs to both constrained and unconstrained firms can help businesses expand. As suggested above, if these policies are overdone, they might entail entry in contexts where it would not have occurred under complete information. Specifically, our equilibrium predictions imply that such policies can deter entry in markets where high demand actually supports the entry of new competitors, or attract entry in contexts where demand is still low.

---

<sup>3</sup> Our results depend on the severity of the capacity constraint. In particular, when capacity constraints are not severe, no type of incumbent significantly benefits from breaking her capacity constraint. In this case, a policy reducing financial costs would only switch the particular pooling equilibrium being played, namely, from one where no type of incumbent expands to one where both firms expand. Hence, under weak capacity constraints our paper suggests that a policy reducing both firms' financial costs is futile, since it does not modify the entry patterns that can arise in the pooling equilibria of the game.

<sup>4</sup> In the last Christmas season, for instance, Bloomberg reported a 5.5% increase in sales among U.S. retailers (see Bloomberg.com on December 28th, 2010), while the Wall Street Journal recorded a 4.2% sales increase among chain-stores (December 21st, 2010), relative to the same period in 2009.

Our equilibrium results hold under relatively general conditions. First, the incumbent's capacity constraint can arise from her high efficiency level or high market demand, indicating that our conclusions can be applied to settings where firms are privately informed about their capacity constraint, regardless of the source of such constraint. Second, the paper's equilibrium predictions are not qualitatively affected whether or not the incumbent's capacity constraint is very severe. To illustrate our results, we present a parametric example with linear demand and constant marginal costs in the appendix.

This paper contributes to both the literature pertaining to capacity constraints under complete information contexts, and that on signaling in entry-deterrence games. On one hand, Dixit's seminal work (1979, 1980) —analyzing the incentives for an incumbent to deter entry by expanding her capacity in a two-period game— has been expanded in other studies where both incumbent and entrant are perfectly informed. Specifically, Ware (1984) examines entry deterrence in a three-stage game, and Formby and Smith (1984) and Mason and Nowell (1992) study the incumbent's incentives to allow entry and then collude with the entrant. The role of capacity constraints as an entry deterrence device has, however, been analyzed using complete information settings, wherein every firm is able to perfectly observe other firms' cost structure. This assumption may not be sensible in certain industries that have been monopolized for long periods of time, where entrants have access to very limited information about the incumbent's cost structure or the precise market demand. Our paper hence contributes to the literature on capacity constraints by relaxing this assumption and allowing for incomplete information.<sup>5</sup>

On the other hand, this paper builds upon the literature on entry deterrence in signaling games, such as Milgrom and Roberts (1982) where the entrant is uncertain about the incumbent's unit costs.<sup>6</sup> In their setting, only the separating equilibrium can survive standard equilibrium refinements. In our model, in contrast, incomplete information about the incumbent's capacity constraint (or market demand) does not necessarily imply that the constrained incumbent experiences greater benefits from investing in additional capacity than the unconstrained incumbent.<sup>7</sup> As the Single-Crossing Property does not

---

<sup>5</sup> Arvan (1986) considers an incomplete information version of Dixit's (1980) model but focuses on type-dependent strategy profiles, unlike our paper that examines both type-dependent and type-independent strategy profiles. In addition, our model allows for capacity constraints to stem from both the incumbent's efficiency level and market demand.

<sup>6</sup> In an extension to Milgrom and Roberts' (1982), Harrington (1986) allows for the possibility that the entrant is uncertain about his own costs after entry. Interestingly, this article shows that when the costs of the entrant and the incumbent are sufficiently positively correlated then Milgrom and Roberts' (1982) results are reversed. That is, the incumbent's production is below the simple monopoly output in order to strategically deter entry. Our model is different from Harrington (1986) because in our setting both firms know each other's costs, but the entrant is uninformed about the incumbent's capacity constraint.

<sup>7</sup> Essentially, the constrained incumbent experiences a larger increase in profits from expanding her facility than her unconstrained counterpart if entry is deterred, but may experience a smaller increase if entry follows. As we show in the paper, this result holds even when the constrained incumbent can finance her expansion at a lower cost than the unconstrained incumbent.

necessarily hold in our model, both separating and pooling equilibria can be sustained in equilibrium and survive equilibrium refinements.

Another related article is Matthews and Mirman (1983), where the incumbent sets prices that can communicate information about market profitability to potential entrants as in our model. However, the actual price in the market (which is also the message observed by the potential entrant) receives a random shock, given that the incumbent sets prices before demand is actually realized. In contrast, we assume that market demand or capacity constraints are perfectly observed by the incumbent across periods.<sup>8</sup> In a recent article, Ridley (2008) analyzes an environment where an informed firm's entry provides a noisy signal about market demand to additional entrants. We consider, however, that the expansion decision of the informed firm may have a favorable effect on her technology, whereas Ridley (2008) assumes that the cost structure is unaffected. This implies that the informative separating equilibrium can arise in our model only if the incumbent's expansion produces strong technological benefits in addition to serving as a signal to potential entrants. Finally, Espinola-Arredondo *et al.* (in press) considers a similar information structure, where the incumbent is perfectly informed about market demand and chooses whether to invest in cost-reducing technologies. In their model, however, the incumbent is never limited by a capacity constraint, and the investment helps the incumbent lower her marginal cost in all units.

The paper is organized as follows. Section 2 presents the model. Section 3 describes our equilibrium predictions. Section 4 examines two extensions to our model, showing that the equilibrium results are qualitatively unaffected. Section 5 elaborates on the policy implications of our results and the last section concludes.

## 2. Model

Consider a market with a monopolist (she) and a potential entrant (he) with inverse demand function  $p(Q)$  which satisfies  $p'(Q) < 0$  and  $p''(Q) \leq 0$ . The monopolist is perfectly informed about her (constant) marginal production costs being low,  $c_L$ , or high,  $c_H$ , where  $p(0) > c_H > c_L \geq 0$ . In order to introduce the effect of the capacity constraint, we consider that the low-cost incumbent's profit-maximizing output exceeds her production capacity,  $\bar{q}$ , whereas that of the high-cost incumbent does not. The monopolist decides whether to expand her facility (which allows the low-cost incumbent to produce her profit-maximizing output in future periods) or to not expand. The time structure of this incomplete information game is described as follows:

---

<sup>8</sup> Bagwell and Ramey (1990) and Albaek and Overgaard (1994) examine a similar entry deterrence in a model where the potential entrant can perfectly observe both the incumbent's pre-entry pricing strategy and its advertising expenditures. In contrast, we restrict the amount of information available to the entrant to the expansion decision of the incumbent.

1. Nature determines the incumbent's marginal costs: high  $c_H$  with probability  $p$ , or low  $c_L$  with probability  $1-p$ . The incumbent privately observes her cost structure, but the entrant does not.
2. After observing her cost structure, the incumbent decides whether to expand her facility.
3. After observing the incumbent's expansion (or no expansion) decision, the entrant updates his beliefs about the incumbent's costs. Let  $\mu(H | Exp)$  and  $\mu(H | NoExp)$  denote the entrant's posterior beliefs about a high-cost incumbent after observing expansion or no expansion, respectively.
4. Given these beliefs, the entrant decides whether to enter the incumbent's market or to remain in a perfectly competitive market with associated zero economic profits.

We assume that expansion is costly and specify by  $K_H$  and  $K_L$  to be the incumbent's expansion costs when her marginal production costs are high and low, respectively. For generality, we do not restrict expansion costs  $K_H$  and  $K_L$ .<sup>9</sup> In order to make the entry decision interesting, we consider that the entrant's marginal costs are high. As a consequence, he has incentives to enter (stay out) when the incumbent's costs are high (low, respectively). In addition, we assume that the entrant must incur a fixed entry cost  $F > 0$  while his entry costs are zero if he remains in the perfectly competitive industry.

**First period.** For a production level  $q_{i,K}^S$  let subscript “ $i,K$ ” indicate firm  $i = \{inc, ent\}$  (incumbent or entrant) when the incumbent's marginal costs are  $K = \{H, L\}$ , whereas superscript  $S$  denotes the particular market structure in which the firm operates  $S = \{M, D\}$ , either monopoly or duopoly. For instance,  $q_{inc,H}^M$  denotes the monopoly profit-maximizing output for the high-cost incumbent. In particular, note that  $q_{inc,H}^M < \bar{q}$ , since this incumbent does not face a capacity constraint, yielding monopoly profits of  $\pi_{inc,H}^{M,NE}$ , where superscript  $NE$  denotes that the incumbent did not expand her facility.<sup>10</sup> In contrast, the low-cost incumbent's profit-maximizing output,  $q_{inc,L}^M$ , satisfies  $q_{inc,L}^M > \bar{q}$  given that she faces a capacity constraint. Because this type of incumbent is affected by her capacity constraint  $\bar{q}$ , we use  $\pi_{inc,L}^{M,NE}(\bar{q})$  to represent her monopoly profits when she does not expand, where  $\pi_{inc,L}^{M,NE}(\bar{q}) < \pi_{inc,L}^{M,E}$ . Intuitively, this allows the capacity constraint to take several forms: from an extreme context where output cannot be further increased beyond  $\bar{q}$ , to milder settings where the incumbent's marginal cost experiences an

---

<sup>9</sup> Expansion costs might be weakly lower for the most efficient incumbent, i.e.,  $K_H \geq K_L$ . Intuitively, this might occur when the incumbent uses a share of previous period profits to finance her expansion decision. We elaborate on this specific case in our discussion of the equilibrium results (Section 5).

<sup>10</sup> Note that superscript  $NE$  is thereafter used in first period profits since the incumbent did not have the chance to expand her facility yet. In our description of output and profit decisions during the second period, however, this superscript can either be  $E$  or  $NE$  to denote that the incumbent expanded (did not expand, respectively).

increase for all units surpassing capacity level  $\bar{q}$ ; as in Dixit (1980). Under both circumstances, nonetheless, the constrained (low cost) incumbent enjoys an increase in her profits when she relaxes (“breaks”) her capacity constraint by expanding her facility, raising profits from  $\pi_{inc,L}^{M,NE}(\bar{q})$  to  $\pi_{inc,L}^{M,E}$ .

**Second period, No entry.** In the second period, if there is no entry and the incumbent does not expand her facility, then the high(low)-cost incumbent’s profits are  $\pi_{inc,H}^{M,NE}(\pi_{inc,L}^{M,NE}(\bar{q}))$ , respectively), coinciding with those in the first period. In contrast, if the high-cost incumbent expands, her profits become  $\pi_{inc,H}^{M,E} - K_H$ , where  $\pi_{inc,H}^{M,E} = \pi_{inc,H}^{M,NE}$ , since this incumbent did not originally face a capacity constraint.<sup>11</sup> The low-cost incumbent, however, faces a capacity constraint  $\bar{q}$  and her expansion decision helps produce her profit-maximizing output  $q_{inc,L}^M > \bar{q}$  (since she breaks her capacity constraint) entailing profits of  $\pi_{inc,L}^{M,E} - K_L$ , where  $\pi_{inc,L}^{M,E} > \pi_{inc,L}^{M,NE}(\bar{q})$ .

**Second period, Entry.** If entry occurs, firms compete as Cournot duopolists. Let us first analyze the case in which the incumbent does not expand her facility. If the incumbent’s costs are high, duopoly profits are  $\pi_{inc,H}^{D,NE}$  and  $\pi_{ent,H}^{D,NE} - F > 0$  for incumbent and entrant, respectively. Intuitively, the entrant obtains a positive profit from entering, since he competes with a high-cost incumbent. If, in contrast, the incumbent’s costs are low, the incumbent produces a duopoly profit-maximizing output of  $q_{inc,L}^{D,NE}$  in the case that she did not expand her facility. In this section, we consider that this incumbent also faces a capacity constraint under duopoly.<sup>12</sup> Intuitively, the capacity constraint she faces is relatively strong, yielding profits of  $\pi_{inc,L}^{D,NE}(\bar{q})$  which are a function of the capacity constraint  $\bar{q}$ . In this case, the entrant produces  $q_{ent,L}^{D,NE}$  with associated profits of  $\pi_{ent,L}^{D,NE} - F$ .

Let us now examine the case where the incumbent expands her facility. The high-cost incumbent’s duopoly profits are  $\pi_{inc,H}^{D,E} - K_H$ , while the entrant’s duopoly profits are  $\pi_{ent,H}^{D,E} - F > 0$ . Intuitively, note that  $\pi_{i,H}^{D,E} = \pi_{i,H}^{D,NE}$  for both firms  $i = \{inc, ent\}$  since the production capacity of the high-cost incumbent is unaffected by her expansion decision. Finally, if the incumbent’s costs are low, her expansion decision helps break her capacity constraint, producing  $q_{inc,L}^{D,E} > \bar{q}$ , and yielding profits

---

<sup>11</sup> This implies that the unconstrained monopolist does not modify her sales after expanding her facility and, as suggested below, her benefits from expansion arise only if entry is deterred.

<sup>12</sup> At the end of section 3 we relax this assumption and show that our equilibrium results are not qualitatively affected. In particular, we allow for the capacity constraint to be binding (not binding) under monopoly (duopoly, respectively). Thus, the capacity constraint will not be as severe as in our current analysis, where it affects the incumbent both under monopoly and duopoly. Note that this assumption can alternatively be interpreted in terms of the efficiency of the low-cost incumbent. Specifically, for a given capacity constraint, a decrease in her marginal cost  $c_L$  implies that the incumbent finds the capacity constraint limiting under both market structures.



$\pi_{inc,L}^{D,E} - K_L$  and  $\pi_{ent,L}^{D,E} - F$  for the incumbent and entrant, respectively. As suggested above, the entrant has incentives to enter the market only when competing with the high-cost incumbent. This implies that entry costs,  $F$ , satisfy  $\pi_{ent,H}^{D,NE} > F > \pi_{ent,L}^{D,NE}$  when the incumbent does not expand<sup>13</sup> and  $\pi_{ent,H}^{D,E} > F > \pi_{ent,L}^{D,E}$  when the incumbent expands.<sup>14</sup>

## 2.1. Expansion benefits

On one hand, the expansion decision produces a direct benefit from enlarging her facility (“breaking” the capacity constraint), which allows the incumbent to produce using her efficient cost structure. Importantly, note that only the low-cost incumbent enjoys this direct benefit, since her production is limited by the capacity constraint, whereas the high-cost incumbent does not. On the other hand, expansion brings, a potential loss in profits if such expansion attracts entry. Let  $BCC_L = \pi_{inc,L}^{M,E} - \pi_{inc,L}^{M,NE}(\bar{q}) > 0$  denote the benefits from breaking the capacity constraint for the low-cost incumbent (direct benefit). Furthermore, let  $PLE_K^E = \pi_{inc,K}^{M,E} - \pi_{inc,K}^{D,E} > 0$  represent the profits loss that the  $K$ -type incumbent suffers due to entry, where  $K = \{H, L\}$ . If despite not expanding her facility, entry follows, the low-cost incumbent experiences a profit loss of  $PLE_L^{NE} = \pi_{inc,L}^{M,NE}(\bar{q}) - \pi_{inc,L}^{D,NE}(\bar{q}) > 0$ .<sup>15</sup> Finally, note that the profit loss from entry for the high-cost incumbent when she expands,  $PLE_H^E = \pi_{inc,H}^{M,E} - \pi_{inc,H}^{D,E}$ , coincides with that when he does not,  $PLE_H^{NE} = \pi_{inc,H}^{M,NE} - \pi_{inc,H}^{D,NE}$ , since  $\pi_{inc,H}^{M,E} = \pi_{inc,H}^{M,NE}$  under monopoly and  $\pi_{inc,H}^{D,E} = \pi_{inc,H}^{D,NE}$  under duopoly.

Hence, if expansion deters entry, the low-cost incumbent benefits from  $BCC_L$ , while the high-cost incumbent obtains no benefits or losses. If, in contrast, expansion does not deter entry, the low-cost incumbent’s  $BCC_L$  benefit is reduced by the profit loss of sharing the market with the entrant, i.e.,

<sup>13</sup> If, in contrast, the duopoly profits that the entrant obtains when competing against a low-cost incumbent who did not expand,  $\pi_{ent,L}^{D,NE}$ , are sufficiently high, then  $\pi_{ent,L}^{D,NE} > F$ . We analyze that extension of the model in section 4.

<sup>14</sup> Nonetheless, the above two conditions can be summarized as  $\pi_{ent,H}^{D,NE} > F > \pi_{ent,L}^{D,NE}$ . In particular, the entrant’s profits satisfy  $\pi_{ent,H}^{D,E} = \pi_{ent,H}^{D,NE}$  since the high-cost incumbent is not affected by the capacity constraint—and therefore the expansion decision does not modify her production capacity in the second period—but  $\pi_{ent,L}^{D,NE} > \pi_{ent,L}^{D,E}$  given that the entrant’s duopoly profits decrease when the low-cost incumbent eliminates her capacity constraint.

<sup>15</sup> Note that capacity constraint  $\bar{q}$  affects the low-cost incumbent both under monopoly and duopoly if she does not expand, i.e.,  $PLE_L^{NE} = \pi_{inc,L}^{M,NE}(\bar{q}) - \pi_{inc,L}^{D,NE}(\bar{q})$ . At the end of section 3 we consider, instead, that the capacity constraint only affects her as a monopolist, yielding a profit loss due to entry of  $PLE_L^{NE} = \pi_{inc,L}^{M,NE}(\bar{q}) - \pi_{inc,L}^{D,NE}$ .

$BCC_L - PLE_L^E < BCC_L$ , whereas the high-cost incumbent only bears the profit loss due to entry, i.e.,  $-PLE_H^E < 0$ . Note that expansion benefits when entry ensues,  $BCC_L - PLE_L^E$ , are only positive if  $BCC_L - PLE_L^E = \pi_{inc,L}^{D,E} - \pi_{inc,L}^{M,NE}(\bar{q}) > 0$ . Intuitively, this condition holds when the low-cost incumbent is relatively limited by her capacity constraint, and thus her monopoly profits before expanding her facility,  $\pi_{inc,L}^{M,NE}(\bar{q})$ , are lower than her duopoly profits after the expansion,  $\pi_{inc,L}^{D,E}$ . This implies that her benefit from breaking the capacity constraint completely offsets the profit loss associated to entry, i.e.,  $BCC_L > PLE_L^E$ . Finally, if despite not expanding entry occurs, the incumbent only experiences a profit loss due to entry of  $-PLE_K^{NE} < 0$  for all  $K = \{H, L\}$ .

**Single-Crossing Property.** The single-crossing property is not necessarily satisfied under all conditions. In particular, if expansion deters entry, the low-cost incumbent obtains larger benefits from expanding her facility,  $BCC_L > 0$ , than the high-cost does, i.e., the latter obtains zero profits from breaking the capacity constraint. When expansion attracts entry, however, the benefits for the low-cost incumbent,  $BCC_L - PLE_L^E$ , are larger than for the high-cost type,  $-PLE_H^E < 0$ , if and only if  $\pi_{inc,L}^{D,E} - \pi_{inc,L}^{M,NE}(\bar{q}) > \pi_{inc,H}^{D,E} - \pi_{inc,H}^{M,E}$ . More precisely, since  $\pi_{inc,H}^{D,E} - \pi_{inc,H}^{M,E} < 0$ , the previous condition is guaranteed to hold as long as  $\pi_{inc,L}^{D,E} > \pi_{inc,L}^{M,NE}(\bar{q})$ .<sup>16</sup> From our previous discussion, this occurs when  $BCC_L - PLE_L^E > 0$  which intuitively implies that the low-cost incumbent's benefit from breaking her capacity constraint outweighs her profit loss from attracting entry.<sup>17</sup>

### 3. Equilibrium Analysis

Before analyzing the set of Perfect Bayesian Equilibria (PBE) of this signaling game, let us introduce some additional notation. In particular, let  $p^{NE}$  denote the probability that makes an entrant indifferent between the expected profits from Cournot competition after no expansion,

---

<sup>16</sup> Note that  $\pi_{inc,L}^{D,E} > \pi_{inc,L}^{M,NE}(\bar{q})$  is a sufficient condition whereas  $\pi_{inc,L}^{D,E} - \pi_{inc,L}^{M,NE}(\bar{q}) > \pi_{inc,H}^{D,E} - \pi_{inc,H}^{M,E}$  is a necessary condition for the single-crossing property to hold. Hence, if  $\pi_{inc,L}^{D,E} > \pi_{inc,L}^{M,NE}(\bar{q})$  is not satisfied, condition  $\pi_{inc,L}^{D,E} - \pi_{inc,L}^{M,NE}(\bar{q}) > \pi_{inc,H}^{D,E} - \pi_{inc,H}^{M,E}$  can still hold if, for instance, the profit loss due to entry for the high-cost incumbent,  $PLE_H^E$ , is relatively large.

<sup>17</sup> A similar argument would also be valid if the incumbent's efficiency was drawn from a continuum of possible levels and not only from two levels, high and low, and if the incumbent were allowed to choose from a continuum of expansion investments. In general, when expansion is followed by entry, the low-cost incumbent does not necessarily obtain a larger benefit from marginally increasing her investment in comparison to a high-cost incumbent.

$p(\pi_{ent,H}^{D,NE} - F) + (1-p)(\pi_{ent,L}^{D,NE} - F)$  and the profits from operating in the alternative market,<sup>18</sup> where superscript *NE* represents that the incumbent *did not expand* her facility. Similarly, let  $p^E$  denote the probability that makes the entrant indifferent between the expected profits from Cournot competition after expansion,  $p(\pi_{ent,H}^{D,E} - F) + (1-p)(\pi_{ent,L}^{D,E} - F)$  and the profits from operating in the alternative market, where superscript *E* represents that the incumbent *expanded* her facility.<sup>19</sup> It is easy to show that  $p^{NE} < p^E$ , intuitively indicating that entry can be sustained under a larger set of priors if the incumbent did not expand, for all  $p > p^{NE}$ , than if she did, for all  $p > p^E$ . Our description of equilibrium outcomes is separated into three regimes according to the priors: low,  $p < p^{NE}$ , intermediate,  $p^{NE} \leq p < p^E$ , and high priors,  $p \geq p^E$ . The following proposition identifies the set of PBE under the first regime.

**Proposition 1.** *When priors are relatively low,  $p < p^{NE}$ , the following strategy profiles can be supported as PBEs of the expansion signaling game:*

1. *A separating equilibrium where the incumbent chooses  $(NoExp_H, Exp_L)$ , and the entrant selects  $(Enter_{NoExp}, NoEnter_{Exp})$  if and only if expansion costs satisfy  $K_H > PLE_H^E$  and  $K_L < BCC_L - PLE_L^E$  and the entrant's beliefs are  $\mu(H | NoExp) = 1$  and  $\mu(H | Exp) = 0$ ;*
2. *A pooling equilibrium with expansion,  $(Exp_H, Exp_L)$ , and the entrant selects  $(Enter_{NoExp}, NoEnter_{Exp})$  if and only if expansion costs satisfy  $K_L < BCC_L + PLE_L^{NE}$  and  $K_H < PLE_H^E$  and the entrant's beliefs are  $\mu(H | Exp) = p < p^E$  and  $\mu(H | NoExp) \geq p^{NE}$ ;*
3. *A pooling PBE where both types of incumbent do not expand their facility  $(NoExp_H, NoExp_L)$  followed by no entry, where either:*
  - a) *the entrant's strategy is  $(NoEnter_{NoExp}, Enter_{Exp})$  given that her off-the-equilibrium beliefs are  $\mu(H | Exp) = \mu \geq p^E$ , for expansion costs satisfying  $K_L > BCC_L - PLE_L^E$  and  $K_H > 0$ ; or*
  - b) *the entrant's strategy is  $(NoEnter_{NoExp}, NoEnter_{Exp})$  given that her off-the-equilibrium beliefs are  $\mu(H | Exp) = \mu < p^E$ , for expansion costs satisfying  $K_L > BCC_L$  and  $K_H > 0$ .*

The following figure depicts the set of expansion costs  $(K_H, K_L)$  under which each of the above PBEs can be sustained. First, when the low-cost expansion costs are relatively low,  $K_L < BCC_L - PLE_L^E$ , but those of

<sup>18</sup> Recall that the entrant obtains zero profits on the alternative perfectly competitive market where information is readily available to any potential entrants.

<sup>19</sup> The expressions for  $p^{NE}$  and  $p^E$  are obtained by solving for  $p$  in these indifference conditions. They are both included in the appendix. We show that  $p^E, p^{NE} \in [0, 1]$ , and that these expressions satisfy  $p^{NE} < p^E$  under all parameter values.

the high-cost incumbent are relatively high, i.e.,  $K_H > PLE_H^E$ , the former expands her facility while the latter does not. Under these expansion costs, hence, information about the incumbent's cost structure is perfectly transmitted to the entrant, deterring her from entering after observing an expansion. Despite the information transmission, however, note that the incumbent's expansion decision can be supported under different parameter conditions than under complete information. Specifically, if the entrant is perfectly informed about the incumbent's costs being low, he is deterred, and hence the low-cost incumbent expands if expansion costs satisfy  $K_L < BCC_L$ , i.e., if expansion costs are lower than the only benefit from expansion under complete information embodied in  $BCC_L$ . In contrast, the high-cost incumbent does not expand since entry ensues and such expansion does not bring any direct benefit. Hence, under complete information the low (high)-cost incumbent expands (does not expand) if  $K_L > BCC_L$  and for any  $K_H > 0$ . Therefore, the entrant's lack of information about the incumbent's type induces the low-cost incumbent to expand her facility under a larger set of expansion costs,  $K_L < BCC_L - PLE_L^E$ , than in the complete information setting,  $K_L < BCC_L$ , since  $BCC_L - PLE_L^E < BCC_L$ . Intuitively, the low-cost incumbent is willing to incur larger expansion costs in order to convey her type to the entrant, thus deterring entry. As a consequence, an increase in the profit loss associated to entry induces the incumbent to expand under higher expansion costs, ultimately enlarging the wedge between the set of parameter values in which expansion is profitable under complete information and those for which expansion is sustained under incomplete information.

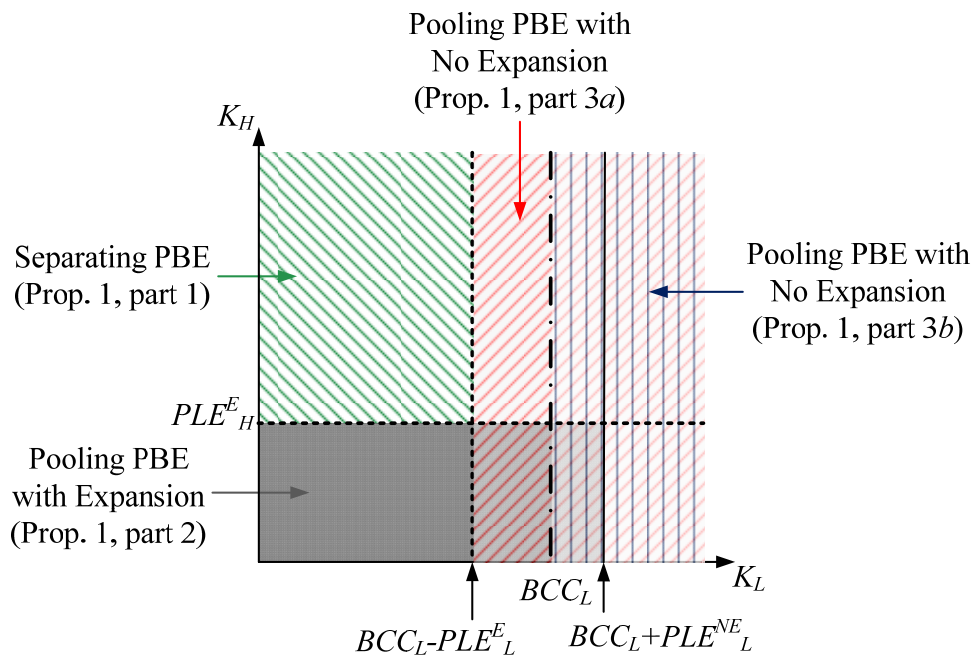


Figure 1: Equilibrium predictions under low priors.

The figure also depicts a pooling equilibrium (described in part 2 of the Proposition 1) whereby both types of incumbent expand their facility given that expansion costs are relatively low, i.e.,  $K_L < BCC_L + PLE_L^{NE}$  and  $K_H < PLE_H^E$ . By expanding, incumbents successfully deter entry, which occurs because prior probability  $p$  is sufficiently low. Hence, the high-cost incumbent expands if the profit loss she avoids by deterring entry is larger than her expansion costs, i.e.,  $K_H < PLE_H^E$ . On the other hand, the low-cost incumbent expands if her expansion cost is lower than the foregone benefits from expansion, which arise not only from the profit loss she avoids by deterring entry but also from the benefits she obtains by breaking the capacity constraint, i.e.,  $K_L < BCC_L + PLE_L^{NE}$ .

Finally, the pooling equilibrium described in part 3 of Proposition 1 examines the case where no type of incumbent expands given the high expansion costs and that the entrant is deterred after observing no expansion. This outcome is supported in both pooling equilibria described in parts 3a and 3b. These equilibria differ, however, in their off-the-equilibrium predictions. In particular, the equilibrium in part 3a considers that, after observing a deviation towards expansion, the entrant believes that the incumbent's costs must be high. These off-the-equilibrium beliefs are, however, not very sensible and do not survive standard equilibrium refinements.<sup>20</sup> If, by contrast, the entrant believes that a deviation towards expansion proceeds from a low-cost incumbent, then he does not enter. In this setting, the incumbent is therefore protected from entry both after expanding and not expanding her facility. Hence, the monopolist expands if her benefits from breaking the capacity constraint ( $BCC_L$  for the low-cost and zero for the high-cost incumbent) are larger than her corresponding expansion costs.

Let us next analyze how an increase in the efficiency of the low-cost incumbent affects our equilibrium results. In particular, a reduction in  $c_L$  reflects a more efficient low-cost incumbent.<sup>21</sup> In particular, a more efficient incumbent becomes more limited by her capacity constraint and, therefore, obtains a larger benefit from expanding her facility, i.e.,  $BCC_L$  raises. Note that the pooling equilibria described in Proposition 1 (part 3) —where the low-cost incumbent does not expand— are sustained under more restrictive parameter conditions as the low-cost incumbent becomes more efficient, i.e., the region corresponding to this equilibrium in figure 1 shrinks. Indeed, a larger efficiency increases her benefits from breaking the capacity constraint,  $BCC_L$ , inducing her to expand under larger expansion costs. In contrast, an increase in the efficiency level of the low-cost incumbent expands the set of parameter values under which we can support the separating equilibrium and the pooling equilibrium

---

<sup>20</sup> Appendix 1 shows that the deviation towards expansion is more likely to originate from the low- than from the high-cost incumbent. Therefore, the pooling equilibrium in part 3a violates the Cho and Kreps' (1987) Intuitive Criterion under most parameter conditions.

<sup>21</sup> This implies that  $BCC_L < PLE_L^{NE}$  when the incumbent is not very efficient ( $c_L$  is relatively high), but  $BCC_L > PLE_L^{NE}$  when her efficiency increases ( $c_L$  is relatively low).

where both types of incumbent expand their facility. Let us next examine equilibrium outcomes when priors are intermediate, i.e.,  $p^{NE} \leq p < p^E$ .

**Proposition 2.** *When priors satisfy  $p^{NE} \leq p < p^E$ , the strategy profiles described in parts 1 and 2 of Proposition 1 can still be supported as PBEs of the expansion signaling game. In addition, a pooling equilibrium with no expansion can be sustained,  $(NoExp_H, NoExp_L)$ , followed by entry (since priors satisfy  $\mu(H | NoExp) = p \geq p^{NE}$ ) if either of the following two cases arises:*

- a) *The entrant's strategy is  $(Enter_{NoExp}, NoEnter_{Exp})$  given that her off-the-equilibrium beliefs are  $\mu(H | Exp) = \mu < p^E$  and expansion costs satisfy  $K_L > BCC_L + PLE_L^E$  and  $K_H > PLE_H^E$ ; or*
- b) *The entrant's strategy is  $(Enter_{NoExp}, Enter_{Exp})$  given that her off-the-equilibrium beliefs are  $\mu(H | Exp) = \mu \geq p^E$  and expansion costs satisfy  $K_L > BCC_L + PLE_L^{NE} - PLE_L^E$  and  $K_H > 0$ .*

The following figure depicts equilibrium outcomes when the prior probability of facing a high-cost incumbent is intermediate.<sup>22</sup>

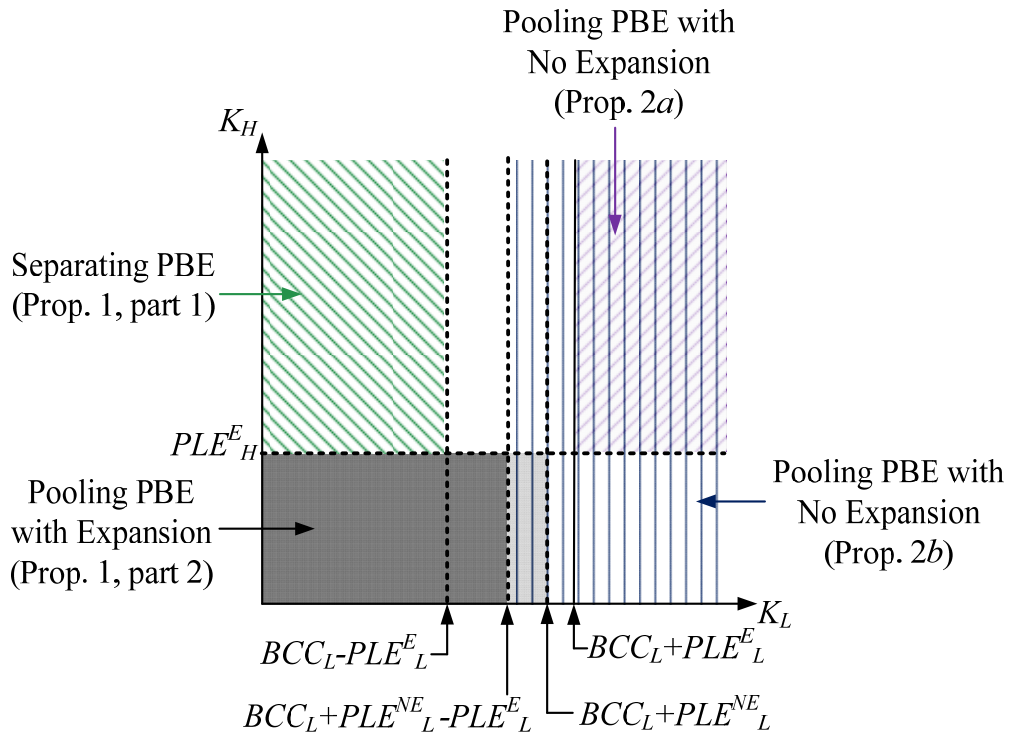


Figure 2: Equilibrium predictions under intermediate priors.

Relative to the set of equilibria when priors are relatively low depicted in figure 1, two equilibrium outcomes can still be supported under intermediate priors: the (fully informative) separating

<sup>22</sup> Note that the figure depicts the case where  $PLE_H^E > PLE_L^{NE}$ . An analogous figure can be constructed otherwise.

equilibrium and the pooling equilibrium where both types of incumbent expand their facility. Nonetheless, two pooling equilibria with no expansion emerge; as described in Proposition 2a and 2b.<sup>23</sup> First, Proposition 2a specifies a pooling equilibrium similar to that in Proposition 1, part 3a. Unlike that equilibrium, however, the entrant is now attracted to the market after observing no expansion (in equilibrium) since priors are relatively higher, both in the equilibria described in Proposition 2a and 2b. First, if entry follows after no expansion but does not otherwise (as described in Proposition 2a), the high-cost incumbent does not expand (as prescribed) if the benefit she would obtain from deterring entry —by deviating towards expansion— is lower than her expansion costs, i.e.,  $K_H > PLE_H^E$ . Similarly, the low-cost incumbent does not expand if the benefit she would obtain from expanding (not only arising from avoiding entry but also from breaking the capacity constraint) is lower than her expansion costs, i.e.,  $K_L > BCC_L + PLE_L^E$ . If, in contrast, the entrant believes that a deviation towards expansion must originate from a high-cost incumbent, then entry follows regardless of the incumbent’s action (as described in Proposition 2b). These off-the-equilibrium beliefs, however, are not very sensible since the low-cost incumbent is more likely to deviate towards expansion than the high-cost incumbent, and indeed violate standard equilibrium refinements.<sup>24</sup> Let us finally examine our equilibrium predictions under relatively high priors, i.e.,  $p \geq p^E$ .

**Proposition 3.** *When priors satisfy  $p \geq p^E$ , only the separating strategy profile described in Proposition 1 (part 1), and the pooling strategy profiles with no expansion specified in Proposition 2a and 2b can be supported as PBEs of the expansion signaling game.*

Therefore, the pooling equilibrium where both types of incumbent expand cannot be sustained when priors are relatively high. Intuitively, the expansion decision by both types of incumbent keeps the entrant “in the dark” about the incumbent’s cost structure and entry is deterred when priors satisfy  $p < p^E$ . When priors are relatively high  $p \geq p^E$ , however, this strategy would attract entry, leading the incumbent to not use it. All other equilibrium outcomes can still be sustained in this context.

**Not severe capacity constraints.** In our previous analysis, we consider that the low-cost incumbent is severely limited by her capacity constraint. In particular, when she does not expand her facility she faces

---

<sup>23</sup> In addition, Appendix 2 shows that, under relatively general conditions, a semiseparating equilibrium can be sustained, whereby one or both types of incumbent randomize their expansion decision.

<sup>24</sup> Appendix 1 shows that the pooling equilibrium described in Proposition 2b violates the Cho and Kreps’ (1987) Intuitive Criterion for all expansion costs lower than the benefits that the low-cost incumbent obtains from breaking her capacity constraint and protecting the market, i.e.,  $K_L < BCC_L + PLE_L^E$

a capacity constraint both as a monopolist and as a duopolist, yielding profits of  $\pi_{inc,L}^{M,NE}(\bar{q})$  and  $\pi_{inc,L}^{D,NE}(\bar{q})$ , respectively. Therefore, the profit loss due to entry, when the low-cost incumbent is severely limited by her capacity constraint was defined as  $PLE_L^{NE} \equiv \pi_{inc,L}^{M,NE}(\bar{q}) - \pi_{inc,L}^{D,NE}(\bar{q})$ . If, however, the capacity constraint affects the incumbent only as a monopolist, then the profit loss can be expressed as  $\pi_{inc,K}^{M,NE}(\bar{q}) - \pi_{inc,K}^{D,NE}$ . Since profits are lower when the capacity constraint is binding,  $\pi_{inc,K}^{D,NE}(\bar{q}) < \pi_{inc,K}^{D,NE}$ , the profit loss is more substantial when the incumbent is severely limited by the capacity constraint than otherwise. Let us evaluate the consequences of a less severe capacity constraint in our equilibrium results. Importantly, our above discussion shows that only the profit loss from entry is reduced.<sup>25</sup> When priors are relatively low, this implies that the set of parameter values supporting the pooling equilibrium with expansion (described in Proposition 1, part 2) shrinks. Intuitively, the low-cost incumbent obtains a smaller benefit from deterring entry, and therefore expands only under cheaper expansion costs. When priors are intermediate or high, a relaxation in the capacity constraint also produces a larger set of expansion costs sustaining the pooling equilibrium in which both types of incumbent do not expand; as described in Proposition 2b.

**Remark.** For simplicity, we focus on the set of pure-strategy PBEs. Appendix 2 elaborates on the properties of equilibria in which either one type of incumbent (or both) randomize their expansion decision, i.e., semiseparating equilibria. Intuitively, this appendix shows that when priors are low, semiseparating equilibria can be sustained under parameter conditions for which pure-strategy PBE already exists. When priors are relatively higher, however, semiseparating equilibria can be supported for regions of expansion costs where no equilibrium could be sustained when firms are restricted to use pure strategies. These semiseparating equilibria, nonetheless, predict that the low-cost incumbent does not necessarily expand her facility with a higher probability than the high-cost incumbent, thus limiting the informative role of the incumbent's expansion decision. Importantly, these equilibria can be sustained when both types of incumbent face relatively low expansion costs. Therefore, a policy that lowers expansion costs for both types of incumbent—which occurs, for instance, if government agencies cannot observe the incumbent's costs—can potentially promote this type of semiseparating equilibria, whereby information is essentially concealed from the entrant.

---

<sup>25</sup> In particular, allowing for the low-cost incumbent to be unaffected by her capacity constraint under duopoly only lowers  $PLE_L^{NE}$  but does not affect  $BCC_L$ ,  $PLE_H^{NE}$  or  $PLE_K^E$ , for all  $K = \{H, L\}$ .



## 4. Extensions

In this section we consider different extensions of our model. First, we show that a similar information transmission can be sustained when the incumbent's constraint arises from a significant market demand rather than from her efficiency level considered above. Furthermore, note that our model does not allow for expansion to serve as an entry deterrence tool, but merely as a device to convey the incumbent's cost-structure to the entrant and thus deter him from the market. Indeed, our previous assumptions consider that the entrant enters (stays out) when the incumbent's costs are high (low, respectively), regardless of her expansion decision. In our second extension we investigate how our equilibrium results are affected if the low-cost incumbent experiences entry when she does not expand, but deters it otherwise.

**Market demand as a capacity constraint.** Consider an analogous signaling model where the incumbent's production costs are common knowledge but market demand is only observed by the incumbent. The entrant, however, only knows the prior probability that demand is high or low. Similarly as in our previous model, the incumbent decides whether to expand her facility, and the entrant, observing the incumbent's decision, chooses to enter the market. In this case, the incumbent experiences a capacity constraint when demand is high, but does not when demand is low.<sup>26</sup> The entrant prefers to enter the market when demand is high but stay out otherwise. Hence, let  $BCC_H \equiv \pi_{inc,H}^{M,E} - \pi_{inc,H}^{M,NE}(\bar{q}) > 0$  denote the benefit that the high-demand incumbent obtains from breaking her capacity constraint. Similarly as in our above setting, let  $PLE_K^E = \pi_{inc,K}^{M,E} - \pi_{inc,K}^{D,E} > 0$  represent the profit loss due to entry that the incumbent suffers after expanding her facility, while  $PLE_K^{NE} = \pi_{inc,K}^{M,NE}(\bar{q}) - \pi_{inc,K}^{D,NE}(\bar{q}) > 0$  denote the profit loss after no expansion, for any demand level  $K = \{H, L\}$ . The following result shows that the strategy profiles specified in propositions 1-3 can also be supported in this information context by switching the type of incumbent who experiences a capacity constraint: from the low-cost incumbent in the previous setting to the high-demand incumbent in this environment. Hence, regions of expansion costs  $(K_H, K_L)$  sustaining every equilibrium are switched, from the low-cost (high-cost) incumbent to the high-demand (low-demand) incumbent, respectively. For instance, the separating equilibrium where only the constrained high-demand incumbent expands can be supported if  $K_H < BCC_H - PLE_H^E$  and  $K_L > PLE_L^E$ .

---

<sup>26</sup> Similarly as in our previous model, each capacity constraint  $\bar{q}$  can be interpreted as a maximum production level that the high-demand incumbent cannot exceed, but also as an increase in the incumbent's marginal costs of production when her output exceeds  $\bar{q}$ , as in Dixit (1980).

**Proposition 4.** *In the expansion signaling game where the potential entrant does not observe market demand, the separating and pooling strategy profiles described in Propositions 1-3 can be sustained as PBEs, where the regions of expansion costs supporting each equilibrium are switched across the two types of incumbent, from the low-cost (high-cost) incumbent to the high-demand (low-demand) incumbent, respectively.*

Thus, our results in the previous section can be extended to different information contexts where the incumbent suffering a capacity constraint conveys her type to the entrant in the separating equilibrium of the game regardless of the source of such constraint. A similar intuition is applicable to the pooling equilibria, whereby the incumbent's actions conceal whether or not she faces a capacity constraint.

**Expansion is not only an informative signal.** Let us finally extend our model to the case where the expansion decision can serve as an entry deterrence device, even under complete information. Similar to our previous assumptions, the high-cost incumbent's capacity does not affect the entrant's decision, since he enters both after observing expansion and no expansion. Unlike our initial setting, however, we consider that if the incumbent's costs are low, the entrant enters if the incumbent is still constrained (no expansion),  $\pi_{ent,L}^{D,NE} > F$ , but stays out otherwise,  $\pi_{ent,L}^{D,E} < F$ , i.e.,  $\pi_{ent,L}^{D,NE} > F > \pi_{ent,L}^{D,E}$ . Under these assumptions, the low-cost incumbent not only needs to convey her type to the entrant, but must also expand if she seeks to deter entry.<sup>27</sup> This result is confirmed in the following corollary.

**Corollary 1.** *In the expansion signaling game where the entrant is uninformed about the incumbent's cost structure and  $\pi_{ent,L}^{D,NE} > F > \pi_{ent,L}^{D,E}$ , all PBEs described in Propositions 1-3 can be sustained, except for the pooling equilibria with no expansion (followed by no entry) specified in Proposition 1 (part 3).*

Intuitively, the low-cost incumbent faces an additional incentive to expand, relative to our previous model: not only she must convey her type to the potential entrant if she wants to deter entry, but she must also expand her facility, i.e.,  $\pi_{ent,L}^{D,NE} > F > \pi_{ent,L}^{D,E}$ . This explains why the pooling equilibrium with no expansion (followed by no entry) specified in Proposition 1 (part 3) cannot be sustained in this setting.

---

<sup>27</sup> Note that in a complete information setting these incentives resemble those in Dixit (1980).

## 5. Discussion and policy implications

Let us next evaluate the welfare properties of the above equilibria. In the separating equilibrium, entry occurs in high-demand markets, but it is deterred in low-demand markets. This implies that the entry pattern described in the separating equilibrium of the game coincides with that arising under complete information.<sup>28</sup> If, instead, the entrant entered into low-demand markets, his overall profits (net of entry costs) would be negative, inducing the entrant to fail in the long run and exit the industry. In contrast, entry in the pooling equilibria does not ensue in similar conditions as under complete information, since it might occur (not occur) when entry is not (is, respectively) profitable given the entrant's lack of accurate information. For instance, entry follows in the pooling equilibrium even when demand is low if priors are sufficiently high, whereas entry does not occur despite demand being actually high when priors are sufficiently low. A similar argument can be extended to the context where the entrant is uninformed about the incumbent's efficiency level.

Our discussion suggests that public agencies should promote separating equilibria in order to support more desirable entry profiles. We can examine under which conditions this type of equilibrium occurs and what policies can facilitate it. The separating equilibrium can be sustained if the expansion costs that the constrained incumbent faces are significantly lower than those of the unconstrained incumbent. Such difference in expansion costs can arise if, for instance, constrained incumbents—either very efficient firms or incumbents operating in high-demand markets—accumulate profits before their expansion decision. Hence, a constrained incumbent could self-finance a larger portion of her expansion than the unconstrained incumbent, thus not having to access capital markets to the same extent as the unconstrained firm. In this case, no government intervention is necessary since, from our previous discussion, entry patterns in the separating equilibrium are similar to those under complete information.

If, however, financial institutions are unable to differentiate among both types of incumbent, firms face similar expansion costs. In such case, our results suggest socially improving policies. Specifically, if expansion costs are symmetric and relatively high, a policy reducing expansion costs can induce a change in incumbents' expansion decision, from a pooling equilibrium where no type of incumbent expands (producing undesirable entry patterns) to a separating equilibrium where only the constrained incumbent expands (generating desirable entry). The following figure illustrates the effect of this policy, as a reduction in the symmetric expansion costs (those along the diagonal  $K_H=K_L$ ) from point  $A$  to  $B$ .<sup>29</sup> Importantly, note that a policy radically reducing firms' expansion costs might produce undesirable outcomes; as depicted by point  $C$  in the figure. In particular, expansion costs are so

---

<sup>28</sup> A similar argument is applicable to the case in which the entrant is uninformed about the incumbent's efficiency.

<sup>29</sup> For simplicity, the figure considers  $p < p^{NE}$ . Analogous figures can be applied to different prior probabilities.

significantly reduced at  $C$  that both types of incumbent choose to expand their facility, hindering the ability of expansion decisions to serve as an informative signal for potential entrants.

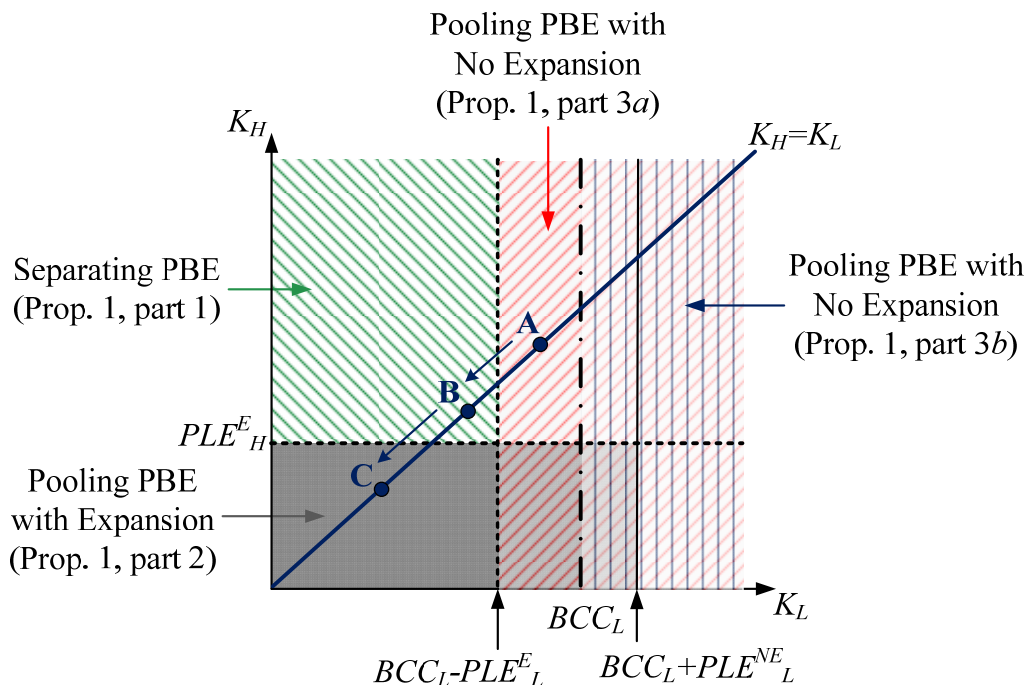


Figure 3: Effects of a policy reducing expansion costs.

Note that our previous result is applicable under all prior probabilities. When priors are relatively low, a symmetric decrease in expansion costs moves the outcome towards the pooling equilibrium where both types of incumbent expand and entry does not ensue. When priors are intermediate and high, such symmetric reduction in expansion costs moves equilibrium outcomes towards a semiseparating equilibrium where the low-cost incumbent does not necessarily expand with a larger probability than the high-cost incumbent, as suggested in section 3. Hence, policies reducing expansion costs do not necessarily induce desirable entry patterns.

In order to promote the separating equilibrium, our model suggests that government agencies should increase the benefits that firms experience when breaking their capacity constraint, e.g., subsidizing their production costs. Such policies, rather than a reduction of firms' fixed costs from expansion, can expand the set of parameter values under which the separating equilibrium arises in our model. Examples about this type of policies abound. Germany and Spain, for instance, were among the first countries to implement subsidies reducing the marginal production cost of firms installing solar cell panels.<sup>30</sup> In our model, such a subsidy would increase the constrained incumbent's post-entry profits,

<sup>30</sup> As documented in *The Economist*, on December 9<sup>th</sup> 2010.

thereby increasing  $BCC_L$  more significantly than  $PLE_K^E$ , ultimately enlarging the set of parameter values supporting the separating equilibrium. A similar argument can be applied for utility discount programs, such as the “Power for Jobs” program in New York State, which also reduces marginal production costs.<sup>31</sup>

Finally, note that the reduction of symmetric expansion costs promotes the separating equilibrium only if  $BCC_L - PLE_L^E > PLE_H^E$ . Intuitively, this occurs when the incumbent’s benefit from breaking her capacity constraint is significant, e.g., the capacity constraint is severe. Otherwise, a policy that reduces expansion costs for both types of incumbents only switches the pooling equilibrium being played, from one where no type of incumbent expands to that in which both types expand.<sup>32</sup> In line with our above discussion, in these contexts a policy reducing marginal production costs might be more appropriate than a subsidy in expansion costs.

## 6. Conclusions

This paper examines entry deterrence and signaling in a context where the incumbent experiences a capacity constraint. We demonstrate that separating and pooling equilibria can be sustained and that most of them survive standard equilibrium refinements. Our results suggest that severe capacity constraints expand the set of parameter values that support the fully-informative separating equilibrium. Otherwise, this set shrinks, leading to an expansion of the set sustaining pooling equilibria. Furthermore, we showed that if financial institutions discriminate constrained and unconstrained incumbents, and the financial costs that these two types of incumbent face when choosing to expand their facility are substantially different, the separating equilibrium can be supported under large parameter values. In this case, information about the incumbent’s cost structure (or market demand) is perfectly transmitted to the entrant, and entry patterns are desirable, calling for no government intervention. If, in contrast, both types of incumbent face similar expansion costs and the capacity constraint is severe, we identify a policy that can help move the industry towards the separating equilibrium (with similar entry patterns to those under complete information), namely, a reduction in expansion costs. Nonetheless, our results also show that this policy should not be overemphasized. Otherwise, such policy would leave undesirable entry patterns unaffected.

Our model offers several extensions for further research. First, the paper considers that the incumbent can only choose one specific investment level in order to expand her facility, which is sufficiently large to eliminate her capacity constraint. In richer settings, however, the incumbent might

---

<sup>31</sup> However, note that in order to guarantee that the entrant stays out of the market where the efficient incumbent operates the above policy might have to be accompanied by an increase in the administrative costs of entry,  $F$ .

<sup>32</sup> Graphically, this implies that cutoff  $BCC_L - PLE_L^E$  crosses cutoff  $PLE_H^E$  above the diagonal  $K_H = K_L$ .

choose among a continuum of investment levels, each of them yielding a different capacity.<sup>33</sup> Second, we consider that potential entrants only observe the incumbent's expansion decision, but are not able to observe the incumbent's output. In some industries, nonetheless, the entrant might observe both incumbent's actions. Entry patterns can hence be different to those in signaling games where the entrant either observes the incumbent's output alone —as in Milgrom and Roberts (1982)— or her expansion decision alone —as in this paper. Specifically, this is due to the fact that the incumbent's expansion decision breaks her capacity constraint in the second period but not in the first. Hence, a constrained incumbent in this setting would not be able to increase her first-period production level beyond  $\bar{q}$  in order to convey her efficiency level to the potential entrant, hampering the role of output as an informative signal. Hence, the introduction of an additional signal, rather than improving information transmission to the entrant, could potentially limit the dissemination of information.

---

<sup>33</sup> Both separating and pooling equilibria might still emerge in this context, since when expansion is followed by entry, the constrained incumbent does not necessarily obtain a larger benefit from marginally increasing her investment than the unconstrained incumbent does, as suggested in our discussion of the single-crossing property.

## APPENDIX

### Appendix 1 - Equilibrium refinement

**Proposition A.** All equilibria identified in Propositions 1 and 2 survive the Cho and Kreps' (1987) Intuitive Criterion, except for:

1. the pooling equilibrium of no expansion followed by  $[NoEnter_{NoExp}, Enter_{Exp}]$  as described in Proposition 1(part 3a), if expansion costs satisfy  $BCC_L > K_L > BCC_L - PLE_L^E$  and  $K_H > 0$ ;
2. the pooling equilibrium of no expansion followed by  $[NoEnter_{NoExp}, NoEnter_{Exp}]$  as described in Proposition 1(part 3b), if expansion costs satisfy  $BCC_L + PLE_L^{NE} > K_L > BCC_L$  and  $PLE_H^{NE} > K_H > 0$ ;
3. the pooling equilibrium of no expansion followed by  $[Enter_{NoExp}, Enter_{Exp}]$  as described in Proposition 2b, if expansion costs satisfy  $BCC_L + PLE_L^E > K_L > BCC_L + PLE_L^{NE} - PLE_L^E$  and  $PLE_H^{NE} > K_H > 0$ ; and if expansion costs satisfy  $BCC_L + PLE_L^E > K_L > BCC_L + PLE_L^{NE} - PLE_L^E$  and  $K_H > PLE_H^{NE}$ .

**Proof.**

**Pooling equilibrium (Proposition 2b):** If the high-cost incumbent deviates towards expansion, the highest payoff she can obtain is  $\pi_{inc,H}^{M,E} - K_H$ , which strictly exceeds her equilibrium payoff of  $\pi_{inc,H}^{D,NE}$  if and only if  $\pi_{inc,H}^{M,E} - \pi_{inc,H}^{D,NE} > K_H$ , and since  $\pi_{inc,H}^{M,E} = \pi_{inc,H}^{M,NE}$  this condition implies  $PLE_H^{NE} \equiv \pi_{inc,H}^{M,NE} - \pi_{inc,H}^{D,NE} > K_H$ . Hence, considering the equilibrium condition for the high-cost incumbent ( $K_H > 0$ ), she deviates towards expansion if and only if  $PLE_H^{NE} > K_H > 0$ . Similarly, if the low-cost incumbent deviates towards expansion, the highest payoff she can obtain is  $\pi_{inc,L}^{M,E} - K_L$ , which strictly exceeds her equilibrium payoff of  $\pi_{inc,L}^{D,NE}$  if and only if  $K_L < \pi_{inc,L}^{M,E} - \pi_{inc,L}^{D,NE}(\bar{q}) \equiv BCC_L + PLE_L^E$ . Considering the equilibrium condition for the low-cost incumbent ( $K_L > BCC_L$ ), she deviates towards expansion if and only if  $BCC_L + PLE_L^{NE} - PLE_L^E < K_L < BCC_L + PLE_L^E$ . Hence, the following cases can arise:

- If  $BCC_L + PLE_L^{NE} - PLE_L^E < K_L < BCC_L + PLE_L^E$  and  $0 < K_H < PLE_H^{NE}$  for the low-cost and high-cost incumbent, respectively, then both types of incumbent deviate towards expansion. Then the entrant's off-the-equilibrium beliefs are updated to  $\mu(H | Exp) = p$ , where  $p^{NE} \leq p < p^E$ , leading the entrant to stay out after observing this expansion. But then both types of incumbent have incentives to deviate towards expansion, and the pooling equilibrium without expansion violates the Intuitive Criterion for all  $BCC_L + PLE_L^{NE} - PLE_L^E < K_L < BCC_L + PLE_L^E$  and  $0 < K_H < PLE_H^{NE}$  and intermediate priors  $p^{NE} \leq p < p^E$ . If, instead, priors are relatively high,  $p \geq p^E$ , the entrant enters after observing the deviation towards expansion. Hence, the high-cost incumbent does not deviate since its profit from deviating towards expansion,  $\pi_{inc,H}^{D,E} - K_H$ , is lower than her equilibrium profit from not expanding,  $\pi_{inc,H}^{D,NE}$ , since  $\pi_{inc,H}^{D,E} = \pi_{inc,H}^{D,NE}$ . Similarly, the low-cost incumbent does not deviate towards expansion either if her profits from doing so,  $\pi_{inc,L}^{D,E} - K_L$ , is lower than her equilibrium profit,  $\pi_{inc,L}^{D,NE}(\bar{q})$  or  $K_L > \pi_{inc,L}^{D,E} - \pi_{inc,L}^{D,NE}(\bar{q}) \equiv BCC_L + PLE_L^{NE} - PLE_L^E$ , which holds in this pooling equilibrium. Therefore, no type of incumbent deviates and the pooling equilibrium without expansion of Proposition 2b survives the Intuitive Criterion for all  $BCC_L + PLE_L^{NE} - PLE_L^E < K_L < BCC_L + PLE_L^E$  and  $0 < K_H < PLE_H^{NE}$  and priors are relatively high, i.e.,  $p \geq p^E$ .

- If  $K_H > PLE_H^{NE}$  but  $BCC_L + PLE_L^{NE} - PLE_L^E < K_L < BCC_L + PLE_L^E$ , then the high-cost incumbent does not deviate towards expansion, but the low-costs incumbent does deviate. The entrant's off-the-equilibrium beliefs (after observing an expansion) become  $\mu(H | Exp) = 0$ , whereas his beliefs after observing no expansion are  $\mu(H | NoExp) = p \in [0,1]$ . Since  $p > p^{NE}$  holds in this equilibrium, the entrant does not enter after observing expansion, but enters otherwise, i.e.,  $[NoEnter_{exp}, Enter_{noexp}]$ . Given this response by the entrant after updating his off-the-equilibrium beliefs, the high-cost incumbent does not deviate towards expansion if  $\pi_{inc,H}^{D,NE}(\bar{q}) > \pi_{inc,H}^{M,E} - K_H$  or  $K_H > PLE_H^{NE}$ , which is satisfied in the case we consider. However, the low-cost incumbent deviates towards expansion since  $\pi_{inc,L}^{D,NE}(\bar{q}) < \pi_{inc,L}^{M,E} - K_L$ , or

$$0 < K_L < \pi_{inc,L}^{M,E} - \pi_{inc,L}^{D,NE}(\bar{q}) \equiv BCC_L + PLE_L^E.$$

Considering, in addition, this incumbent's equilibrium conditions ( $BCC_L + PLE_L^{NE} - PLE_L^E < K_L$ ), the above condition becomes  $BCC_L < K_L < BCC_L + PLE_L^E$ , which indeed holds in the case we consider. As a consequence, the low-cost incumbent deviates towards expansion. Hence, the pooling equilibrium (NoExp<sub>H</sub>, NoExp<sub>L</sub>) with (Enter<sub>NoExp</sub>, Enter<sub>Exp</sub>), as described in Proposition 2b, *violates* the Intuitive Criterion when expansion costs satisfy  $K_H > PLE_H^{NE}$  and  $BCC_L + PLE_L^{NE} - PLE_L^E < K_L < BCC_L + PLE_L^E$ .

- If  $K_L > BCC_L + PLE_L^{NE}$  and  $K_H > PLE_H^{NE}$ , then no incumbent deviates towards expansion. This implies that the entrant does not update his beliefs and therefore he responds by using the prescribed strategy (Enter<sub>Exp</sub>, Enter<sub>NoExp</sub>). Hence, the pooling (NoExp<sub>H</sub>, NoExp<sub>L</sub>) with (Enter<sub>Exp</sub>, Enter<sub>NoExp</sub>) survives the Intuitive Criterion.
- If  $K_L > BCC_L + PLE_L^{NE}$  but  $0 < K_H < PLE_H^{NE}$ , then the low-cost incumbent does not deviate towards expansion, but the high-cost incumbent does deviate. The entrant's off-the-equilibrium beliefs become  $\mu(H | Exp) = 1$ , whereas his beliefs after observing no expansion (in equilibrium) are  $\mu(H | NoExp) = p \in [0,1]$ . Since  $p > p^{NE}$  holds in this equilibrium, the entrant enters after observing expansion, and also enters after observing no expansion, i.e.,  $[Enter_{Exp}, Enter_{NoExp}]$ . Hence, the entrant's strategy coincides with that in equilibrium, and therefore both types of incumbent's equilibrium strategy are unaffected. Thus, this pooling equilibrium survives the Intuitive Criterion under expansion costs  $K_L > BCC_L + PLE_L^{NE}$  and  $0 < K_H < PLE_H^{NE}$ .

**Pooling equilibrium (Proposition 1, Part 3a):** If the low-cost incumbent deviates towards expansion the highest payoff she can obtain is  $\pi_{inc,L}^{M,E} - K_L$ , which strictly exceeds her equilibrium payoff of  $\pi_{inc,L}^{M,NE}$  if and only if  $K_L < \pi_{inc,L}^{M,E} - \pi_{inc,L}^{M,NE} \equiv BCC_L$ . Combining this condition with the parameter values under which this equilibrium is supported ( $K_L > BCC_L - PLE_L^E$ ), we obtain that the low-cost incumbent deviates towards expansion if  $BCC_L > K_L > BCC_L - PLE_L^E$ . Similarly, if the high-cost incumbent deviates towards expansion, the highest payoff she can obtain is  $\pi_{inc,H}^{M,E} - K_H$ , which does not exceed her equilibrium payoff of  $\pi_{inc,H}^{M,NE}$  since  $\pi_{inc,H}^{M,E} - K_H > \pi_{inc,H}^{M,NE}$  implies  $0 = \pi_{inc,H}^{M,E} - \pi_{inc,H}^{M,NE} > K_H$ . As consequence, the high-cost incumbent does not deviate under any parameter values. The following two cases can hence arise:

- If  $BCC_L > K_L > BCC_L - PLE_L^E$  only the low-cost incumbent has incentives to deviate towards expansion, which helps the entrant restrict his off-the-equilibrium beliefs to  $\mu(H | Exp) = 0$ . These beliefs induce no entry after observing expansion (and no entry after expansion either since  $\mu(H | NoExp) = p < p^{NE}$  in this equilibrium), i.e.,  $[NoEnter_{NoExp}, NoEnter_{Exp}]$ . Given this strategy for the entrant, the low-cost incumbent deviates towards expansion since  $\pi_{inc,L}^{M,E} - K_L > \pi_{inc,L}^{M,NE}(\bar{q})$  given that  $K_L < BCC_L$  holds in this case. In contrast,



the high-cost incumbent does not deviate towards expansion since  $\pi_{inc,H}^{M,E} - K_H < \pi_{inc,H}^{M,NE}$  for all  $K_H > 0$ . Therefore, only the low-cost incumbent deviates towards expansion, and the pooling PBE where (NoExp<sub>H</sub>, NoExp<sub>L</sub>) with (NEnter<sub>NoExp</sub>, Enter<sub>Exp</sub>), as described in Proposition 1, Part 3a, *violates* the Intuitive Criterion if expansion costs satisfy  $BCC_L > K_L > BCC_L - PLE_L^E$  and  $K_H > 0$ .

- If, instead,  $K_L > BCC_L > BCC_L - PLE_L^E$ , then no type of incumbent has incentives to deviate towards no expansion. Hence, the entrant's beliefs are unaffected, his strategy still coincides with that in the pooling equilibrium, i.e.,  $[NEnter_{NoExp}, Enter_{Exp}]$ , and this pooling equilibrium survives the Intuitive Criterion.

**Pooling equilibrium (Proposition 2a):** If the low-cost incumbent deviates towards expansion the highest payoff she can obtain is  $\pi_{inc,L}^{M,E} - K_L$ , which strictly exceeds her equilibrium payoff of  $\pi_{inc,L}^{D,NE}(\bar{q})$  if and only if  $K_L < \pi_{inc,L}^{M,E} - \pi_{inc,L}^{D,NE}(\bar{q}) \equiv BCC_L - PLE_L^E$ . This inequality, however, contradicts the parameter condition for the low-cost incumbent supporting this pooling PBE. As a consequence, she does not deviate towards expansion. Similarly, if the high-cost incumbent deviates towards expansion, the highest payoff she can obtain is  $\pi_{inc,H}^{M,E} - K_H$ , which strictly exceeds her equilibrium payoff of  $\pi_{inc,H}^{D,NE}$  if and only if  $K_H < \pi_{inc,H}^{M,E} - \pi_{inc,H}^{D,NE} \equiv PLE_H^E$ . This inequality also contradicts the parameter condition for the high-cost incumbent supporting this pooling PBE. Therefore, she does not deviate towards expansion either. Hence, no type of incumbent has incentives to deviate towards expansion, and the pooling equilibrium (NoExp<sub>H</sub>, NoExp<sub>L</sub>) with (E<sub>NEXP</sub>, NE<sub>EXP</sub>) survives the Intuitive Criterion.

**Pooling equilibrium (Proposition 1b):** If the low-cost incumbent deviates towards expansion the highest payoff she can obtain is  $\pi_{inc,L}^{M,E} - K_L$ , which strictly exceeds her equilibrium payoff of  $\pi_{inc,L}^{M,NE}$  if and only if  $K_L < \pi_{inc,L}^{M,E} - \pi_{inc,L}^{M,NE}(\bar{q}) \equiv BCC_L$ . This inequality, however, contradicts the parameter condition for the high-cost incumbent supporting this pooling PBE ( $K_L > BCC_L$ ). As a consequence, she does not deviate towards expansion. Similarly, if the high-cost incumbent deviates towards expansion, the highest payoff she can obtain is  $\pi_{inc,H}^{M,E} - K_H$ , which does not exceed her equilibrium payoff of  $\pi_{inc,H}^{M,NE}(\bar{q})$  for any  $K_H > 0$ . Therefore, the high-cost incumbent does not deviate towards expansion either, and the pooling equilibrium (NoExp<sub>H</sub>, NoExp<sub>L</sub>) with (NoEntry<sub>NEXP</sub>, NoEntry<sub>EXP</sub>) survives the Intuitive Criterion.

**Pooling equilibrium (Proposition 1, Part 3b):** If the high-cost incumbent deviates towards no expansion, the highest payoff she can obtain is  $\pi_{inc,H}^{M,NE}$ , which strictly exceeds her equilibrium payoff of  $\pi_{inc,H}^{M,E} - K_H$  for any  $K_H > 0$ , i.e.,  $\pi_{inc,H}^{M,NE} > \pi_{inc,H}^{M,E} - K_H$ , or  $K_H > \pi_{inc,H}^{M,E} - \pi_{inc,H}^{M,NE} = 0$ . Considering the equilibrium condition for the high-cost incumbent ( $K_H < PLE_H^{NE}$ ), this implies that she deviates towards no expansion if and only if  $0 < K_H < PLE_H^{NE}$ . Regarding the low-cost incumbent, if she deviates towards no expansion the highest payoff she can obtain is  $\pi_{inc,L}^{M,NE}(\bar{q})$ , which strictly exceeds her equilibrium payoff of  $\pi_{inc,L}^{M,E} - K_L$  if and only if  $K_L > \pi_{inc,L}^{M,E} - \pi_{inc,L}^{M,NE}(\bar{q}) \equiv BCC_L$ . Considering the equilibrium condition for the low-cost incumbent ( $K_L < BCC_L + PLE_L^{NE}$ ), she deviates towards no expansion if and only if  $BCC_L < K_L < BCC_L + PLE_L^{NE}$ . Hence, the following cases can arise:

- If  $BCC_L < K_L < BCC_L + PLE_L^{NE}$  and  $0 < K_H < PLE_H^{NE}$ , then both incumbents deviate towards no expansion, and the entrant's off-the-equilibrium beliefs become  $\mu(H | NoExp) = p$ , where  $p < p^{NE}$  (the pooling equilibrium of Proposition 1, part 3b holds only for relatively low priors). The entrant hence stays out after observing a deviation towards no expansion. Therefore, both types of incumbents have incentives to deviate towards no expansion if  $\pi_{inc,H}^{M,NE} > \pi_{inc,H}^{M,E} - K_H$ , or  $K_H > \pi_{inc,H}^{M,E} - \pi_{inc,H}^{M,NE} = 0$  for the high-cost incumbent and  $\pi_{inc,L}^{M,NE}(\bar{q}) > \pi_{inc,L}^{M,E} - K_L$ , or  $K_L > \pi_{inc,L}^{M,E} - \pi_{inc,L}^{M,NE}(\bar{q}) \equiv BCC_L$  for the low-cost incumbent. Since both

conditions on the expansion costs of the low and high-cost incumbent are satisfied, both types of incumbent deviate towards no expansion, and the pooling equilibrium with expansion of Proposition 1 (part 3b) violates the Intuitive Criterion for all:  $BCC_L < K_L < BCC_L + PLE_L^{NE}$  and  $0 < K_H < PLE_H^{NE}$ .

- If, instead,  $K_L < BCC_L < BCC_L + PLE_L^{NE}$  but  $0 < K_H < PLE_H^{NE}$ , then the low-cost incumbent does not deviate towards no expansion, while the high-cost incumbent does deviate. The entrant's off-the-equilibrium beliefs become  $\mu(H | NExp) = 1$ , whereas his beliefs after observing expansion are  $\mu(H | Exp) = p \in [0, 1]$ .

Since in this equilibrium priors satisfy  $p < p^E$ , the entrant stays out after observing expansion but enters after observing no expansion, i.e.,  $(NoEnter_{Exp}, Enter_{NExp})$ . The optimal response by the entrant after updating his beliefs, however, coincides with his response in this pooling equilibrium. As a consequence, the incumbent's expansion decision is unaffected, and we can conclude that this pooling equilibrium survives the Intuitive Criterion if expansion costs satisfy  $K_L < BCC_L < BCC_L + PLE_L^{NE}$  and  $0 < K_H < PLE_H^{NE}$ .

## Appendix 2 – Semiseparating equilibria

**Proposition B.** *The following strategy profiles can be supported as semi-separating PBEs of the game:*

1. *A strategy profile where the incumbent expands her facility when her costs are low,  $p_L=1$ , but expands with probability  $p_H \in (0,1)$  when her costs are high, where  $p_H = \frac{p^E}{1-p^E} \frac{1-p}{p}$ . In this equilibrium, after observing no expansion the entrant enters,  $s=1$ , and after observing expansion, the entrant enters with probability  $r = 1 - \frac{K_H}{PLE_H^E}$ , given beliefs  $\mu(H | NoExp) = 1$  and  $\mu(H | Exp) = p^E$ . This equilibrium can be only supported if priors are relatively high,  $p > p^E$ , and expansion costs satisfy  $0 < K_H < \min\{PLE_H^E, K_H^1\}$ , where  $K_H^1 \equiv -\frac{B}{PLE_L^E} + \frac{PLE_H^E}{PLE_L^E} K_L$  and  $B \equiv PLE_H^E [BCC_L + (PLE_L^{NE} - PLE_L^E)]$ .*
2. *A strategy profile where the incumbent expands her facility when her costs are high,  $p_H=1$ , but expands with probability  $p_L \in (0,1)$  when her costs are low, where  $p_L = \frac{p}{1-p} \frac{1-p^E}{p^E}$ . In this equilibrium, after observing no expansion the entrant stays out,  $s=0$ , and after observing expansion, the entrant enters with probability  $r = 1 - \frac{K_H}{PLE_H^E}$ , given beliefs  $\mu(H | NoExp) = 0$  and  $\mu(H | Exp) = p^E$ . This equilibrium can be only supported if priors are relatively high,  $p > p^E$ , and expansion costs satisfy  $K_L < PLE_L^E$ .*
3. *A strategy profile where the high-cost incumbent expands with probability  $p_H = \frac{p^E(p-p^{NE})}{(p^E-p^{NE})p}$ , and the low-*

*cost incumbent expands with probability  $p_L = \frac{(1-p^E)(p-p^{NE})}{(p^E-p^{NE})(1-p)}$ , where  $p_H, p_L \in (0,1)$  After observing*

*expansion (no expansion) the entrant enters with probability  $r$  ( $s$ , respectively), where*

$$r = \left[ K_L - BCC_L - \frac{K_H \times PLE_L^{NE}}{PLE_H^E} \right] \times \frac{1}{PLE_L^{NE} - PLE_L^E} \quad \text{and}$$

$$s = \left[ K_L - BCC_L - \frac{K_H \times PLE_L^E}{PLE_H^E} \right] \times \frac{1}{PLE_L^{NE} - PLE_L^E}$$

given beliefs  $\mu(H | NoExp) = p^{NE}$  and  $\mu(H | Exp) = p^E$ . This equilibrium can be only supported if priors are intermediate,  $p^{NE} \geq p > p^E$ , and: (1) expansion costs satisfy  $0 < K_H < \frac{C}{PLE_L^{NE}}$  and  $0 < K_L < \frac{PLE_L^E}{PLE_H^E} K_H + \frac{B}{PLE_H^E}$  if  $PLE_L^{NE} > PLE_L^E$ ; (2) expansion costs satisfy  $K_H > 0$  for all  $K_L \leq BCC_L$  and  $K_H > \frac{C}{PLE_L^E}$  otherwise if  $PLE_L^{NE} \leq PLE_L^E$ , where  $B \equiv PLE_H^E [BCC_L + (PLE_L^{NE} - PLE_L^E)]$  and  $C \equiv PLE_H^E (K_L - BCC_L)$ .

4. A strategy profile where the incumbent does not expand her facility when her costs are high,  $p_H=0$ , but expands with probability  $p_L \in (0,1)$  when her costs are low, where  $p_L = \frac{p^{NE} - p}{p^{NE}(1-p)}$ . In this equilibrium, after observing expansion the entrant stays out,  $r=0$ , and after observing no expansion the entrant enters with probability  $s = \frac{BCC_L}{PLE_L^E} + \frac{K_L}{PLE_L^E}$ , given beliefs  $\mu(H | Exp) = 0$  and  $\mu(H | NoExp) = p^{NE}$ . This equilibrium can be only supported if priors are relatively low,  $p < p^{NE}$ , and expansion costs satisfy  $PLE_L^E - BCC_L > K_L > \frac{PLE_L^E}{PLE_H^E} K_H - BCC_L$ .

5. A strategy profile where the incumbent does not expand her facility when her costs are low,  $p_L=0$ , but expands with probability  $p_H \in (0,1)$  when her costs are high, where  $p_H = \frac{p - p^{NE}}{p(1-p^{NE})}$ . In this equilibrium, after observing expansion the entrant enters,  $r=0$ , and after observing no expansion the entrant enters with probability  $s = \frac{K_H}{PLE_H^E}$ , given beliefs  $\mu(H | Exp) = 1$  and  $\mu(H | NoExp) = p^{NE}$ . This equilibrium can be only supported for priors,  $p > p^{NE}$ , and expansion costs satisfying

$$PLE_H^E > K_H > \frac{PLE_H^E}{PLE_L^E} K_L + \frac{BCC_L \times PLE_H^E}{PLE_L^E}$$

### Proof.

**$p_L=1$  and  $p_H \in (0,1)$ .** In this equilibrium, the entrant's beliefs after observing no expansion become  $\mu(H | NoExp) = 1$ , which leads him to enter since  $\pi_{ent,H}^{D,NE} > F$ . In the case that the entrant observes expansion, he mixes if his beliefs  $\mu(H | Exp)$  satisfy

$$\mu(H | Exp) \times (\pi_{ent,H}^{D,E} - F) + (1 - \mu(H | Exp)) (\pi_{ent,L}^{D,E} - F) = 0$$

and solving for  $\mu(H | Exp)$ , we obtain  $\mu(H | Exp) = \frac{F - \pi_{ent,H}^{D,E}}{\pi_{ent,L}^{D,E} - \pi_{ent,H}^{D,E}} \equiv p^E$ . We can now use the entrant's

posterior beliefs  $\mu(H | Exp) = p^E$  in order to find the probability,  $p_H$ , with which the high-cost incumbent randomizes, by using Bayes' rule as follows

$$\mu(H | Exp) = p^E = \frac{p \times p_H}{(p \times p_H) + (1 - p)}$$

Solving for  $p_H$  we obtain  $p_H = \frac{p^E}{(1-p^E)} \times \frac{(1-p)}{p}$ , where  $p_H \in (0,1)$  for all  $p^E < p$ . In addition, note that  $p_H$  is

decreasing in  $p$  since  $\frac{\partial p_H}{\partial p} = \frac{p^E}{(p^E - 1) \times p^2} < 0$ , starting from  $\lim_{p \rightarrow p^E} p_H = 1$  and converging to  $\lim_{p \rightarrow 1} p_H = 0$ .

Regarding the incumbent, when her costs are high, she mixes as prescribed,  $p_H \in (0,1)$ , if and only if

$$r \times (\pi_{inc,H}^{D,E} - K_H) + (1-r) \times (\pi_{inc,H}^{M,E} - K_H) = \pi_{inc,H}^{D,NE}$$

where  $r$  is the probability with which the entrant enters after observing expansion. Solving for  $r$ , we obtain

$$r = \frac{\pi_{inc,H}^{M,E} - \pi_{inc,H}^{D,NE} - K_H}{\pi_{inc,H}^{M,E} - \pi_{inc,H}^{D,E}}$$

and since  $PLE_H^E = \pi_{inc,H}^{M,E} - \pi_{inc,H}^{D,E}$  and  $\pi_{inc,H}^{D,NE} = \pi_{inc,H}^{D,E}$ , we can rewrite this probability as  $r = 1 - \frac{K_H}{PLE_H^E}$ , where

$r \in (0,1)$  only if expansion costs satisfy  $K_H < PLE_H^E$ . On the other hand, the low-cost incumbent expands as prescribed ( $p_L=1$ ) if and only if

$$r \times (\pi_{inc,L}^{D,E} - K_L) + (1-r) \times (\pi_{inc,L}^{M,E} - K_L) > \pi_{inc,L}^{D,NE}$$

which implies

$$r > \frac{\pi_{inc,L}^{M,E} - \pi_{inc,L}^{D,NE} - K_L}{\pi_{inc,L}^{M,E} - \pi_{inc,L}^{D,E}} = \frac{BCC_L + PLE_L^{NE} - K_L}{PLE_L^E} \equiv \hat{r},$$

Hence, probability  $r$  must satisfy  $r > \hat{r}$  which implies  $K_H < -\frac{B}{PLE_L^E} + \frac{PLE_H^E}{PLE_L^E} K_L \equiv K_H^1$  where

$B \equiv PLE_H^E [BCC_L + (PLE_L^{NE} - PLE_L^E)]$ . Hence, this semiseparating strategy profile can be supported as an

equilibrium if expansion costs satisfy  $0 < K_H < \min\{PLE_H^E, K_H^1\}$  and priors are relatively high, i.e.  $p > p^E$ .

**$p_H=1$  and  $p_L \in (0,1)$ .** In this equilibrium, the entrant's posterior beliefs after observing no expansion are  $\mu(H | NoExp) = 0$ , which leads him to stay out since  $\pi_{ent,L}^{D,NE} < F$ . In the case that the entrant observes expansion, he mixes if his beliefs  $\mu(H | Exp)$  satisfy

$$\mu(H | Exp) \times (\pi_{ent,H}^{D,E} - F) + (1 - \mu(H | Exp)) (\pi_{ent,L}^{D,E} - F) = 0$$

and solving for  $\mu(H | Exp)$ , we obtain  $\mu(H | Exp) = p^E$ . Using Bayes' rule,

$$\mu(H | Exp) = p^E = \frac{p}{p + (1-p)p_L}$$

and solving for  $p_L$  we obtain  $p_L = \frac{p}{1-p} \times \frac{1-p^E}{p^E}$ , where  $p_L \in (0,1)$  for all  $p < p^E$ . In addition,  $p_L$  is increasing

in  $p$  since  $\frac{\partial p_L}{\partial p} = \frac{1-p^E}{p^E(1-p)^2} > 0$ , starts at  $\lim_{p \rightarrow 0} p_L = 0$  and converges to  $\lim_{p \rightarrow p^E} p_L = 1$ .

Regarding the incumbent, when her costs are high, she expands as prescribed ( $p_H=1$ ) if and only if

$$r \times (\pi_{inc,H}^{D,E} - K_H) + (1-r) \times (\pi_{inc,H}^{M,E} - K_H) > \pi_{inc,H}^{M,NE}$$

solving for  $r$ , and using the property that  $\pi_{inc,H}^{M,NE} = \pi_{inc,H}^{M,E}$ , we find

$$r > \frac{-K_H}{\pi_{inc,H}^{M,E} - \pi_{inc,H}^{D,E}} = -\frac{K_H}{PLE_H^E} \equiv \tilde{r}$$

where cutoff  $\tilde{r} < 0$  for all parameter values. On the other hand, the low-cost incumbent randomizes as prescribed,  $p_L \in (0,1)$ , if and only if

$$r \times (\pi_{inc,L}^{D,E} - K_L) + (1-r) \times (\pi_{inc,L}^{M,E} - K_L) = \pi_{inc,L}^{D,E}.$$

which implies

$$r = \frac{\pi_{inc,L}^{M,E} - \pi_{inc,L}^{D,E} - K_L}{\pi_{inc,L}^{M,E} - \pi_{inc,L}^{D,E}} = 1 - \frac{K_L}{PLE_L^E},$$

where  $r \in (0,1)$  if  $K_L < PLE_L^E$ . Finally, since cutoff  $\tilde{r} < 0$ ,  $r > \tilde{r}$  under all parameter values. Hence, this semiseparating strategy profile can be supported as an equilibrium if expansion costs satisfy  $K_L < PLE_L^E$  for low and intermediate priors, i.e.,  $p < p^E$ .

**$p_L, p_H \in (0,1)$ .** In this equilibrium, after observing an expansion, the entrant is indifferent between entering and not entering the incumbent's market if and only if his posterior beliefs  $\mu(H | Exp)$  satisfy

$$\mu(H | Exp) \times (\pi_{ent,H}^{D,E} - F) + (1 - \mu(H | Exp)) (\pi_{ent,L}^{D,E} - F) = 0$$

and solving for  $\mu(H | Exp)$ , we obtain  $\mu(H | Exp) = \frac{F - \pi_{ent,H}^{D,E}}{\pi_{ent,L}^{D,E} - \pi_{ent,H}^{D,E}} \equiv p^E$ . We can then use the entrant's

posterior beliefs  $\mu(H | Exp) = p^E$  in order to find probability,  $p_H$ , with which the incumbent randomizes when her costs are high, by using Bayes' rule, as follows

$$\mu(H | Exp) = p^E = \frac{p \times p_H}{(p \times p_H) + ((1-p) \times p_L)}$$

Solving for  $p_H$  we obtain  $p_H(p_L) = \frac{p^E \times p_L \times (1-p)}{p \times (1-p^E)}$ . Similarly, after observing that the incumbent does not

expand, the entrant is indifferent between entering and not entering the incumbent's market if and only if his posterior beliefs  $\mu(H | NoExp)$  satisfy

$$\mu(H | NoExp) \times (\pi_{ent,H}^{D,NE} - F) + (1 - \mu(H | NoExp)) (\pi_{ent,L}^{D,NE} - F) = 0$$

and solving for  $\mu(H | NoExp)$  we obtain  $\mu(H | NoExp) = \frac{F - \pi_{ent,L}^{D,NE}}{\pi_{ent,H}^{D,NE} - \pi_{ent,L}^{D,NE}} \equiv p^{NE}$ . We can hence use the entrant's

posterior beliefs  $\mu(H | NoExp) = p^{NE}$  in order to find probability,  $p_L$ , with which the incumbent randomizes when her costs are low, by using Bayes' rule, as follows

$$\mu(H | NoExp) = p^{NE} = \frac{p \times (1-p_H)}{(p \times (1-p_H)) + ((1-p) \times (1-p_L))}$$

Solving for  $p_L$  we obtain  $p_L(p_H) = \frac{p^{NE} + (p_H - 1)p - p_H \times p^{NE} \times p}{p^{NE} \times (1-p)}$ . Solving for  $p_H$  and  $p_L$  simultaneously, we

obtain  $p_L = \frac{(1-p^E) \times (p - p^{NE})}{(p^E - p^{NE}) \times (1-p)}$  and  $p_H = \frac{p^E \times (p - p^{NE})}{(p^E - p^{NE}) \times p}$ . First, note that  $p_L \geq 0$  if and only if  $p \geq p^{NE}$ ,

given that  $p^E > p^{NE}$  and  $p, p^E, p^{NE} \in (0,1)$  under all parameter values. In addition,  $p_L < 1$  for all  $p < p^E$ .

Therefore,  $p_L \in (0,1)$  if and only if priors are intermediate, i.e.,  $p^{NE} \leq p < p^E$ . Second, note that  $p_H \geq 0$  if and only if  $p \geq p^{NE}$ . Furthermore,  $p_H < 1$  for all  $p < p^E$ . Therefore,  $p_H \in (0,1)$  only if priors are intermediate, i.e.,  $p^{NE} \leq p < p^E$ . We can therefore conclude that under intermediate priors both types of incumbent randomize their expansion decisions,  $p_H, p_L \in (0,1)$ , where note that  $p_H \geq p_L$  for all  $p < p^E$ , which holds in this regime of intermediate priors. We next show that these probabilities are both increasing in  $p$ , since

$$\frac{\partial p_L}{\partial p} = \frac{(1-p^E)(1-p^{NE})}{(1-p)^2(p^E - p^{NE})} > 0 \quad \text{and} \quad \frac{\partial p_H}{\partial p} = \frac{p^{NE} p^E}{p^2(p^E - p^{NE})} > 0.$$

Finally, note that at the lower bound of  $p \in [p^{NE}, p^E)$ , the incumbent's randomization becomes

$$\lim_{p \rightarrow p^{NE}} p_L = \lim_{p \rightarrow p^{NE}} p_H = 0 \quad \text{whereas at the upper bound, we obtain} \quad \lim_{p \rightarrow p^E} p_L = \lim_{p \rightarrow p^E} p_H = 1.$$

Let us now examine the incumbent's strategy in this equilibrium. If the high-cost incumbent expands with probability  $p_H \in (0,1)$ , as prescribed, it must be that the entrant makes her indifferent between expanding and not expanding her facility,

$$r \times (\pi_{inc,H}^{D,E} - K_H) + (1-r) \times (\pi_{inc,H}^{M,E} - K_H) = s \times \pi_{inc,H}^{D,NE} + (1-s) \times \pi_{inc,H}^{M,NE},$$

where  $r$  and  $s$  are the probability with which the entrant enters after observing expansion and no expansion, respectively. Solving for  $r$ , we obtain

$$r(s) = \frac{\pi_{inc,H}^{M,E} - \pi_{inc,H}^{M,NE} - K_H}{\pi_{inc,H}^{M,E} - \pi_{inc,H}^{D,E}} + s \left( \frac{\pi_{inc,H}^{M,NE} - \pi_{inc,H}^{D,NE}}{\pi_{inc,H}^{M,E} - \pi_{inc,H}^{D,E}} \right)$$

and since  $\pi_{inc,H}^{M,E} = \pi_{inc,H}^{M,NE}$  and  $\pi_{inc,H}^{D,E} = \pi_{inc,H}^{D,NE}$ , the above expression simplifies to  $r(s) = s - \frac{K_H}{PLE_H^E}$ .

Similarly, when the incumbent's costs are low, the entrant makes the incumbent indifferent between expanding and not expanding her facility,

$$r \times (\pi_{inc,L}^{D,E} - K_L) + (1-r) \times (\pi_{inc,L}^{M,E} - K_L) = s \times \pi_{inc,L}^{D,NE} + (1-s) \times \pi_{inc,L}^{M,NE}.$$

Solving for  $s$ , we obtain

$$s(r) = \frac{\pi_{inc,L}^{M,NE} + K_L - \pi_{inc,L}^{M,E}}{\pi_{inc,L}^{M,NE} - \pi_{inc,L}^{D,NE}} - r \left( \frac{\pi_{inc,L}^{D,E} - \pi_{inc,L}^{M,E}}{\pi_{inc,L}^{M,NE} - \pi_{inc,L}^{D,NE}} \right),$$

which simplifies to  $s(r) = \frac{K_L - BCC_L}{PLE_L^{NE}} + r \left( \frac{PLE_L^E}{PLE_L^{NE}} \right)$ . Solving for probabilities  $s$  and  $r$  simultaneously, we obtain

$$s = \left[ K_L - BCC_L - \frac{K_H \times PLE_L^E}{PLE_H^E} \right] \times \frac{1}{PLE_L^{NE} - PLE_L^E}$$

$$r = \left[ K_L - BCC_L - \frac{K_H \times PLE_L^{NE}}{PLE_H^E} \right] \times \frac{1}{PLE_L^{NE} - PLE_L^E}$$

First, note, that probability  $s$  is positive if and only if  $PLE_L^{NE} > PLE_L^E$  and  $\frac{PLE_H^E \times (K_L - BCC_L)}{PLE_L^E} > K_H$  hold.

Secondly,  $s < 1$  if and only if

$$K_H > \frac{PLE_H^E}{PLE_L^E} K_L - \frac{PLE_H^E \times (BCC_L + (PLE_L^{NE} - PLE_L^E))}{PLE_L^E} \quad (1)$$

Similarly, note that probability  $r$  is positive if and only if  $PLE_L^{NE} > PLE_L^E$  and  $\frac{PLE_H^E \times (K_L - BCC_L)}{PLE_L^{NE}} > K_H$  hold.

Finally, note that  $r < 1$  if and only if

$$K_H > \frac{PLE_H^E}{PLE_L^{NE}} K_L - \frac{PLE_H^E (BCC_L + (PLE_L^{NE} - PLE_L^E))}{PLE_L^{NE}} \quad (2)$$

Let us first analyze the case in which condition  $PLE_L^{NE} > PLE_L^E$  holds. In this case, probabilities  $r, s \in (0, 1)$  if expansion costs satisfy

$$\frac{C}{PLE_L^E} > K_H > \frac{PLE_H^E}{PLE_L^E} K_L - \frac{B}{PLE_L^E} \quad \text{and} \quad \frac{C}{PLE_L^{NE}} > K_H > \frac{PLE_H^E}{PLE_L^{NE}} K_L - \frac{B}{PLE_L^{NE}}$$

where  $B \equiv PLE_H^E [BCC_L + (PLE_L^{NE} - PLE_L^E)]$  and  $C \equiv PLE_H^E (K_L - BCC_L)$ .

First, note that  $\frac{C}{PLE_L^E} > \frac{C}{PLE_L^{NE}}$  since  $PLE_L^{NE} > PLE_L^E$  in the case we consider. Hence  $\frac{C}{PLE_L^{NE}} > K_H$  is more

restrictive than  $\frac{C}{PLE_L^E} > K_H$ . In order to rank expressions  $\frac{PLE_H^E}{PLE_L^E} \times K_L - \frac{B}{PLE_L^E}$  and  $\frac{PLE_H^E}{PLE_L^{NE}} \times K_L - \frac{B}{PLE_L^{NE}}$ ,

note that vertical intercepts satisfy  $0 > \frac{B}{PLE_L^{NE}} > \frac{B}{PLE_L^E}$  and the equation in condition (2) is flatter than that in (1)

since  $\frac{PLE_H^E}{PLE_L^E} > \frac{PLE_H^E}{PLE_L^{NE}}$ . Hence, condition (1) is more restrictive than (2). Thus, we only need to use

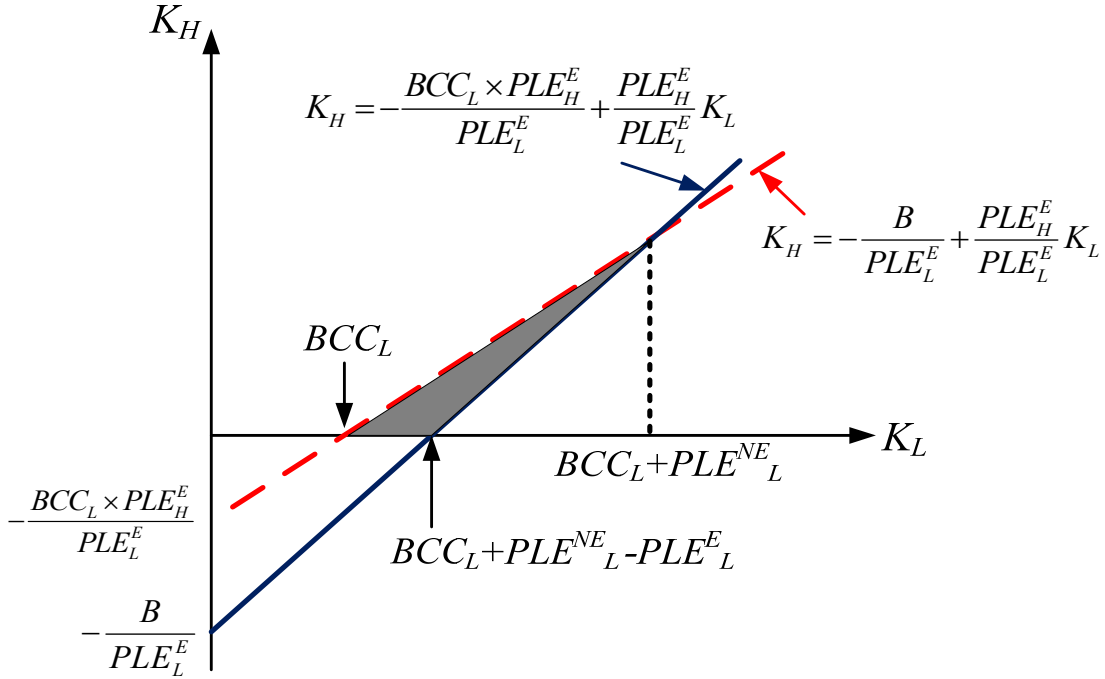
$$K_H > \frac{PLE_H^E}{PLE_L^E} K_L - \frac{B}{PLE_L^E}$$

Regarding condition  $\frac{C}{PLE_L^{NE}} > K_H$ , note that it can be expressed as

$$\frac{PLE_H^E}{PLE_L^{NE}} K_L - \frac{BCC_L \times PLE_H^E}{PLE_L^{NE}} > K_H$$

which is flatter than condition (1) since  $\frac{PLE_H^E}{PLE_L^{NE}} < \frac{PLE_H^E}{PLE_L^E}$ , and the vertical intercept is also smaller (absolute value)

than that of condition (1) since  $BCC_L \times PLE_H^E < B$  and  $PLE_L^{NE} < PLE_L^E$ , as depicted in the following figure.



Hence, when condition  $PLE_L^{NE} > PLE_L^E$  holds, this semiseparating equilibrium with  $p_H, p_L \in (0,1)$  can be supported for intermediate priors  $p^{NE} \geq p > p^E$  and expansion costs satisfying

$$0 < K_H < \frac{C}{PLE_L^{NE}} \quad \text{and} \quad 0 < K_L < \frac{PLE_L^E}{PLE_H^E} K_H + \frac{B}{PLE_H^E}.$$

If, in contrast,  $PLE_L^{NE} < PLE_L^E$  holds, probabilities  $r, s \in (0,1)$  if expansion costs satisfy

$$K_H > \frac{C}{PLE_L^E} \quad \text{and} \quad K_H > \frac{PLE_H^E}{PLE_L^E} K_L - \frac{B}{PLE_L^E}$$

and

$$K_H > \frac{C}{PLE_L^{NE}} \quad \text{and} \quad K_H > \frac{PLE_H^E}{PLE_L^{NE}} K_L - \frac{B}{PLE_L^{NE}}$$

From our previous discussion,  $\frac{C}{PLE_L^E} > \frac{C}{PLE_L^{NE}}$ . In addition,  $K_H > \frac{C}{PLE_L^E}$  is more restrictive than

$K_H > \frac{PLE_H^E}{PLE_L^E} K_L - \frac{B}{PLE_L^E}$  since both expressions have the same slope but the former originates at a higher vertical

intercept than the latter since  $BCC_L \times PLE_H^E < B$ . Therefore, when  $PLE_L^{NE} < PLE_L^E$  holds this semiseparating equilibrium with  $p_H, p_L \in (0,1)$  can be supported for intermediate priors  $p^{NE} \geq p > p^E$  and expansion costs satisfying

$$K_H > 0 \quad \text{for all} \quad K_L \leq BCC_L, \quad \text{and} \quad K_H > \frac{C}{PLE_L^E} \quad \text{otherwise.}$$

**$p_H = 0$  and  $p_L \in (0,1)$ .** We now check other semiseparating strategy profiles where either type of incumbent does not expand her facility. Let us first analyze the case where  $p_H = 0$  and  $p_L \in (0,1)$ . In this case, the entrant's posterior



beliefs become  $\mu(H | Exp) = 0$  after observing an expansion, which leads him to stay out since  $\pi_{ent,L}^{D,E} < F$ . In the case that the entrant observes no expansion from the incumbent, the entrant mixes if and only if his beliefs  $\mu(H | NoExp)$  satisfy

$$\mu(H | NoExp) \times (\pi_{ent,H}^{D,NE} - F) + (1 - \mu(H | NoExp)) (\pi_{ent,L}^{D,NE} - F) = 0$$

and solving for  $\mu(H | NoExp)$ , we obtain  $\mu(H | NoExp) = p^{NE}$ . We can hence use Bayes' rule to find

$$\mu(H | NoExp) = p^{NE} = \frac{p}{p + (1-p)(1-p_L)}$$

Solving for  $p_L$  we obtain  $p_L = \frac{p^{NE} - p}{p^{NE}(1-p)}$ , where  $p_L \in (0,1)$  for low priors, i.e.,  $p < p^{NE}$ . In addition,  $p_L$  is

decreasing in  $p$  since  $\frac{\partial p_L}{\partial p} = \frac{p^{NE} - 1}{p^{NE}(1-p)^2} < 0$  starting at  $\lim_{p \rightarrow p^{NE}} p_L = 0$  and converging to  $\lim_{p \rightarrow 0} p_L = 1$ .

Regarding the high-cost incumbent, she does not expand as prescribed,  $p_H = 0$ , if

$$\pi_{inc,H}^{M,E} - K_H < s \times \pi_{inc,H}^{D,NE} + (1-s) \times \pi_{inc,H}^{M,NE}.$$

Solving for  $s$ , we obtain

$$s > \frac{\pi_{inc,H}^{M,NE} - \pi_{inc,H}^{M,E}}{\pi_{inc,H}^{M,NE} - \pi_{inc,H}^{D,NE}} + \frac{K_H}{\pi_{inc,H}^{M,NE} - \pi_{inc,H}^{D,NE}}$$

and since  $PLE_H^{NE} = \pi_{inc,H}^{M,NE} - \pi_{inc,H}^{D,NE}$ ,  $\pi_{inc,H}^{M,NE} = \pi_{inc,H}^{M,E}$  and  $PLE_H^{NE} = PLE_H^E$ , then this expression reduced to

$s > \frac{K_H}{PLE_H^E} \equiv \hat{s}$ . On the other hand, the low-cost incumbent mixes as prescribed,  $p_L \in (0,1)$ , if and only if

$$\pi_{inc,L}^{M,E} - K_L = s \times \pi_{inc,L}^{D,NE} + (1-s) \times \pi_{inc,L}^{M,NE}.$$

Solving for  $s$ , we obtain

$$s = \frac{\pi_{inc,L}^{M,NE} - \pi_{inc,L}^{D,E}}{\pi_{inc,L}^{M,NE} - \pi_{inc,L}^{D,NE}} + \frac{K_L}{\pi_{inc,L}^{M,NE} - \pi_{inc,L}^{D,NE}},$$

and since  $BCC_L = \pi_{inc,L}^{M,NE} - \pi_{inc,L}^{M,E}$  and  $PLE_L^{NE} = \pi_{inc,L}^{M,NE} - \pi_{inc,L}^{D,NE}$ , this expression becomes

$$s = \frac{BCC_L}{PLE_L^E} + \frac{K_L}{PLE_L^E} \equiv \tilde{s}$$

where probability cutoff  $\tilde{s}$  satisfies  $\tilde{s} \in (0,1)$  if  $K_L < PLE_L^E - BCC_L$ . Hence, we need  $\tilde{s} > \hat{s}$ , or

$K_L > \frac{PLE_L^E}{PLE_H^E} K_H - BCC_L$ . Hence, this semiseparating equilibrium can be sustained for relatively low priors,

$p < p^{NE}$ , and expansion costs satisfying  $PLE_L^E - BCC_L > K_L > \frac{PLE_L^E}{PLE_H^E} K_H - BCC_L$ .

**$p_L = 0$  and  $p_H \in (0,1)$ .** Let us finally check the strategy profile where only the high-cost incumbent randomizes and the low-cost incumbent does not expand. In this case, the entrant's posterior beliefs after observing expansion are  $\mu(H | Exp) = 1$ , which leads him to enter since  $\pi_{ent,H}^{D,E} > F$ . In the case that the entrant observes no expansion from the incumbent, the entrant mixes if his beliefs  $\mu(H | NoExp)$  are such that

$$\mu(H | NoExp) \times (\pi_{ent,H}^{D,NE} - F) + (1 - \mu(H | NoExp)) (\pi_{ent,L}^{D,NE} - F) = 0$$

and solving for  $\mu(H | NoExp)$ , we obtain  $\mu(H | NoExp) = p^{NE}$ . Using Bayes' rule we have

$$\mu(H | NoExp) = p^{NE} = \frac{p(1-p_H)}{p(1-p_H) + (1-p)}$$

and solving for  $p_H$  we obtain  $p_H = \frac{p-p^{NE}}{p(1-p^{NE})}$ , where  $p_H \in (0,1)$  for all  $p > p^{NE}$ . In addition,  $p_H$  is increasing

in  $p$  since  $\frac{\partial p_H}{\partial p} = \frac{p^{NE}}{(1-p^{NE}) \times p^2} > 0$ , starting at  $\lim_{p \rightarrow p^{NE}} p_H = 0$  and converging to  $\lim_{p \rightarrow 1} p_H = 1$ .

Regarding the low-cost incumbent, she does not expand as prescribed ( $p_L=0$ ) if and only if

$$\pi_{inc,L}^{M,E} - K_L < s \times \pi_{inc,L}^{D,NE} + (1-s) \times \pi_{inc,L}^{M,NE},$$

and solving for  $s$ , we find

$$s > \frac{BCC_L}{PLE_L^E} + \frac{K_L}{PLE_L^E} \equiv \tilde{s}$$

On the other hand, the high-cost incumbent mixes as prescribed,  $p_H \in (0,1)$ , if and only if

$$\pi_{inc,H}^{M,E} - K_H = s \times \pi_{inc,H}^{D,NE} + (1-s) \pi_{inc,H}^{M,NE}.$$

Solving for  $s$ , we obtain  $s = \frac{K_H}{PLE_H^E} \equiv \hat{s}$ , where  $\hat{s} \in (0,1)$  for all  $K_H < PLE_H^E$ . Hence, we need that  $\hat{s} > \tilde{s}$ , or

$K_H > \frac{PLE_H^E}{PLE_L^E} K_L + \frac{BCC_L \times PLE_H^E}{PLE_L^E}$ . Hence, this semiseparating strategy profile can be sustained for priors

$p > p^{NE}$  and expansion costs satisfying

$$PLE_H^E > K_H > \frac{PLE_H^E}{PLE_L^E} K_L + \frac{BCC_L \times PLE_H^E}{PLE_L^E}.$$

### Appendix 3 – Parametric examples

**Capacity constraint arising from efficiency.** Consider a linear inverse demand curve  $p(Q)=1-Q$ , and constant marginal costs  $c_H=1/5$  for the high-cost incumbent (and the entrant) and  $c_L=0$  (only for the low-cost incumbent). In addition, assume a capacity constraint  $\bar{q}=1/8$  that the low-cost incumbent cannot exceed. Then, for the high-cost incumbent we have  $\pi_{inc,H}^{M,NE} = \pi_{inc,H}^{M,E} = 4/25$  and  $\pi_{inc,H}^{D,NE} = \pi_{inc,H}^{D,E} = 16/225$  under monopoly and duopoly, respectively. For the low-cost incumbent, monopoly profits grow from  $\pi_{inc,L}^{M,NE}(\bar{q}) = 7/64$  to  $\pi_{inc,L}^{M,E} = 1/4$  as a result of expansion, and so do duopoly profits, from  $\pi_{inc,L}^{D,NE}(\bar{q}) = 27/320$  to  $\pi_{inc,L}^{D,E} = 4/25$ . Therefore, cutoff expansion costs are  $BCC_L = \frac{1}{4} - \frac{7}{64} = \frac{9}{64}$ ,  $PLE_L^E = \frac{1}{4} - \frac{4}{25} = \frac{9}{100}$ ,  $PLE_L^E = \frac{7}{64} - \frac{27}{320} = \frac{1}{40}$  for the low-cost incumbent and  $PLE_H^E = PLE_H^{NE} = \frac{4}{25} - \frac{16}{225} = \frac{4}{45}$  for the high-cost incumbent.

**Capacity constraint arising from high demand.** Consider a linear inverse demand curve, either high  $p^H(Q)=1-Q$  or low  $p^L(Q)=1/3-Q$ , and constant marginal costs  $c=0$  for both incumbent and entrant. In addition, assume that the incumbent only faces a capacity constraint,  $\bar{q}=1/8$ , when operating in a high-demand market since she cannot produce an output exceeding  $\bar{q}$ . In this setting, the (unconstrained) low-demand incumbent's profits are  $\pi_{inc,L}^{M,NE} = \pi_{inc,L}^{M,E} = 1/4$  and  $\pi_{inc,L}^{D,NE} = \pi_{inc,L}^{D,E} = 1/81$  under monopoly and duopoly, respectively. For the constrained high-demand incumbent, monopoly profits grow from  $\pi_{inc,H}^{M,NE}(\bar{q}) = 3/16$  to  $\pi_{inc,H}^{M,E} = 1/4$  as a result of expansion, and similarly for duopoly profits, which increase from  $\pi_{inc,H}^{D,NE}(\bar{q}) = 5/48$  to  $\pi_{inc,H}^{D,E} = 1/9$ . Hence, cutoff expansion

costs are  $BCC_H = \frac{1}{16}$ ,  $PLE_H^E = \frac{5}{36}$ ,  $PLE_H^{NE} = \frac{1}{12}$  for the high-demand incumbent and  $PLE_L^{NE} = PLE_L^E = \frac{77}{324}$  for the low-demand incumbent.

### **Proof of Propositions 1, 2 and 3:**

**Pooling equilibrium with no expansion.** Let us investigate if the strategy profile  $\{NoExp_H, NoExp_L\}$  can be supported as a pooling PBE of this signaling game. First, the entrant's beliefs are  $\mu(H | NExp) = p$  after observing no expansion (in equilibrium) and  $\mu(H | Exp) = \mu \in [0, 1]$  after observing expansion (off-the-equilibrium). Given these beliefs after observing *no expansion* (in equilibrium) the entrant enters if and only if

$$p(\pi_{ent,H}^{D,NE} - F) + (1-p)(\pi_{ent,L}^{D,NE} - F) \geq 0,$$

where the right-hand side represents the entrant's profits from staying in the perfectly competitive market (with zero profits). Solving for  $p$ , we obtain that the entrant enters if  $p \geq \frac{F - \pi_{ent,L}^{D,NE}}{\pi_{ent,H}^{D,NE} - \pi_{ent,L}^{D,NE}} \equiv p^{NE}$ . Note that this cutoff is positive and smaller than one,  $1 > p^{NE} > 0$ , since entry costs,  $F$ , satisfy  $\pi_{ent,H}^{D,NE} > F > \pi_{ent,L}^{D,NE}$  by definition. Hence, after observing no expansion (in equilibrium) the entrant enters the market if  $p \geq p^{NE}$  and stays out otherwise. Similarly, after observing expansion (off-the-equilibrium), the entrant enters if and only if

$$\mu(\pi_{ent,H}^{D,E} - F) + (1-\mu)(\pi_{ent,L}^{D,E} - F) \geq 0$$

Solving for  $\mu$ , we find that the entrant enters if  $\mu \geq \frac{F - \pi_{ent,L}^{D,E}}{\pi_{ent,H}^{D,E} - \pi_{ent,L}^{D,E}} \equiv p^E$ . Note that this cutoff is positive and smaller

than one,  $1 > p^E > 0$ , since  $\pi_{ent,H}^{D,E} > F > \pi_{ent,L}^{D,E}$  is satisfied by definition. Indeed,  $\pi_{ent,H}^{D,NE} = \pi_{ent,H}^{D,E} > F > \pi_{ent,L}^{D,NE} > \pi_{ent,L}^{D,E}$  given that the entrant's profits are not affected by the (unconstrained) high-cost incumbent decision to expand,  $\pi_{ent,H}^{D,NE} = \pi_{ent,H}^{D,E}$ , and the entrant's profits are higher when the low-cost incumbent does not expand than when she does,  $\pi_{ent,L}^{D,NE} > \pi_{ent,L}^{D,E}$ . Hence, after observing expansion (off-the-equilibrium) the entrant enters if  $\mu \geq p^E$  and stays out otherwise. Finally, note that  $p^{NE} < p^E$ . Indeed,

$$p^{NE} \equiv \frac{F - \pi_{ent,L}^{D,NE}}{\pi_{ent,H}^{D,NE} - \pi_{ent,L}^{D,NE}} < \frac{F - \pi_{ent,L}^{D,E}}{\pi_{ent,H}^{D,E} - \pi_{ent,L}^{D,E}} \equiv p^E, \text{ solving for the entry cost, } F, \text{ we obtain}$$

$$F < \frac{\pi_{ent,H}^{D,E} \pi_{ent,L}^{D,NE} - \pi_{ent,L}^{D,E} \pi_{ent,H}^{D,NE}}{(\pi_{ent,L}^{D,NE} - \pi_{ent,L}^{D,E}) - (\pi_{ent,H}^{D,NE} - \pi_{ent,H}^{D,E})}$$

and since  $\pi_{ent,H}^{D,NE} = \pi_{ent,H}^{D,E}$  we can reduce the above expression to  $F < \pi_{ent,H}^{D,E}$ , which holds by definition. Given the entrant's strategies let us now analyze the incumbent:

- If  $p < p^{NE}$  and  $\mu \geq p^E$  then the entrant does not enter after observing no expansion (in equilibrium) but enters otherwise. Hence, the low-cost incumbent prefers to not expand (as prescribed) if and only if  $\pi_{inc,L}^{M,NE}(\bar{q}) > \pi_{inc,L}^{D,E} - K_L$ , or  $K_L > \pi_{inc,L}^{D,E} - \pi_{inc,L}^{M,NE}(\bar{q})$ , where  $BCC_L - PLE_L^E \equiv \pi_{inc,L}^{D,E} - \pi_{inc,L}^{M,NE}(\bar{q})$ . Similarly, the high-cost incumbent does not expand if  $\pi_{inc,H}^{M,NE} > \pi_{inc,H}^{D,E} - K_H$ , or  $K_H > \pi_{inc,H}^{D,E} - \pi_{inc,H}^{M,NE} < 0$  is satisfied, which holds for any expansion cost  $K_H > 0$ . Thus, the strategy profile in which both types of incumbent do not

expand their facility can be supported as a pooling PBE in the signaling game if  $K_L > BCC_L - PLE_L^E$ ; as described in Proposition 1, Part 3a.

- If  $p < p^{NE}$  and  $\mu < p^E$  then the entrant does not enter after observing any action from the incumbent. Therefore, the low-cost monopolist prefers to not expand (as prescribed) if and only if  $\pi_{inc,L}^{M,NE}(\bar{q}) > \pi_{inc,L}^{M,E} - K_L$ , or  $K_L > \pi_{inc,L}^{M,E} - \pi_{inc,L}^{M,NE}(\bar{q}) \equiv BCC_L$ . Similarly, the high-cost incumbent prefers to not expand since  $\pi_{inc,H}^{M,NE} > \pi_{inc,H}^{M,E} - K_H$  or  $K_H > \pi_{inc,H}^{M,E} - \pi_{inc,H}^{M,NE} < 0$  which is satisfied for any  $K_H > 0$ . Thus, this strategy profile can be sustained as a pooling PBE in the signaling game if expansion costs satisfy  $K_L > BCC_L$ ; as described in Proposition 1, Part 3b.
- If  $p \geq p^{NE}$  and  $\mu < p^E$  then the entrant enters after observing no expansion (in equilibrium) but does not enter otherwise. Hence, the low-cost incumbent does not expand (attracting entry) if and only if  $\pi_{inc,L}^{D,NE}(\bar{q}) > \pi_{inc,L}^{M,E} - K_L$ , or  $K_L > \pi_{inc,L}^{M,E} - \pi_{inc,L}^{D,NE}(\bar{q}) \equiv BCC_L + PLE_L^E$ . Similarly, the high-cost incumbent does not expand if and only if  $\pi_{inc,H}^{D,NE} > \pi_{inc,H}^{M,E} - K_H$ , or  $K_H > \pi_{inc,H}^{M,E} - \pi_{inc,H}^{D,NE} = PLE_H^E$ . Thus, this strategy profile can be supported as a pooling PBE in the signaling game under expansion costs  $K_L > BCC_L + PLE_L^E$  and  $K_H > PLE_H^E$ ; as described in Proposition 2a, and Proposition 3.
- If  $p \geq p^{NE}$  and  $\mu \geq p^E$  then the entrant enters after observing any action from the incumbent. Therefore, the low-cost incumbent does not expand (as prescribed) if and only if  $\pi_{inc,L}^{D,NE}(\bar{q}) > \pi_{inc,L}^{D,E} - K_L$ , or  $K_L > \pi_{inc,L}^{D,E} - \pi_{inc,L}^{D,NE}(\bar{q}) \equiv BCC_L + PLE_L^{NE} - PLE_L^E$ . Similarly, the high-cost incumbent does not expand since  $\pi_{inc,H}^{D,NE} > \pi_{inc,H}^{D,E} - K_H$ , or  $K_H > \pi_{inc,H}^{D,E} - \pi_{inc,H}^{D,NE} = 0$ , which holds for any  $K_H > 0$ . Thus, this strategy profile can be supported as a pooling PBE for expansion costs  $K_L > BCC_L + PLE_L^{NE} - PLE_L^E$  and  $K_H > 0$ ; as described in Proposition 2b, and Proposition 3.

**Separating equilibrium.** Let us now consider the separating strategy profile where only the low-cost incumbent expands, i.e.,  $\{NotExpand_H, Expand_L\}$ . First, entrant's updated beliefs become  $\mu(H | NExp) = 1$  and  $\mu(H | Exp) = 0$ . Given these beliefs, the entrant enters after observing no expansion since  $\pi_{ent,H}^{D,NE} - F > 0$ , or  $\pi_{ent,H}^{D,NE} > F$ , which satisfies our initial assumptions. On the other hand, after observing expansion the entrant stays out since  $\pi_{ent,L}^{D,E} - F < 0$ , or  $\pi_{ent,L}^{D,E} < F$ , which also holds by definition. Therefore, given the entrant's responses, the high-cost incumbent does not expand (as prescribed) if and only if  $\pi_{inc,H}^{D,NE} > \pi_{inc,H}^{M,E} - K_H$ , or  $K_H > \pi_{inc,H}^{M,E} - \pi_{inc,H}^{D,NE}$ . Since  $\pi_{inc,H}^{M,E} - \pi_{inc,H}^{D,NE} = \pi_{inc,H}^{M,E} - \pi_{inc,H}^{D,E} \equiv PLE_H^E$  given that  $\pi_{inc,H}^{D,NE} = \pi_{inc,H}^{D,E}$ , we can then conclude that the high-cost incumbent does not expand if  $K_H > PLE_H^E$ . In contrast, the low-cost incumbent expands (as prescribed) if and only if  $\pi_{inc,L}^{D,NE}(\bar{q}) < \pi_{inc,L}^{M,E} - K_L$  or  $K_L < \pi_{inc,L}^{M,E} - \pi_{inc,L}^{D,NE}(\bar{q}) \equiv BCC_L - PLE_L^E$ . Thus, this strategy profile can be sustained as a separating PBE for expansion costs  $K_H > PLE_H^E$  and  $K_L < BCC_L - PLE_L^E$ ; as described in Proposition 1 (Part 1) and Proposition 3.

For completeness, let us check that the opposite separating strategy profile  $\{Exp_H, NoExp_L\}$  cannot be supported as a PBE of the signaling game. In this case, the entrant's updated beliefs become  $\mu(H | NExp) = 0$  and  $\mu(H | Exp) = 1$ . Given these beliefs, the entrant enters after observing expansion since  $\pi_{ent,H}^{D,E} - F > 0$  or  $\pi_{ent,H}^{D,E} > F$ , which holds by definition. However, the entrant does not enter after observing no expansion given that  $\pi_{ent,L}^{D,NE} - F < 0$ , or  $\pi_{ent,L}^{D,NE} < F$ , which is satisfied by definition. Given the entrant's responses, the low-cost

incumbent does not expand (as prescribed) if and only if  $\pi_{inc,L}^{M,NE}(\bar{q}) > \pi_{inc,L}^{D,E} - K_L$ , or  $K_L > \pi_{inc,L}^{D,E} - \pi_{inc,L}^{M,NE}(\bar{q})$ . On the other hand, the high-cost incumbent expands (as prescribed) if  $\pi_{inc,H}^{M,NE} < \pi_{inc,H}^{D,E} - K_H$ , or  $K_H < \pi_{inc,H}^{D,E} - \pi_{inc,H}^{M,NE} = \pi_{inc,H}^{D,E} - \pi_{inc,H}^{M,E} < 0$  (since  $\pi_{inc,H}^{M,E} = \pi_{inc,H}^{M,NE}$ ), which cannot hold for any  $K_H > 0$ . Thus, this strategy profile cannot be supported as a separating PBE of the signaling game.

**Pooling equilibrium with expansion.** Let us investigate if the pooling strategy profile in which both types of incumbent expand their facility, i.e.,  $\{Exp_H, Exp_L\}$ , can be supported as a PBE of the signaling game. First, the entrant's beliefs are  $\mu(H | Exp) = p$  after observing expansion (in equilibrium) and  $\mu(H | NExp) = \gamma \in [0, 1]$  after observing no expansion (off-the-equilibrium). Given these beliefs, after observing expansion, the entrant enters if

$$p(\pi_{ent,H}^{D,E} - F) + (1-p)(\pi_{ent,L}^{D,E} - F) \geq 0$$

Solving for  $p$ , we obtain  $p \geq \frac{F - \pi_{ent,L}^{D,E}}{\pi_{ent,H}^{D,E} - \pi_{ent,L}^{D,E}} \equiv p^E$ , where  $0 \leq p^E \leq 1$  from our above discussion. Hence, entry

ensues after observing expansion if  $p > p^E$ , but does not otherwise. If, instead, the entrant observes *no* expansion, she enters if

$$\gamma(\pi_{ent,H}^{D,NE} - F) + (1-\gamma)(\pi_{ent,L}^{D,NE} - F) \geq 0,$$

Solving for  $\gamma$ , we find  $\gamma \geq \frac{F - \pi_{ent,L}^{D,NE}}{\pi_{ent,H}^{D,NE} - \pi_{ent,L}^{D,NE}} \equiv p^{NE}$ , where  $0 \leq p^{NE} \leq 1$  holds from our above discussion. Therefore,

the entrant enters after observing no expansion if  $\gamma \geq p^{NE}$ , but does not otherwise. Given the entrant's strategies let us now examine the incumbent:

- If  $p \geq p^E$  and  $\gamma \geq p^{NE}$  then the entrant enters both after observing expansion and no expansion. Therefore, the low-cost incumbent expands (as prescribed) if and only if  $\pi_{inc,L}^{D,NE}(\bar{q}) < \pi_{inc,L}^{D,E} - K_L$ , or  $K_L < \pi_{inc,L}^{D,E} - \pi_{inc,L}^{D,NE}(\bar{q})$ . However, the high-cost incumbent does not expand since  $\pi_{inc,H}^{D,NE} > \pi_{inc,H}^{D,E} - K_H$ , or  $K_H > \pi_{inc,H}^{D,E} - \pi_{inc,H}^{D,NE} = 0$ , which holds for any expansion costs  $K_H > 0$ . Thus, the strategy profile in which both types of incumbent expand cannot be supported as a pooling PBE of the signaling game.
- If  $p \geq p^E$  but  $\gamma < p^{NE}$  then the entrant enters after observing expansion (in equilibrium), but does not enter otherwise. Hence, the low-cost incumbent expands (as prescribed) if and only if  $\pi_{inc,L}^{M,NE}(\bar{q}) < \pi_{inc,L}^{D,E} - K_L$ , or  $K_L < \pi_{inc,L}^{D,E} - \pi_{inc,L}^{M,NE}(\bar{q}) \equiv BCC_L - PLE_L^E$ . In contrast, the high-cost incumbent does not expand since  $\pi_{inc,H}^{M,NE} > \pi_{inc,H}^{D,E} - K_H$ , or  $K_H > \pi_{inc,H}^{D,E} - \pi_{inc,H}^{M,NE} < 0$ , which holds for all expansion costs  $K_H > 0$ . Therefore, this strategy profile cannot be sustained as a pooling PBE of the signaling game.
- If  $p < p^E$  but  $\gamma \geq p^{NE}$  then the entrant does not enter after observing expansion (in equilibrium), but enters otherwise. Hence, the low-cost incumbent expands (as prescribed) if and only if  $\pi_{inc,L}^{D,NE}(\bar{q}) < \pi_{inc,L}^{M,E} - K_L$ , or  $K_L < \pi_{inc,L}^{M,E} - \pi_{inc,L}^{D,NE}(\bar{q})$ . Since

$$BCC_L + PLE_L^{NE} = (\pi_{inc,L}^{M,E} - \pi_{inc,L}^{M,NE}(\bar{q})) + (\pi_{inc,L}^{M,NE}(\bar{q}) - \pi_{inc,L}^{D,NE}(\bar{q})) = \pi_{inc,L}^{M,E} - \pi_{inc,L}^{D,NE}(\bar{q}),$$

the low-cost incumbent expands if  $K_L < BCC_L + PLE_L^{NE}$ . Similarly, the high-cost incumbent expands (as prescribed) if and only if  $\pi_{inc,H}^{D,NE} < \pi_{inc,H}^{M,E} - K_H$ , or  $K_H < \pi_{inc,H}^{M,E} - \pi_{inc,H}^{D,NE} \equiv PLE_H^{NE}$ , since  $\pi_{inc,H}^{M,NE} = \pi_{inc,H}^{M,E}$  and  $\pi_{inc,H}^{D,NE} = \pi_{inc,H}^{D,E}$  for the unconstrained high-cost incumbent under monopoly and duopoly, respectively. Thus, this

strategy profile can be supported as a pooling PBE under expansion costs  $K_L < BCC_L + PLE_L^{NE}$  and  $K_H < PLE_H^E$ ; as described in Proposition 1, Part 2.

- If  $p < p^E$  and  $\gamma < p^{NE}$  then the entrant does not enter after observing any action from the incumbent. Therefore, the low-cost incumbent expands (as prescribed) if and only if  $\pi_{inc,L}^{M,NE}(\bar{q}) < \pi_{inc,L}^{M,E} - K_L$ , or  $K_L < \pi_{inc,L}^{M,E} - \pi_{inc,L}^{M,NE}(\bar{q}) \equiv BCC_L$ . In contrast, the high-cost incumbent does not expand since  $\pi_{inc,H}^{M,NE} > \pi_{inc,H}^{M,E} - K_H$ , or  $K_H > \pi_{inc,H}^{M,E} - \pi_{inc,H}^{M,NE} = 0$ , which holds for any expansion cost  $K_H > 0$ . Thus, this strategy profile cannot be sustained as a pooling PBE.

### **Proof of Proposition 4:**

**Pooling equilibrium with no expansion.** Let us investigate if the pooling strategy profile  $\{NoExp_H, NoExp_L\}$  can be supported as a pooling PBE of this signaling game. First, the entrant's beliefs are  $\mu(H | NExp) = p$  after observing no expansion (in equilibrium) and  $\mu(H | Exp) = \mu \in [0, 1]$  after observing expansion (off-the-equilibrium). Given these beliefs after observing *no expansion* (in equilibrium) the entrant enters if and only if

$$p(\pi_{ent,H}^{D,NE} - F) + (1-p)(\pi_{ent,L}^{D,NE} - F) \geq 0,$$

where the right-hand side represents the entrant's profits from staying in the perfectly competitive market (with zero profits). Solving for  $p$ , we obtain that the entrant enters if  $p \geq \frac{F - \pi_{ent,L}^{D,NE}}{\pi_{ent,H}^{D,NE} - \pi_{ent,L}^{D,NE}} \equiv p^{NE}$ . Note that this cutoff is

positive and smaller than one,  $1 > p^{NE} > 0$ , since entry costs,  $F$ , satisfy  $\pi_{ent,H}^{D,NE} > F > \pi_{ent,L}^{D,NE}$  by definition. Hence, after observing no expansion (in equilibrium) the entrant enters the market if  $p \geq p^{NE}$  and stays out otherwise. Similarly, after observing expansion (off-the-equilibrium), the entrant enters if and only if

$$\mu(\pi_{ent,H}^{D,E} - F) + (1-\mu)(\pi_{ent,L}^{D,E} - F) \geq 0$$

Solving for  $\mu$ , we find that the entrant enters if  $\mu \geq \frac{F - \pi_{ent,L}^{D,E}}{\pi_{ent,H}^{D,E} - \pi_{ent,L}^{D,E}} \equiv p^E$ . Note that this cutoff is positive and smaller

than one,  $1 > p^E > 0$ , since  $\pi_{ent,H}^{D,E} > F > \pi_{ent,L}^{D,E}$  is satisfied by definition. Indeed,  $\pi_{ent,H}^{D,NE} > \pi_{ent,H}^{D,E} > F > \pi_{ent,L}^{D,NE} = \pi_{ent,L}^{D,E}$  given that the entrant's profits are not affected by the (unconstrained) low-demand incumbent decision to expand,  $\pi_{ent,L}^{D,NE} = \pi_{ent,L}^{D,E}$ , and the entrant's profits are higher when the high-demand incumbent does not expand than when she does,  $\pi_{ent,H}^{D,NE} > \pi_{ent,H}^{D,E}$ . Hence, after observing expansion (off-the-equilibrium) the entrant enters if  $\mu \geq p^E$  and stays out otherwise. Finally, note that  $p^{NE} < p^E$  as it shown in the proof of Proposition 1.

Given the entrant's strategies let us now analyze the incumbent:

- If  $p < p^{NE}$  and  $\mu \geq p^E$  then the entrant does not enter after observing no expansion (in equilibrium) but enters otherwise. Hence, the high-demand incumbent prefers to not expand (as prescribed) if and only if  $\pi_{inc,H}^{M,NE}(\bar{q}) > \pi_{inc,H}^{D,E} - K_H$ , or  $K_H > \pi_{inc,H}^{D,E} - \pi_{inc,H}^{M,NE}(\bar{q})$ , where  $BCC_H - PLE_H^E \equiv \pi_{inc,H}^{D,E} - \pi_{inc,H}^{M,NE}(\bar{q})$ . Similarly, the low-demand incumbent does not expand if  $\pi_{inc,L}^{M,NE} > \pi_{inc,L}^{D,E} - K_L$ , or  $K_L > \pi_{inc,L}^{D,E} - \pi_{inc,L}^{M,NE} < 0$  is satisfied, which holds for any expansion cost  $K_L > 0$ . Thus, the strategy profile in which both types of incumbent

do not expand their facility can be supported as a pooling PBE in the signaling game if  $K_H > BCC_H - PLE_H^E$

- If  $p < p^{NE}$  and  $\mu < p^E$  then the entrant does not enter after observing any action from the incumbent. Therefore, the high-demand monopolist prefers to not expand (as prescribed) if and only if  $\pi_{inc,H}^{M,NE}(\bar{q}) > \pi_{inc,H}^{M,E} - K_H$ , or  $K_H > \pi_{inc,H}^{M,E} - \pi_{inc,H}^{M,NE}(\bar{q}) \equiv BCC_H$ . Similarly, the low-demand incumbent prefers to not expand since  $\pi_{inc,L}^{M,NE} > \pi_{inc,L}^{M,E} - K_L$  or  $K_L > \pi_{inc,L}^{M,E} - \pi_{inc,L}^{M,NE} < 0$  which is satisfied for any  $K_L > 0$ . Thus, this strategy profile can be sustained as a pooling PBE in the signaling game if expansion costs satisfy  $K_H > BCC_H$ .
- If  $p \geq p^{NE}$  and  $\mu < p^E$  then the entrant enters after observing no expansion (in equilibrium) but does not enter otherwise. Hence, the high-demand incumbent does not expand (attracting entry) if and only if  $\pi_{inc,H}^{D,NE}(\bar{q}) > \pi_{inc,H}^{M,E} - K_H$ , or  $K_H > \pi_{inc,H}^{M,E} - \pi_{inc,H}^{D,NE}(\bar{q}) \equiv BCC_H + PLE_H^E$ . Similarly, the low-demand incumbent does not expand if and only if  $\pi_{inc,L}^{D,NE} > \pi_{inc,L}^{M,E} - K_L$ , or  $K_L > \pi_{inc,L}^{M,E} - \pi_{inc,L}^{D,NE} = PLE_L^E$ . Thus, this strategy profile can be supported as a pooling PBE in the signaling game under expansion costs  $K_H > BCC_H + PLE_H^E$  and  $K_L > PLE_L^E$ .
- If  $p \geq p^{NE}$  and  $\mu \geq p^E$  then the entrant enters after observing any action from the incumbent. Therefore, the high-demand incumbent does not expand (as prescribed) if and only if  $\pi_{inc,H}^{D,NE}(\bar{q}) > \pi_{inc,H}^{D,E} - K_H$ , or  $K_H > \pi_{inc,H}^{D,E} - \pi_{inc,H}^{D,NE}(\bar{q}) \equiv BCC_H + PLE_H^{NE} - PLE_H^E$ . Similarly, the low-demand incumbent does not expand since  $\pi_{inc,L}^{D,NE} > \pi_{inc,L}^{D,E} - K_L$ , or  $K_L > \pi_{inc,L}^{D,E} - \pi_{inc,L}^{D,NE} = 0$ , which holds for any  $K_L > 0$ . Thus, this strategy profile can be supported as a pooling PBE for expansion costs  $K_H > BCC_H + PLE_H^{NE} - PLE_H^E$  and  $K_L > 0$ .

**Separating equilibrium.** Let us now consider the separating strategy profile where only the high-demand incumbent expands, i.e.,  $\{Expand_H, NotExpand_L\}$ . First, entrant's updated beliefs become  $\mu(H | NExp) = 0$  and  $\mu(H | Exp) = 1$ . Given these beliefs, the entrant enters after observing expansion since  $\pi_{ent,H}^{D,E} - F > 0$ , or  $\pi_{ent,H}^{D,E} > F$ . On the other hand, after observing no expansion the entrant stays out since  $\pi_{ent,L}^{D,NE} - F < 0$ , or  $\pi_{ent,L}^{D,NE} < F$ , which also holds by definition. Therefore, given the entrant's responses, the low-demand incumbent does not expand (as prescribed) if and only if  $\pi_{inc,L}^{D,NE} > \pi_{inc,L}^{M,E} - K_L$ , or  $K_L > \pi_{inc,L}^{M,E} - \pi_{inc,L}^{D,NE}$ . Since  $\pi_{inc,L}^{M,E} - \pi_{inc,L}^{D,NE} = \pi_{inc,L}^{M,E} - \pi_{inc,L}^{D,E} \equiv PLE_L^E$  given that  $\pi_{inc,L}^{D,NE} = \pi_{inc,L}^{D,E}$ , we can then conclude that the high-demand incumbent does not expand if  $K_L > PLE_L^E$ . In contrast, the high-demand incumbent expands (as prescribed) if and only if  $\pi_{inc,H}^{D,NE}(\bar{q}) < \pi_{inc,H}^{M,E} - K_H$  or  $K_H < \pi_{inc,H}^{M,E} - \pi_{inc,H}^{D,NE}(\bar{q}) \equiv BCC_H - PLE_H^E$ . Thus, this strategy profile can be sustained as a separating PBE for expansion costs  $K_L > PLE_L^E$  and  $K_H < BCC_H - PLE_H^E$ .

For completeness, let us check that the opposite separating strategy profile  $\{NoExp_H, Exp_L\}$  cannot be supported as a PBE of the signaling game. In this case, the entrant's updated beliefs become  $\mu(H | NExp) = 1$  and  $\mu(H | Exp) = 0$ . Given these beliefs, the entrant does not enter after observing expansion since  $\pi_{ent,H}^{D,E} - F < 0$  or  $\pi_{ent,H}^{D,E} < F$ , which holds by definition. However, the entrant enters after observing no expansion given that  $\pi_{ent,L}^{D,NE} - F > 0$ , or  $\pi_{ent,L}^{D,NE} > F$ , which is satisfied by definition. Given the entrant's responses, the high-demand incumbent does not expand (as prescribed) if and only if  $\pi_{inc,H}^{M,NE}(\bar{q}) > \pi_{inc,H}^{D,E} - K_H$ , or  $K_H > \pi_{inc,H}^{D,E} - \pi_{inc,H}^{M,NE}(\bar{q})$ . On the other hand, the low-demand incumbent expands (as prescribed) if  $\pi_{inc,L}^{M,NE} < \pi_{inc,L}^{D,E} - K_L$ , or

$K_L < \pi_{inc,L}^{D,E} - \pi_{inc,L}^{M,NE} = \pi_{inc,L}^{D,E} - \pi_{inc,L}^{M,E} < 0$  (since  $\pi_{inc,H}^{M,E} = \pi_{inc,H}^{M,NE}$ ), which cannot hold for any  $K_L > 0$ . Thus, this strategy profile cannot be supported as a separating PBE of the signaling game.

**Pooling equilibrium with expansion.** Let us investigate if the pooling strategy profile in which both types of incumbent expand their facility, i.e.,  $\{Exp_H, Exp_L\}$ , can be supported as a PBE of the signaling game. First, the entrant's beliefs are  $\mu(H | Exp) = p$  after observing expansion (in equilibrium) and  $\mu(H | NExp) = \gamma \in [0, 1]$  after observing no expansion (off-the-equilibrium). Given these beliefs, after observing expansion, the entrant enters if

$$p(\pi_{ent,H}^{D,E} - F) + (1-p)(\pi_{ent,L}^{D,E} - F) \geq 0$$

Solving for  $p$ , we obtain  $p \geq \frac{F - \pi_{ent,H}^{D,E}}{\pi_{ent,L}^{D,E} - \pi_{ent,H}^{D,E}} \equiv p^E$ , where  $0 \leq p^E \leq 1$  from our above discussion. Hence, entry

ensues after observing expansion if  $p \geq p^E$ , but does not otherwise. If, instead, the entrant observes *no* expansion, she enters if

$$\gamma(\pi_{ent,H}^{D,NE} - F) + (1-\gamma)(\pi_{ent,L}^{D,NE} - F) \geq 0,$$

Solving for  $\gamma$ , we find  $\gamma \geq \frac{F - \pi_{ent,H}^{D,NE}}{\pi_{ent,L}^{D,NE} - \pi_{ent,H}^{D,NE}} \equiv p^{NE}$ , where  $0 \leq p^{NE} \leq 1$  holds from our above discussion. Therefore,

the entrant enters after observing no expansion if  $\gamma \geq p^{NE}$ , but does not otherwise. Given the entrant's strategies let us now examine the incumbent:

- If  $p \geq p^E$  and  $\gamma \geq p^{NE}$  then the entrant enters both after observing expansion and no expansion. Therefore, the high-demand incumbent expands (as prescribed) if and only if  $\pi_{inc,H}^{D,NE}(\bar{q}) < \pi_{inc,H}^{D,E} - K_H$ , or  $K_H < \pi_{inc,H}^{D,E} - \pi_{inc,H}^{D,NE}(\bar{q})$ . However, the low-demand incumbent does not expand since  $\pi_{inc,L}^{D,NE} > \pi_{inc,L}^{D,E} - K_L$ , or  $K_L > \pi_{inc,L}^{D,E} - \pi_{inc,L}^{D,NE} = 0$ , which holds for any expansion costs  $K_L > 0$ . Thus, the strategy profile in which both types of incumbent expand cannot be supported as a pooling PBE of the signaling game.
- If  $p \geq p^E$  but  $\gamma < p^{NE}$  then the entrant enters after observing expansion (in equilibrium), but does not enter otherwise. Hence, the high-demand incumbent expands (as prescribed) if and only if  $\pi_{inc,H}^{M,NE}(\bar{q}) < \pi_{inc,H}^{D,E} - K_H$ , or  $K_H < \pi_{inc,H}^{D,E} - \pi_{inc,H}^{M,NE}(\bar{q}) \equiv BCC_H - PLE_H^E$ . In contrast, the low-demand incumbent does not expand since  $\pi_{inc,L}^{M,NE} > \pi_{inc,L}^{D,E} - K_L$ , or  $K_L > \pi_{inc,L}^{D,E} - \pi_{inc,L}^{M,NE} < 0$ , which holds for all expansion costs  $K_L > 0$ . Therefore, this strategy profile cannot be sustained as a pooling PBE of the signaling game.
- If  $p < p^E$  but  $\gamma \geq p^{NE}$  then the entrant does not enter after observing expansion (in equilibrium), but enters otherwise. Hence, the high-demand incumbent expands (as prescribed) if and only if  $\pi_{inc,H}^{D,NE}(\bar{q}) < \pi_{inc,H}^{M,E} - K_H$ , or  $K_H < \pi_{inc,H}^{M,E} - \pi_{inc,H}^{D,NE}(\bar{q})$ . Since

$$BCC_H + PLE_H^{NE} = (\pi_{inc,H}^{M,E} - \pi_{inc,H}^{M,NE}(\bar{q})) + (\pi_{inc,H}^{M,NE}(\bar{q}) - \pi_{inc,H}^{D,NE}(\bar{q})) = \pi_{inc,H}^{M,E} - \pi_{inc,H}^{D,NE}(\bar{q}),$$

the high-demand incumbent expands if  $K_H < BCC_H + PLE_H^{NE}$ . Similarly, the low-demand incumbent expands (as prescribed) if and only if  $\pi_{inc,L}^{D,NE} < \pi_{inc,L}^{M,E} - K_L$ , or  $K_L < \pi_{inc,L}^{M,E} - \pi_{inc,L}^{D,NE} \equiv PLE_L^{NE}$ , since  $\pi_{inc,L}^{M,NE} = \pi_{inc,L}^{M,E}$  and  $\pi_{inc,L}^{D,NE} = \pi_{inc,L}^{D,E}$  for the unconstrained low-demand incumbent under monopoly and duopoly, respectively. Thus, this strategy profile can be supported as a pooling PBE under expansion costs  $K_H < BCC_H + PLE_H^{NE}$  and  $K_L < PLE_L^{NE}$ .



- If  $p < p^E$  and  $\gamma < p^{NE}$  then the entrant does not enter after observing any action from the incumbent. Therefore, the high-demand incumbent expands (as prescribed) if and only if  $\pi_{inc,H}^{M,NE}(\bar{q}) < \pi_{inc,H}^{M,E} - K_H$ , or  $K_H < \pi_{inc,H}^{M,E} - \pi_{inc,H}^{M,NE}(\bar{q}) \equiv BCC_H$ . In contrast, the low-demand incumbent does not expand since  $\pi_{inc,L}^{M,NE} > \pi_{inc,L}^{M,E} - K_L$ , or  $K_L > \pi_{inc,L}^{M,E} - \pi_{inc,L}^{M,NE} = 0$ , which holds for any expansion cost  $K_L > 0$ . Thus, this strategy profile cannot be sustained as a pooling PBE.

### ***Proof of Corollary 1:***

**Pooling equilibrium with no expansion.** Let us investigate if the strategy profile  $\{NoExp_H, NoExp_L\}$  can be supported as a pooling PBE of this signaling game. First, the entrant's beliefs are  $\mu(H|NExp) = p$  after observing no expansion (in equilibrium) and  $\mu(H|Exp) = \mu \in [0,1]$  after observing expansion (off-the-equilibrium). Given these beliefs after observing *no expansion* (in equilibrium) the entrant enters if and only if

$$p(\pi_{ent,H}^{D,NE} - F) + (1-p)(\pi_{ent,L}^{D,NE} - F) \geq 0,$$

where the right-hand side represents the entrant's profits from staying in the perfectly competitive market (with zero profits). Solving for  $p$ , we obtain that the entrant enters if  $p \geq \frac{F - \pi_{ent,L}^{D,NE}}{\pi_{ent,H}^{D,NE} - \pi_{ent,L}^{D,NE}} \equiv p^{NE}$ . Note that this cutoff is

negative since  $\pi_{ent,H}^{D,NE} > \pi_{ent,L}^{D,NE} > F$  by definition, i.e., the entrant enters if the low-cost incumbent does not expand, but stays out otherwise. Hence, the entrant enters after observing no expansion, for all parameter values. After observing expansion (off-the-equilibrium), the entrant enters if and only if

$$\mu(\pi_{ent,H}^{D,E} - F) + (1-\mu)(\pi_{ent,L}^{D,E} - F) \geq 0$$

Solving for  $\mu$ , we find that the entrant enters if  $\mu \geq \frac{F - \pi_{ent,L}^{D,E}}{\pi_{ent,H}^{D,E} - \pi_{ent,L}^{D,E}} \equiv p^E$ . Note that this cutoff is positive and smaller

than one,  $1 > p^{NE} > 0$ , since  $\pi_{ent,H}^{D,E} > F > \pi_{ent,L}^{D,E}$  is satisfied by definition. Indeed,  $\pi_{ent,H}^{D,NE} = \pi_{ent,H}^{D,E} > F > \pi_{ent,L}^{D,NE} > \pi_{ent,L}^{D,E}$  given that the entrant's profits are not affected by the (unconstrained) high-cost incumbent decision to expand,  $\pi_{ent,H}^{D,NE} = \pi_{ent,H}^{D,E}$ , and the entrant's profits are higher when the low-cost incumbent does not expand than when she does,  $\pi_{ent,L}^{D,NE} > \pi_{ent,L}^{D,E}$ . Hence, after observing expansion (off-the-equilibrium) the entrant enters if  $\mu \geq p^E$  and stays out otherwise.

Given the entrant's strategies let us now analyze the incumbent:

- If  $\mu < p^E$  then the entrant enters after observing no expansion (in equilibrium) but does not enter otherwise. Hence, the low-cost incumbent does not expand (as prescribed, attracting entry) if and only if  $\pi_{inc,L}^{D,NE}(\bar{q}) > \pi_{inc,L}^{M,E} - K_L$ , or  $K_L > \pi_{inc,L}^{M,E} - \pi_{inc,L}^{D,NE}(\bar{q}) \equiv BCC_L + PLE_L^E$ . Similarly, the high-cost incumbent does not expand if and only if  $\pi_{inc,H}^{D,NE} > \pi_{inc,H}^{M,E} - K_H$ , or  $K_H > \pi_{inc,H}^{M,E} - \pi_{inc,H}^{D,NE} = PLE_H^E$ . Thus, this strategy profile can be supported as a pooling PBE in the signaling game under expansion costs  $K_L > BCC_L + PLE_L^E$  and  $K_H > PLE_H^E$ ; as described in Proposition 2a.
- If  $\mu \geq p^E$  then the entrant enters after observing any action from the incumbent. Therefore, the low-cost incumbent does not expand (as prescribed) if and only if  $\pi_{inc,L}^{D,NE}(\bar{q}) > \pi_{inc,L}^{D,E} - K_L$ , or  $K_L > \pi_{inc,L}^{D,E} - \pi_{inc,L}^{D,NE}(\bar{q}) \equiv BCC_L + PLE_L^{NE} - PLE_L^E$ . Similarly, the high-cost incumbent does not expand since  $\pi_{inc,H}^{D,NE} > \pi_{inc,H}^{D,E} - K_H$ , or  $K_H > \pi_{inc,H}^{D,E} - \pi_{inc,H}^{D,NE} = 0$ , which holds for any  $K_H > 0$ . Thus, this strategy profile

can be supported as a pooling PBE for expansion costs  $K_L > BCC_L + PLE_L^{NE} - PLE_L^E$  and  $K_H > 0$ ; as described in Proposition 2b.

**Separating equilibrium.** Let us now consider the separating strategy profile where only the low-cost incumbent expands, i.e.,  $\{NotExpand_H, Expand_L\}$ . First, entrant's updated beliefs become  $\mu(H | NExp) = 1$  and  $\mu(H | Exp) = 0$ . Given these beliefs, the entrant enters after observing no expansion since  $\pi_{ent,H}^{D,NE} - F > 0$ , or  $\pi_{ent,H}^{D,NE} > F$ , which satisfies our initial assumptions. On the other hand, after observing expansion the entrant stays out since  $\pi_{ent,L}^{D,E} - F < 0$ , or  $\pi_{ent,L}^{D,E} < F$ , which also holds by definition. Therefore, given the entrant's responses, the high-cost incumbent does not expand (as prescribed) if and only if  $\pi_{inc,H}^{D,NE} > \pi_{inc,H}^{M,E} - K_H$ , or  $K_H > \pi_{inc,H}^{M,E} - \pi_{inc,H}^{D,NE}$ . Since  $\pi_{inc,H}^{M,E} - \pi_{inc,H}^{D,NE} = \pi_{inc,H}^{M,E} - \pi_{inc,H}^{D,E} \equiv PLE_H^E$  given that  $\pi_{inc,H}^{D,NE} = \pi_{inc,H}^{D,E}$ , we can then conclude that the high-cost incumbent does not expand if  $K_H > PLE_H^E$ . In contrast, the low-cost incumbent expands (as prescribed) if and only if  $\pi_{inc,L}^{D,NE}(\bar{q}) < \pi_{inc,L}^{M,E} - K_L$  or  $K_L < \pi_{inc,L}^{M,E} - \pi_{inc,L}^{D,NE}(\bar{q}) \equiv BCC_L - PLE_L^E$ . Thus, this strategy profile can be sustained as a separating PBE for expansion costs  $K_H > PLE_H^E$  and  $K_L < BCC_L - PLE_L^E$ ; as described in Proposition 1, Part 1.

For completeness, let us check that the opposite separating strategy profile  $\{Exp_H, NoExp_L\}$  cannot be supported as a PBE of the signaling game. In this case, the entrant's updated beliefs become  $\mu(H | NExp) = 0$  and  $\mu(H | Exp) = 1$ . Given these beliefs, the entrant enters after observing expansion since  $\pi_{ent,H}^{D,E} - F > 0$  or  $\pi_{ent,H}^{D,E} > F$ , which holds by definition. Similarly, the entrant enters after observing no expansion given that  $\pi_{ent,L}^{D,NE} - F > 0$ , or  $\pi_{ent,L}^{D,NE} > F$ , which is satisfied by definition. Given the entrant's responses, the low-cost incumbent does not expand (as prescribed) if and only if  $\pi_{inc,L}^{D,NE}(\bar{q}) > \pi_{inc,L}^{D,E} - K_L$ , or  $K_L > \pi_{inc,L}^{D,E} - \pi_{inc,L}^{D,NE}(\bar{q})$ . On the other hand, the high-cost incumbent expands (as prescribed) if  $\pi_{inc,H}^{D,NE} < \pi_{inc,H}^{D,E} - K_H$ , or  $K_H < \pi_{inc,H}^{D,E} - \pi_{inc,H}^{D,NE} = \pi_{inc,H}^{D,E} - \pi_{inc,H}^{D,E} = 0$  (since  $\pi_{inc,H}^{D,NE} = \pi_{inc,H}^{D,E}$ ), which cannot hold for any  $K_H > 0$ . Thus, this strategy profile cannot be supported as a separating PBE of the signaling game.

**Pooling equilibrium with expansion.** Let us investigate if the pooling strategy profile in which both types of incumbent expand their facility, i.e.,  $\{Exp_H, Exp_L\}$ , can be supported as a PBE of the signaling game. First, the entrant's beliefs are  $\mu(H | Exp) = p$  after observing expansion (in equilibrium) and  $\mu(H | NExp) = \gamma \in [0, 1]$  after observing no expansion (off-the-equilibrium). Given these beliefs, after observing expansion, the entrant enters if

$$p(\pi_{ent,H}^{D,E} - F) + (1-p)(\pi_{ent,L}^{D,E} - F) \geq 0$$

Solving for  $p$ , we obtain  $p \geq \frac{F - \pi_{ent,L}^{D,E}}{\pi_{ent,H}^{D,E} - \pi_{ent,L}^{D,E}} \equiv p^E$ , where  $0 \leq p^E \leq 1$  from our above discussion. Hence, entry

ensues after observing expansion if  $p > p^E$ , but does not otherwise. If, instead, the entrant observes *no* expansion, he enters if

$$\gamma(\pi_{ent,H}^{D,NE} - F) + (1-\gamma)(\pi_{ent,L}^{D,NE} - F) \geq 0,$$

Solving for  $\gamma$ , we find  $\gamma \geq \frac{F - \pi_{ent,L}^{D,NE}}{\pi_{ent,H}^{D,NE} - \pi_{ent,L}^{D,NE}} \equiv p^{NE}$ , where  $p^{NE} < 0$  holds from our above discussion. Therefore, the

entrant enters after observing no expansion under all parameter values. Given the entrant's strategies let us now examine the incumbent:

- If  $p \geq p^E$  then the entrant enters both after observing expansion and no expansion. Therefore, the low-cost incumbent expands (as prescribed) if and only if  $\pi_{inc,L}^{D,NE}(\bar{q}) < \pi_{inc,L}^{D,E} - K_L$ , or  $K_L < \pi_{inc,L}^{D,E} - \pi_{inc,L}^{D,NE}(\bar{q})$ . However, the high-cost incumbent does not expand since  $\pi_{inc,H}^{D,NE} > \pi_{inc,H}^{D,E} - K_H$ , or  $K_H > \pi_{inc,H}^{D,E} - \pi_{inc,H}^{D,NE} = 0$ , which holds for any expansion costs  $K_H > 0$ . Thus, the strategy profile in which both types of incumbent expand cannot be supported as a pooling PBE of the signaling game.
- If  $p < p^E$  then the entrant does not enter after observing expansion (in equilibrium), but enters otherwise. Hence, the low-cost incumbent expands (as prescribed) if and only if  $\pi_{inc,L}^{D,NE}(\bar{q}) < \pi_{inc,L}^{M,E} - K_L$ , or  $K_L < \pi_{inc,L}^{M,E} - \pi_{inc,L}^{D,NE}(\bar{q})$ . Since

$$BCC_L + PLE_L^{NE} = (\pi_{inc,L}^{M,E} - \pi_{inc,L}^{M,NE}(\bar{q})) + (\pi_{inc,L}^{M,NE}(\bar{q}) - \pi_{inc,L}^{D,NE}(\bar{q})) = \pi_{inc,L}^{M,E} - \pi_{inc,L}^{D,NE}(\bar{q}),$$

the low-cost incumbent expands if  $K_L < BCC_L + PLE_L^{NE}$ . Similarly, the high-cost incumbent expands (as prescribed) if and only if  $\pi_{inc,H}^{D,NE} < \pi_{inc,H}^{M,E} - K_H$ , or  $K_H < \pi_{inc,H}^{M,E} - \pi_{inc,H}^{D,NE} \equiv PLE_H^{NE}$ , since  $\pi_{inc,H}^{M,NE} = \pi_{inc,H}^{M,E}$  and  $\pi_{inc,H}^{D,NE} = \pi_{inc,H}^{D,E}$  for the unconstrained high-cost incumbent under monopoly and duopoly, respectively. Thus, this strategy profile can be supported as a pooling PBE under expansion costs  $K_L < BCC_L + PLE_L^{NE}$  and  $K_H \geq K_L$ ; as described in Proposition 1, Part 2.

## References

- [1] Arvan, Lanny (1986) "Sunk Capacity Costs, Long-Run Fixed Costs, and Entry Deterrence under Complete and Incomplete Information," *The RAND Journal of Economics*, vol. 17, pp. 105-1211.
- [2] Albaek, S. and Overgaard, P.B. (1994) "Advertising and pricing to deter or accommodate entry when demand is unknown: Comment," *International Journal of Industrial Organization*, vol.12, pp. 83-87.
- [3] Bagwell, K. and Ramey, G. (1990) "Advertising and pricing to deter or accommodate entry when demand is unknown," *International Journal of Industrial Organization*, vol. 8, pp. 93- 113.
- [4] Cho, I. and Kreps, D. (1987) "Signaling games and stable equilibrium," *Quarterly Journal of Economics*, vol. 102, pp. 179-222.
- [5] Dixit, A. (1979) "A model of duopoly suggesting a theory of entry barriers," *Bell Journal of Economics*, vol. 10, no. 1, pp. 20-32.
- [6] Dixit, A. (1980) "The role of investment in entry-deterrence," *Economic Journal*, vol. 90, pp. 95-106.
- [7] Formby, J. and W. J. Smith (1984) "Collusion, Entry, and Market Shares." *Review of Industrial Organization*, pp. 15-25.
- [8] Espinola-Arredondo, A., E. Gal-Or and F. Munoz-Garcia (in press) "When Should a Firm Expand its Business? The Signaling Implications of Business Expansion," *International Journal of Industrial Organization*, forthcoming.
- [9] Harrington, J. (1986) "Limit pricing when the entrant is uncertain about its cost function," *Econometrica*, vol. 54, pp. 429-437.
- [10] Mason, C. and C. Nowell (1992) "Entry, Collusion, and Capacity Constraints," *Southern Economic Journal*, Vol. 58, No. 4, pp. 1002-1014.
- [11] Matthews, S. and L. Mirrman (1983) "Equilibrium limit pricing: the effects of private information and stochastic demand," *Econometrica*, vol. 51, pp. 981-996.
- [12] Milgrom, P. and J. Roberts (1982) "Limit pricing and entry under incomplete information," *Econometrica*, vol. 50, pp. 443-66.
- [13] Ridley, D. (2008) "Herding versus Hotelling: Market entry with costly information," *Journal of Economics and Management Strategy*, vol. 17, pp. 607-631.
- [14] *The Economist* (2010) "Shining a Light. Solar cells are getting cheaper as subsidies subside," December 9<sup>th</sup>.
- [15] Ware, R. (1984) "Sunk Costs and Strategic Commitment: A Proposed Three-Stage Equilibrium," *The Economic Journal*, vol. 94, no. 374, pp. 370-378.