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Environmental Policy in a Linear City Model of Product Differentiation*

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Abstract

This paper analyzes how a tax/subsidy policy affects consumers' behavior when choosing between green (pollution free goods) and conventional products and its effects on welfare when some consumers have strong preferences for green goods. We develop a three stage complete information game, using the Hotelling's linear city model. We show that when products are identical in all respects except in their environmental properties, a tax/subsidy policy performs better than the case without policy. Our efficiency comparisons suggest that under a setting of horizontal product differentiation a tax/subsidy (either on consumers or polluting firms) produces higher social welfare than the absence of policy. Moreover, the proportion of consumers who prefer green products affects the welfare gains from a subsidy or tax policy.

Keywords: Green products, environmental policy, horizontal product differentiation

JEL classification: H23, L5, Q58

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1 Introduction

Environmentally friendly products also referred to as "green" products, are goods and services considered to inflict minimal or no harm on the environment, such as nontoxic detergents, mercury-free batteries, organic food, hybrid cars and clean energy. This paper analyzes settings where firms produce homogeneous goods, however, their environmental properties are significantly different. Specifically, we study the case in which the production of a green good is less harmful to the environment than a brown product. For instance, some firms emit little or zero emissions during their production process[1]. A green product is an impure public good in our setting, through which public goods can be provided privately (Cornes and Sandler, 1996; Kotchen, 2006). In this paper we consider that consumers who favor green products experience an additional benefit from their purchase decision. In particular, they are concerned about how the production process potentially affects the environment and benefit not only from using the good but also from its environmental characteristics. Nevertheless, as shown by Eriksson (2004) and Rodriguez-Ibeas (2007) such consumers' environmental idealism is not enough to lead to significant reductions in pollution. They also suggest that environmental policies should be considered to mitigate global pollution since consumers' environmental concerns are not sufficient. In this respect, our paper studies how a tax/subsidy policy can promote the consumption of green products in a context of horizontal product differentiation. In addition, we investigate the welfare consequences of this policy.

The paper develops a three-stage complete information game. In the first stage, the government decides the optimal policy (tax or subsidy). In the second stage firms compete à la Bertrand, finally, in the third stage of the game buyers choose their optimal consumption of either a "green" or a conventional "brown" product. We consider the case where two firms produce either brown or green goods. These two products provide the same base utility when they are consumed. The production process of a brown good harms the environment, whereas, green products do not damage the environment, i.e. they are pollution-free goods. Moreover, we assume that consumers are not homogeneous, that is, they have different preferences for green and brown products and their ultimate purchasing decision is affected by price and searching costs. The government in our model maximizes the social welfare which explicitly considers the environmental damage generated by the firm producing a brown good. The two different policies analyzed in this paper influence prices by modifying firms' marginal cost and thus influence consumers' behavior. In our study we aim to answer the following questions: 1) What is the optimal environmental policy when consumers are heterogeneous in their preferences for green products? and (2) how is such policy affected by the degree of heterogeneity among consumers?

We show that when products are differentiated by their environmental properties a tax/subsidy

[1] The list of companies that have taken steps to reduce carbon emissions includes I.B.M., Nike, Coca-Cola and BP. Google, Yahoo and Dell are among the companies that have vowed to become "carbon neutral." (See the New York Times, January 21st, 2009)

policy increases social welfare. Specifically, our efficiency comparisons suggest that under a setting of horizontal product differentiation a tax/subsidy (either on consumers or polluting firms) produces a larger social welfare than no policy. Moreover, the proportion of consumers who prefer green products also affects the welfare gains from a tax or subsidy policy. Some studies have previously analyzed government's influence on market behavior. Bansal and Gangopadhyay (2003) study the effects of both uniform policies on firms and regulations that discriminate between firms based on their environmental quality in the presence of environmentally aware consumers. They find that uniform and discriminatory subsidies reduce total pollution and enhance aggregate welfare, while taxes have the opposite properties. Cremer and Thisse (1999) analyze a vertical differentiation model considering firms' entry behavior. They show that a commodity tax may have a significant impact on the market structure and it may be welfare-improving. Unlike this literature, which considers vertical product differentiation, our paper examines a setting of horizontal product differentiation. As a consequence, this study provides a simplified way to analyze the impact of a policy on social welfare when consumers only care about the degree of pollution of a good during its production process. Our model does not need to assume that quality and environmental properties are related, unlike the previous literature. The final products generated by two firms are exactly the same, however, the externality (emissions) produced during the production process is different.

In addition to tax/subsidy policies, other policy instruments have also been discussed by several authors, such as emission standards or emission permits. For instance, like Cremer and Thisse (1999), Moraga-Gonzalez and Padro-Fumero (2002) consider a vertical product differentiation model, where firms' demand is exogenous. They show that despite the existence of an emission standard, industrial aggregate emissions can still be increased and social welfare reduced as a consequence of the standard. Similarly, Parry et. al. (1999) assessed the welfare effects of a revenue-neutral carbon tax and carbon emission permits using a vertically differentiated product model, taking into account pre-existing tax distortions in factor markets. They found that a carbon tax performs better than emissions permits and it is welfare improving under a larger set of parameter values.

The structure of the paper is as follows, section 2 describes the model, section 3 analyzes the general case without policies, section 4 analyzes the case of a tax/subsidy policy and section 5 contains some concluding remarks.

2 Model

Assume two types of consumers, green (G) and neutral (N). Green consumers account for λ proportion of the population while the proportion of neutral buyers is $1-\lambda$, where $\lambda \in (0,1)$. Similar to the linear city model of horizontal product differentiation we represent the length of the city from zero to one. Consumers are uniformly located along this segment and their location is denoted by $x \in (0,1)$, which represents the distance from the left end of the city. Assume that the total amount of consumers is $M > 0$. In addition, There are two firms located at each end of the city, firm 1 and firm 2, producing a green product and a brown product respectively. Firm 1 is at the left end and firm 2 at the right. Consider green and brown products being similar except for their impact on the environmental quality. For instance, if both firms produce detergent, both types of consumer obtain the same utility from using this product. However, the production of detergent by firm 1 does not contaminate the environment while the production process of firm 2 is not environmentally friendly and generates environmental damage. We also assume that each consumer buys one unit of good. The consumers' utility is described as follow,

$$\begin{aligned} u_i &= v - p_1 - rx + \widetilde{G} \text{ and } u_i = v - p_2 - r(1-x) + \widetilde{G} \\ \widetilde{G} &= \left[\begin{array}{c} g \text{ if } i = G \text{ and } p_1 \\ 0 \text{ if } i = N \text{ and } p_1 \\ 0 \text{ if } i = G, N \text{ and } p_2 \end{array} \right. \end{aligned}$$

when consumer i is green ($i = G$) she obtains a utility v from consuming a green or brown product and an additional utility g if this good is produced by firm 1. In addition, p_1 and p_2 denote the price of a green and a brown product, respectively, where both prices are strictly positive. The cost or disutility per x unit of distance traveled by the consumer when buying either product is $r \in \mathbb{R}^+$. The neutral buyer ($i = N$) only obtains a utility v from both types of products. In particular, this type of buyer does not experience an additional utility from a good produced using clean technology. Firm 1 and 2 have the following profit functions,

$$\begin{aligned} \pi_1 &= (p_1 - c_1)(Q_{1G}(p_1, p_2) + Q_{1N}(p_1, p_2)) \\ \pi_2 &= (p_2 - c_2)(Q_{2G}(p_1, p_2) + Q_{2N}(p_1, p_2)) \end{aligned}$$

where Q_{ji} represents firm j's demand function from buyer i and c_j represents firm j's marginal cost, where $j = \{1, 2\}$ and $i = \{G, N\}$. Assume linear cost functions and marginal costs satisfy $c_1 > c_2$, intuitively firm 1 incurs a higher cost from the reduction of emissions while firm 2's abatement efforts are zero[2]. In our model fixed costs are equal to zero.

In addition, the government seeks to implement an environmental policy, either a tax, t, or subsidy, s, which induces the consumption of an environmentally friendly product. The optimal policy maximizes the following social welfare function,

$$SW = CS + PS + Gov - Env$$

where CS is the consumer surplus including green and neutral buyers, PS represents the producer surplus considering profits from firms 1 and 2, Env is the environmental damage produced by firm 2 and Gov denotes the government revenue (expenditure) from the tax (subsidy, respectively). We

[2]See Bansal, S. and S. Gangopadhyay (2004), Conrad, K. (2005), Cremer, H. and J. Thisse (1999)and Eriksson, C. (2004).

consider that the production of one unit of the brown good produces one unit of emission, hence, $Env = d \times (Q_{2G}(p_1, p_2) + Q_{2N}(p_1, p_2))$ where d represents the marginal environmental damage. The time structure of the three-stage complete information game is as follows:

1. The government implements an environmental policy (either a tax or subsidy) that maximizes social welfare;
2. Firms simultaneously decide the price that maximizes their profit function; and
3. Green and neutral buyers choose whether to consume green or brown products.

3 General case (no policy)

In the case where the government does not implement an environmental policy, our model is reduced to a two stage game. First, let us solve the second stage of the game, where consumers choose which good to buy. A green consumer is indifferent between buying a green or a brown product when $v - p_1 - rx + g = v - p_2 - r(1-x)$. Therefore, the distance x that makes green buyers indifferent between consuming a green or brown product is $\widehat{x}_1 = \frac{p_2 - p_1 + g + r}{2r}$. Similarly, the indifference point for neutral buyers is $\widehat{x}_2 = \frac{p_2 - p_1 + r}{2r}$. Lemma 1 describes the demand from both types of consumers.

Lemma 1. *For a given pair of prices (p_1, p_2), firm 1's demand is,*

$$Q_{1G}(p_1,p_2) = \begin{cases} 0 \text{ if } p_1 > p_2 + g + r \\ \frac{p_2 - p_1 + g + r}{2r} \times \lambda M \text{ if } p_2 + g - r \leq p_1 \leq p_2 + g + r \\ \lambda M \text{ if } p_1 < p_2 + g - r \end{cases}$$

$$Q_{1N}(p_1,p_2) = \begin{cases} 0 \text{ if } p_1 > p_2 + r \\ \frac{-p_1 + p_2 + r}{2r} \times (1-\lambda) \times M \text{ if } p_2 - r \leq p_1 \leq p_2 + r \\ (1-\lambda) \times M \text{ if } p_1 < p_2 - r \end{cases}$$

and firm 2's demand is

$$Q_{2G}(p_1,p_2) = \begin{cases} 0 \text{ if } p_2 > p_1 + r - g \\ \frac{p_1 - p_2 + r - g}{2r} \times \lambda M \text{ if } p_1 - g - r \leq p_2 \leq p_1 + r - g \\ \lambda M \text{ if } p_2 < p_1 - g - r \end{cases}$$

$$Q_{2N}(p_1,p_2) = \begin{cases} 0 \text{ if } p_2 > p_1 + r \\ \frac{p_1 - p_2 + r}{2r} \times (1-\lambda) \times M \text{ if } p_1 - r \leq p_2 \leq p_1 + r \\ (1-\lambda) \times M \text{ if } p_2 < p_1 - r \end{cases}$$

The above lemma describes buyer's behavior for different ranges of prices. Notice that a green buyer does not consume firm 1's product if the difference in prices does not compensate the additional utility she obtains from consuming an environmentally friendly good (net of the travel cost, r). Similarly, the neutral consumer buys a green product if the price of the brown good exceeds the price of the green product, after considering the travel cost.

Firms identify their best response given the price charged by the other firm. That is, firm 1 has to decide its optimal price given its rival's strategy. Let us restrict our analysis to the case where $p_1 \in [p_2 - r, p_2 + g + r]$ and $p_2 \in [p_1 - g - r, p_1 + r]$. The shaded areas in Figure 1 describe the regions for the interior solution for green and neutral buyers.

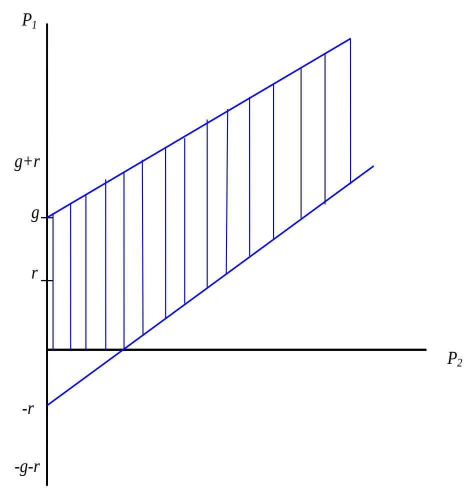

Figure 1: interior solutions.

Therefore, firms j's maximization problem is,

$$\max_{p_j} \pi_j = (p_j - c_j)(Q_{jG}(p_j, p_k) + Q_{jN}(p_j, p_k)) \tag{1}$$

where $j \neq k$. Therefore, optimal prices and firms' profits are,

$$p_1^* = \frac{3r + 2c_1 + c_2 + g\lambda}{3} \text{ and } \pi_1^* = \frac{M(3r - c_1 + c_2 + g\lambda)^2}{18r} \tag{2}$$

$$p_2^* = \frac{3r + c_1 + 2c_2 - g\lambda}{3} \text{ and } \pi_2^* = \frac{M(3r + c_1 - c_2 - g\lambda)^2}{18r} \tag{3}$$

Prices and profits of both types of firms are affected by the proportion of consumers, λ, who value the environmental benefits from green products. Notice that firms 1's profits (firm 2's) increase (decrease) with the proportion of consumers who value environmental products (larger λ) and with the additional benefits associated to green products (larger g). Additionally, as expected, we also find that the travel cost positively affects firms' profits. Finally, using (2) and (3) we obtain the optimal firms' demand,

$$Q_{1G}^{*} = \frac{3r - c_1 + c_2 - 2g\lambda + 3g}{6r}\lambda M \text{ and } Q_{2G}^{*} = \frac{3r + c_1 - c_2 + 2g\lambda - 3g}{6r}\lambda M \tag{4}$$

$$Q_{1N}^{*} = \frac{3r - c_1 + c_2 - 2g\lambda}{6r}(1-\lambda)M \text{ and } Q_{2N}^{*} = \frac{3r + c_1 - c_2 + 2g\lambda}{6r}(1-\lambda)M \tag{5}$$

The demand for green products is positively related with the utility that the consumer derives from protecting the environment, g. This result suggests that, for a given population, an increase on consumers' awareness for conservation of the environment positively affects the demand for goods produced by environmentally friendly firms. We next discuss our results when the government imposes a tax/subsidy policy.

4 Tax/Subsidy Policy

Let us now analyze the case where the government decides to promote the consumption of green products by setting a tax t. Hence, firm 2 pays an emission fee t per unit produced. The solution of the third stage of the game is similar to the one described in the previous section, given that consumers are not affected by this policy. However, the second stage of the game is substantially different, since firm 2 not only incurs a cost c_2 from producing the brown good but also an additional cost from its emissions. Firm 2's maximization problem becomes,

$$\max_{p_2} \pi_2 = (p_2 - c_2 - t)(Q_{2G} + Q_{2N}) \tag{6}$$

The optimal prices when a tax t is set by the government are identified in lemma 2.

Lemma 2. *The optimal prices of firm 1 and 2 in the second stage of the game where the government sets a tax t are, respectively,*

$$p_{1t}^{*} = \frac{3r + 2c_1 + c_2 + g\lambda + t}{3} \text{ and} \tag{7}$$

$$p_{2t}^{*} = \frac{3r + c_1 + 2c_2 - g\lambda + 2t}{3} \tag{8}$$

Higher taxes increase the price charged by both firms. Note that when the proportion of consumers who prefers green products is close to zero, then only the tax can potentially induce the consumption of environmentally friendly products. In fact, if the tax offsets the difference in marginal costs, $c_1 - c_2$, the good produced by firm 1 is cheaper than the brown product. In addition, firm 2's profits are affected by the tax. In particular, the tax favours firm 1's profits, increasing them by an amount equal to the tax. The following proposition states the main result about the first stage of the game, when the regulator sets an optimal tax, and below we elaborate on its intuition and comparative statics.

Proposition 1. *In the three stage complete information game, the optimal tax on firm 2 is*

$$t^* = (2g\lambda + 3d) - 2(c_1 - c_2)$$

In particular the optimal tax increases in the environmental damage and in consumers' benefit from the green product. In order to revert the environmental damage, the government sets a higher tax which induces the consumption of environmentally friendly products. In addition, The tax is decreasing in firm 1's cost disadvantage, $c_1 - c_2$. Intuitively, when both firms are equally efficient in the production of green and brown products, the tax only internalizes the negative effect of pollution and the utility benefits from the green product. In contrast, when the production of the green firm is more costly, $c_1 > c_2$, the tax must also help firm 1's competitiveness.

We next investigate the case where the tax is imposed on consumers of brown products. Since the tax is now paid by consumers, their utility functions are as follows,

$$\begin{aligned} u_i &= v - p_1 - rx + \overline{G} \text{ and } u_i = v - p_2 - r(1-x) + \overline{G} \\ \overline{G} &= \left[\begin{array}{c} g \text{ if } i = G \text{ and } p_1 \\ 0 \text{ if } i = N \text{ and } p_1 \\ -\widetilde{t} \text{ if } i = G, N \text{ and } p_2 \end{array} \right. \end{aligned}$$

Note that both types of consumers pay a tax $\widetilde{t}$ when buying a good produced by firm 2. Therefore, the demand for both firms is substantially different from those presented in Lemma 1. In the presence of a tax $\widetilde{t}$ on brown purchases, the distance x that makes green buyers indifferent between consuming a green or brown product is $\widetilde{x}_1 = \frac{p_2 - p_1 + g + r + \widetilde{t}}{2r}$. Similarly, the indifference point for neutral buyers is $\widetilde{x}_2 = \frac{p_2 - p_1 + r + \widetilde{t}}{2r}$.

Lemma 3 *For a given pair of prices (p_1, p_2), firm 1's demand is,*

$$\begin{aligned} Q_{1G}(p_1, p_2) &= \left[\begin{array}{c} 0 \text{ if } p_1 > p_2 + g + r + \tilde{t} \\ \frac{p_2 - p_1 + g + r + \tilde{t}}{2r} \times \lambda M \text{ if } p_2 + g - r + \tilde{t} \leq p_1 \leq p_2 + g + r + \tilde{t} \\ \lambda M \text{ if } p_1 < p_2 + g - r + \tilde{t} \end{array} \right. \\ Q_{1N}(p_1, p_2) &= \left[\begin{array}{c} 0 \text{ if } p_1 > p_2 + r + \tilde{t} \\ \frac{-p_1 + p_2 + r + \tilde{t}}{2r} \times (1-\lambda) \times M \text{ if } p_2 - r + \tilde{t} \leq p_1 \leq p_2 + r + \tilde{t} \\ (1-\lambda) \times M \text{ if } p_1 < p_2 - r + \tilde{t} \end{array} \right. \end{aligned}$$

and firm 2's demand is

$$Q_{2G}(p_1,p_2) = \begin{cases} 0 \text{ if } p_2 > p_1 + r - g - \tilde{t} \\ \frac{p_1 - p_2 + r - g - \tilde{t}}{2r} \times \lambda M \text{ if } p_1 - g - r - \tilde{t} \leq p_2 \leq p_1 + r - g - \tilde{t} \\ \lambda M \text{ if } p_2 < p_1 - g - r - \tilde{t} \end{cases}$$

$$Q_{2N}(p_1,p_2) = \begin{cases} 0 \text{ if } p_2 > p_1 + r - \tilde{t} \\ \frac{p_1 - p_2 + r - \tilde{t}}{2r} \times (1-\lambda) \times M \text{ if } p_1 - r - \tilde{t} \leq p_2 \leq p_1 + r - \tilde{t} \\ (1-\lambda) \times M \text{ if } p_2 < p_1 - r - \tilde{t} \end{cases}$$

When the regulator sets a tax on consumers buying brown goods relative prices are affected, ultimately promoting the consumption of green products. More neutral buyers are induced to consume green products as the relative price decrease, after considering taxes and traveling costs. In addition, more green buyers choose green products since brown goods became more expensive with the tax.

The following proposition specifies the optimal tax set on consumers.

Proposition 2. *In the three stage complete information game, the optimal tax on the consumers of the brown product is* $\widetilde{t}^* = t^*$.

Note that the optimal tax described in proposition 2 coincides with that identified in proposition 1. This tax ensures that those consumers who are indifferent between buying green and brown goods decide to purchase the environmental product given the extra cost imposed by a tax $\widetilde{t}$. Note that when the proportion of consumers preferring green goods increases, the tax increases and consumers become more heavily penalized for buying the brown product. Intuitively, the negative externality produced by firm 2 is mainly internalized by the neutral buyers who decide to consume a polluting good (brown product).

We next analyze the implementation of a subsidy, s. In this setting both types of consumers receive the subsidy s when buying a good produced by firm 1. Therefore, in the presence of a subsidy, the distance x that makes green buyers indifferent between consuming a green or brown product is now represented by $\overline{x}_1 = \frac{p_2 - p_1 + g + r + s}{2r}$. Similarly, the indifference point for neutral buyers is $\overline{x}_2 = \frac{p_2 - p_1 + r + s}{2r}$. Let us now examine the optimal subsidy resulting from the maximization of social welfare.

Proposition 3. *In the three stage complete information game, the optimal subsidy to consumers for their purchases of green products is* $s^* = t^*$.

Hence, a subsidy policy that makes the green product relatively cheaper is equivalent to a tax that makes the brown product more expensive. We next analyze the welfare comparison between the different environmental policies.

Corollary 1. *The social welfare when firm 2 (or a brown consumer) pays a tax, t^*, or firm 1 (or a green consumer) receives a subsidy, s^*, is strictly higher than for any other tax and subsidy levels, including $t = 0$ and $s = 0$.*

Hence, Corollary 1 implies that if two products are only differentiated in their environmental properties (i.e. in terms of emissions levels), the tax/subsidy policy increases the social welfare since it induces a higher proportion of neutral consumers to buy from firm 1, ultimately reducing the environmental damage. This result highlights the welfare-maximizing advantages of a tax/subsidy policy when there is horizontal product differentiation, and suggests that the implementation of environmental policies are indeed relevant when consumers are not homogeneous. The mere existence of a group of green buyers does not adequately address the negative externality produced by firm 2, while the implementation of environmental policies achieves higher social welfare.

5 Conclusions

This paper studies a three-stage complete information game in order to analyze how different policies affect consumers' behavior when choosing between green (pollution-free goods) and conventional brown products. Specifically, we study the effects of a pollution tax and a subsidy on welfare. Firms' demand is endogenous in our linear product differentiation model. Consumers are divided into two different groups, those who favor environmentally friendly production process and those who do not care about the protection of the environment. We show that, when products are differentiated in their environmental properties, the introduction of a tax/subsidy policy increases social welfare. In addition, the proportion of green buyers and the environmental damage induce more stringent policies. However, despite the consumers preferences, when a green good is too expensive to produce it maybe unprofitable to promote its consumption, and as a consequence the tax/subsidy is relatively small or nonexistent.

From a policy perspective, when consumers differently value the environmental properties of a product and, in addition, goods are identical except for their negative effects on the environment, a tax/subsidy policy increases welfare. However, this policy is specifically targeting the group of consumers (or firms) who do not assign any value to the environmental protection. Hence, the promotion of environmental awareness among consumers can potentially eliminate the role of the tax/subsidy policy and the costs bear from its implementation. In other words, the higher the proportion of consumers who obtain an additional benefit from green products the lower the need of this type of environmental policy. Sartzetakis et al. (2009) show that consumers' preferences changing over time through education can improve social welfare. Therefore, further research is necessary to analyze the complementarities between policies that increase consumers' awareness for the environment and tax/subsidy policies.

6 Appendix

6.1 General case

Let us identify the social welfare when no environmental policy (either tax or subsidy) is implemented by the government. The social welfare is defined as follows,

$$SW = CS + PS - Env$$

where the consumer surplus is composed by green (G) and neutral (N) buyers, $CS = CS_G + CS_N$. In addition, green buyers choose between goods produces by firm 1 and firm 2. Hence, $CS_G = CS_{1G} + CS_{2G}$ and $CS_N = CS_{1N} + CS_{2N}$. Finally, consumer surplus can be written as,

$$CS_G = CS_{1G} + CS_{2G} = \lambda M \left[\int_0^{\widehat{x}_1} (v - p_1 - rx + g)dx + \int_{\widehat{x}_1}^{1} (v - p_2 - r(1-x))dx \right]$$

and

$$CS_N = CS_{1N} + CS_{2N} = \lambda M \left[\int_0^{\widehat{x}_2} (v - p_1 - rx)dx + \int_{\widehat{x}_2}^{1} (v - p_2 - r(1-x))dx \right]$$

Producer surplus is composed by both firms' profits, $PS = \pi_1^* + \pi_2^* = \frac{M(3r - c_1 + c_2 + g\lambda)^2}{18r} + \frac{M(3r + c_1 - c_2 - g\lambda)^2}{18r}$ and the environmental damage is denoted by $Env = d \times (Q_{2G}(p_1, p_2) + Q_{2N}(p_1, p_2))$, where d represent the magnitude of environmental damage and $Q_{2G}(p_1, p_2) + Q_{2N}(p_1, p_2)$ is the total amount of emissions produced by firm 2. Where

$$Q_{2G}^* = \frac{3r + c_1 - c_2 + 2g\lambda - 3g}{6r} \lambda M \text{ and } Q_{2N}^* = \frac{3r + c_1 - c_2 + 2g\lambda}{6r} (1 - \lambda) M$$

Hence the total social welfare in the general case, SW_{GC}, can be denoted as,

$$SW_{GC} = \frac{M}{36r} \left[\begin{array}{c} 5(c_1 - c_2)^2 - 9r(2c_1 + 2c_2 + r - 4v) - g\lambda(10c_1 - 10c_2 + 8g\lambda - 9g - 18r - 4\lambda) \\ -6d(3r + c_1 - c_2 - g\lambda) \end{array} \right]$$

6.2 Proposition 1

First, in lemma 1 we solve the third stage of the game. Note that the tax is imposed on firm 2. Hence, firm 1's demand is,

$$Q_{1G}(p_1,p_2) = \left[\begin{array}{c} 0 \text{ if } p_1 > p_2 + g + r \\ \frac{p_2 - p_1 + g + r}{2r} \times \lambda M \text{ if } p_2 + g - r \leq p_1 \leq p_2 + g + r \\ \lambda M \text{ if } p_1 < p_2 + g - r \end{array} \right.$$

$$Q_{1N}(p_1,p_2) = \left[\begin{array}{c} 0 \text{ if } p_1 > p_2 + r \\ \frac{-p_1 + p_2 + r}{2r} \times M \times (1-\lambda) \text{ if } p_2 - r \leq p_1 \leq p_2 + r \\ M \times (1-\lambda) \text{ if } p_1 < p_2 - r \end{array} \right.$$

and firm 2's demand is

$$Q_{2G}(p_1,p_2) = \left[\begin{array}{c} 0 \text{ if } p_2 > p_1 + r - g \\ \frac{p_1 - p_2 + r - g}{2r} \times \lambda M \text{ if } p_1 - g - r \leq p_2 \leq p_1 + r - g \\ \lambda M \text{ if } p_2 < p_1 - g - r \end{array} \right.$$

$$Q_{2N}(p_1,p_2) = \left[\begin{array}{c} 0 \text{ if } p_2 > p_1 + r \\ \frac{p_1 - p_2 + r}{2r} \times M \times (1-\lambda) \text{ if } p_1 - r \leq p_2 \leq p_1 + r \\ M \times (1-\lambda) \text{ if } p_2 < p_1 - r \end{array} \right.$$

In the second stage of the game firms compete à la Bertrand. Hence, each firm maximizes the following maximization problem,

$$\max_{p_1} \pi_1 = (p_1 - c_1)(Q_{1G}(p_1,p_2) + Q_{1N}(p_1,p_2)$$

Similarly, firm 2 solves the following maximization problem,

$$\max_{p_2} \pi_2 = (p_2 - c_2 - t)(Q_{2G}(p_1,p_2) + Q_{2N}(p_1,p_2))$$

Therefore, optimal prices and firms' profits are,

$$\begin{aligned} p_{1t}^* &= \frac{3r + 2c_1 + c_2 + g\lambda + t}{3} \\ p_{2t}^* &= \frac{3r + c_1 + 2c_2 - g\lambda + 2t}{3} \end{aligned}$$

Hence, the demand of neutral and green consumers are,

$$\begin{aligned} Q_{1G}^* &= \frac{3r - c_1 + c_2 - 2g\lambda + 3g + t}{6r} \times \lambda M \\ Q_{2G}^* &= \frac{3r + c_1 - c_2 + 2g\lambda - 3g - t}{6r} \times \lambda M \end{aligned}$$

and the neutral buyer's demand is

$$
\begin{aligned}
Q^*_{1N} &= \frac{3r - c_1 + c_2 - 2g\lambda + t}{6r} \times M \times (1-\lambda) \\
Q^*_{2N} &= \frac{3r + c_1 - c_2 + 2g\lambda - t}{6r} \times M \times (1-\lambda)
\end{aligned}
$$

Substituting optimal prices and demand for green and brown products we obtain firms' profits,

$$
\begin{aligned}
\pi^*_1(t) &= \frac{M(3r - c_1 + c_2 + g\lambda + t)^2}{18r} \\
\pi^*_2(t) &= \frac{M(3r + c_1 - c_2 - g\lambda - t)^2}{18r}
\end{aligned}
$$

Finally, in the first stage of the game the government sets the optimal tax,

$$\max_t SW(t) = CS(t) + PS(t) + Tax - Env(t)$$

where the above maximization problem can be written as,

$$
\begin{aligned}
\max_t SW(t) &= CS_G(t) + CS_N(t) + \pi_1(t) + \pi_2(t) + t \times (Q_{2G}(p_1, p_2) + Q_{2N}(p_1, p_2)) \\
&\quad - d \times (Q_{2G}(p_1, p_2) + Q_{2N}(p_1, p_2))
\end{aligned}
$$

where

$$CS_G(t) = CS_{1G} + CS_{2G}(t) = \lambda M \left[\int_0^{\widetilde{x}_1} (v - p_1 - rx + g)dx + \int_{\widetilde{x}_1}^1 (v - p_2 - r(1-x))dx \right]$$

and

$$CS_N(t) = CS_{1N} + CS_{2N}(t) = \lambda M \left[\int_0^{\widetilde{x}_2} (v - p_1 - rx)dx + \int_{\widetilde{x}_2}^1 (v - p_2 - r(1-x))dx \right]$$

therefore the social welfare is,

$$\max_t SW(t) = SW_{GC} + \frac{tM}{36r}(-4c_1 + 4c_2 + 4g\lambda + 6d - t)$$

and solving the firts order conditions (F.O.C) with respect to t we obtain the optimal tax,

$$t^* = (2g\lambda + 3d) - 2(c_1 - c_2)$$

Finally, substituting t^* into the $SW(t)$ we obtain,

$$SW_t^{*}= SW_{GC}+\frac{M}{36r}((2g\lambda+3d)-2(c_1-c_2))^2$$

6.3 Proposition 2

First, in lemma 3 we solve the third stage of the game. Note that the tax, $\tilde{t}$, is imposed on consumers of the good produced by firm 2. Hence, firm 1's demand is,

$$Q_{1G}(p_1,p_2) = \left[\begin{array}{c} 0 \text{ if } p_1> p_2+g+r+\tilde{t} \\ \frac{p_2-p_1+g+r+\tilde{t}}{2r}\times\lambda M \text{ if } p_2+g-r+\tilde{t}\leq p_1\leq p_2+g+r+\tilde{t} \\ \lambda M \text{ if } p_1< p_2+g-r+\tilde{t} \end{array}\right.$$

$$Q_{1N}(p_1,p_2) = \left[\begin{array}{c} 0 \text{ if } p_1> p_2+r+\tilde{t} \\ \frac{-p_1+p_2+r+\tilde{t}}{2r}\times M\times(1-\lambda) \text{ if } p_2-r+\tilde{t}\leq p_1\leq p_2+r+\tilde{t} \\ M\times(1-\lambda) \text{ if } p_1< p_2-r+\tilde{t} \end{array}\right.$$

and firm 2's demand is

$$Q_{2G}(p_1,p_2) = \left[\begin{array}{c} 0 \text{ if } p_2> p_1+r-g-\tilde{t} \\ \frac{p_1-p_2+r-g-\tilde{t}}{2r}\times\lambda M \text{ if } p_1-g-r-\tilde{t}\leq p_2\leq p_1+r-g-\tilde{t} \\ \lambda M \text{ if } p_2< p_1-g-r-\tilde{t} \end{array}\right.$$

$$Q_{2N}(p_1,p_2) = \left[\begin{array}{c} 0 \text{ if } p_2> p_1+r-\tilde{t} \\ \frac{p_1-p_2+r-\tilde{t}}{2r}\times M\times(1-\lambda) \text{ if } p_1-r-\tilde{t}\leq p_2\leq p_1+r-\tilde{t} \\ M\times(1-\lambda) \text{ if } p_2< p_1-r-\tilde{t} \end{array}\right.$$

In the second stage of the game firms compete à la Bertrand. Hence, each firm maximizes the following maximization problem,

$$\max_{p_1}\pi_1 = (p_1-c_1)(Q_{1G}(p_1,p_2)+Q_{1N}(p_1,p_2))$$

Similarly, firm 2 solves the following maximization problem,

$$\max_{p_2}\pi_2= (p_2-c_2)(Q_{2G}(p_1,p_2)+Q_{2N}(p_1,p_2))$$

Therefore, optimal prices and firms' profits are,

$$p_{1\tilde{t}}^{*} = \frac{3r+2c_1+c_2+g\lambda+\tilde{t}}{3}$$
$$p_{2\tilde{t}}^{*} = \frac{3r+c_1+2c_2-g\lambda+2\tilde{t}}{3}$$

Hence, the demand of neutral and green consumers are,

$$
\begin{aligned}
Q^*_{1G} &= \frac{3r - c_1 + c_2 - 2g\lambda + 3g + \tilde{t}}{6r} \times \lambda M \\
Q^*_{2G} &= \frac{3r + c_1 - c_2 + 2g\lambda - 3g - \tilde{t}}{6r} \times \lambda M
\end{aligned}
$$

and the neutral buyer's demand is

$$
\begin{aligned}
Q^*_{1N} &= \frac{3r - c_1 + c_2 - 2g\lambda + \tilde{t}}{6r} \times M \times (1-\lambda) \\
Q^*_{2N} &= \frac{3r + c_1 - c_2 + 2g\lambda - \tilde{t}}{6r} \times M \times (1-\lambda)
\end{aligned}
$$

Substituting optimal prices and demand for green and brown products we obtain firms' profits,

$$
\begin{aligned}
\pi^*_1(\tilde{t}) &= \frac{M(3r - c_1 + c_2 + g\lambda + \tilde{t})^2}{18r} \\
\pi^*_2(\tilde{t}) &= \frac{M(3r + c_1 - c_2 - g\lambda - \tilde{t})^2}{18r}
\end{aligned}
$$

Finally, in the first stage of the game the government sets the optimal tax,

$$
\max_{\tilde{t}} SW(\tilde{t}) = CS(\tilde{t}) + PS(\tilde{t}) + Tax - Env(\tilde{t})
$$

where the above maximization problem can be written as,

$$
\begin{aligned}
\max_{\tilde{t}} SW(\tilde{t}) &= CS_G(\tilde{t}) + CS_N(\tilde{t}) + \pi_1(\tilde{t}) + \pi_2(\tilde{t}) + t \times (Q_{2G}(p_1,p_2) + Q_{2N}(p_1,p_2)) \\
&\quad - d \times (Q_{2G}(p_1,p_2) + Q_{2N}(p_1,p_2))
\end{aligned}
$$

where

$$
CS_G(\tilde{t}) = CS_{1G} + CS_{2G}(\tilde{t}) = \lambda M \left[\int_0^{\overline{x}_1} (v - p_1 - rx + g)dx + \int_{\overline{x}_1}^{1} (v - p_2 - r(1-x) - \tilde{t})dx \right]
$$

and

$$
CS_N(\tilde{t}) = CS_{1N} + CS_{2N}(\tilde{t}) = \lambda M \left[\int_0^{\overline{x}_2} (v - p_1 - rx)dx + \int_{\overline{x}_2}^{1} (v - p_2 - r(1-x) + \tilde{t})dx \right]
$$

therefore the social welfare is,

$$\max_{\tilde{t}} SW(\tilde{t}) = SW_{GC} + \frac{\tilde{t}M}{36r}(36r - 4c_1 + 4c_2 + 4g\lambda + 6d - \tilde{t})$$

and solving the F.O.C with respect to $\tilde{t}$ we obtain the optimal tax,

$$\tilde{t}^* = (2g\lambda + 3d) - 2(c_1 - c_2)$$

Finally, substituting $\tilde{t}^*$ into the $SW(\tilde{t})$we obtain,

$$SW^*_{\tilde{t}} = SW_{GC} + \frac{M}{36r}((2g\lambda + 3d) - 2(c_1 - c_2))^2$$

6.4 Proposition 3

In the presence of a subsidy to consumers buying green goods, the distance x that makes green buyers indifferent between consuming a green or brown product is now represented by $\overline{x}_1 = \frac{p_2 - p_1 + g + r + s}{2r}$. Similarly, the indifferent point for neutral buyers is $\overline{x}_2 = \frac{p_2 - p_1 + r + s}{2r}$. Firm 1's demand given a pair of prices (p_1, p_2) is,

$$Q_{1G}(p_1, p_2) = \left[\begin{array}{c} 0 \text{ if } p_1 > p_2 + g + r + s \\ \frac{p_2 - p_1 + g + r + s}{2r} \times \lambda M \text{ if } p_2 + g - r + s \leq p_1 \leq p_2 + g + r + s \\ \lambda M \text{ if } p_1 < p_2 + g - r + s \end{array} \right.$$

$$Q_{1N}(p_1, p_2) = \left[\begin{array}{c} 0 \text{ if } p_1 > p_2 + r + s \\ \frac{-p_1 + p_2 + r + s}{2r} \times M \times (1 - \lambda) \text{ if } p_2 - r + s \leq p_1 \leq p_2 + r + s \\ M \times (1 - \lambda) \text{ if } p_1 < p_2 - r + s \end{array} \right.$$

and firm 2's demand is

$$Q_{2G}(p_1, p_2) = \left[\begin{array}{c} 0 \text{ if } p_2 > p_1 + r - g - s \\ \frac{p_1 - p_2 + r - g - s}{2r} \times \lambda M \text{ if } p_1 - g - r - s \leq p_2 \leq p_1 + r - g - s \\ \lambda M \text{ if } p_2 < p_1 - g - r - s \end{array} \right.$$

$$Q_{2N}(p_1, p_2) = \left[\begin{array}{c} 0 \text{ if } p_2 > p_1 + r - s \\ \frac{p_1 - p_2 + r - s}{2r} \times M \times (1 - \lambda) \text{ if } p_1 - r - s \leq p_2 \leq p_1 + r - s \\ M \times (1 - \lambda) \text{ if } p_2 < p_1 - r - s \end{array} \right.$$

In the second stage of the game firms compete in Bertrand. Hence, each firm maximizes the following maximization problem,

$$\max_{p_1} \pi_1 = (p_1 - c_1)(Q_{1G}(p_1, p_2) + Q_{1N}(p_1, p_2))$$

Similarly, firm 2 solves the following maximization problem,

$$\max_{p_2} \pi_2 = (p_2 - c_2)(Q_{2G}(p_1, p_2) + Q_{2N}(p_1, p_2))$$

Therefore, optimal prices and firms' profits are,

$$\begin{aligned} p_{1s}^* &= \frac{3r + 2c_1 + c_2 + g\lambda + s}{3} \\ p_{2s}^* &= \frac{3r + c_1 + 2c_2 - g\lambda - s}{3} \end{aligned}$$

Hence, the demand of neutral and green consumers are,

$$\begin{aligned} Q_{1G}^* &= \frac{3r - c_1 + c_2 - 2g\lambda + 3g + s}{6r} \times \lambda M \\ Q_{2G}^* &= \frac{3r + c_1 - c_2 + 2g\lambda - 3g - s}{6r} \times \lambda M \end{aligned}$$

and the neutral buyer's demand is

$$\begin{aligned} Q_{1N}^* &= \frac{3r - c_1 + c_2 - 2g\lambda + s}{6r} \times M \times (1 - \lambda) \\ Q_{2N}^* &= \frac{3r + c_1 - c_2 + 2g\lambda - s}{6r} \times M \times (1 - \lambda) \end{aligned}$$

Substituting optimal prices and demand for green and brown products we obtain firms' profits,

$$\begin{aligned} \pi_1^*(s) &= \frac{M(3r - c_1 + c_2 + g\lambda + s)^2}{18r} \\ \pi_2^*(t) &= \frac{M(3r + c_1 - c_2 - g\lambda - s)^2}{18r} \end{aligned}$$

Finally, in the first stage of the game the government sets the optimal subsidy,

$$\max_s SW(s) = CS(s) + PS(s) - Subsidy - Env(s)$$

where the above maximization problem can be written as,

$$\max_s SW(s) = CS_G(s) + CS_N(s) + \pi_1(s) + \pi_2(s) - s \times (Q_{1G} + Q_{1N}) - d \times (Q_{2G} + Q_{2N})$$

where

$$CS_G(s) = CS_{1G}(s) + CS_{2G} = \lambda M \left[\int_0^{\vec{x}_1} (v - p_1 - rx + g + s)dx + \int_{\vec{x}_1}^{1} (v - p_2 - r(1-x))dx \right]$$

and

$$CS_N(s) = CS_{1N}(s) + CS_{2N} = \lambda M \left[\int_0^{\vec{x}_2} (v - p_1 - rx + s)dx + \int_{\vec{x}_2}^{1} (v - p_2 - r(1-x))dx \right]$$

therefore the social welfare is,

$$\max_s SW(s) = SW_{GC} + \frac{sM}{36r}(-4c_1 + 4c_2 + 4g\lambda + 6d - s)$$

and solving the F.O.C with respect to s we obtain the optimal subsidy,

$$s^* = -2c_1 + 2c_2 + 2g\lambda + 3d$$

Finally, substituting s^* into the $SW(s)$ we obtain,

$$SW_s^* = SW_{GC} + \frac{M}{36r}((2g\lambda + 3d) - 2(c_1 - c_2))^2$$

We next compare social welfare when no policy is in place versus the case where the consumer of a green product receives a subsidy,

$$SW_s^* > SW_{GC}$$

since,

$$SW_{GC} + \frac{M}{36r}((2g\lambda + 3d) - 2(c_1 - c_2))^2 > SW_{GC}$$

and

$$\frac{M}{36r}((2g\lambda + 3d) - 2(c_1 - c_2))^2 > 0 \text{ for all values of } c_1, c_2, g, \lambda, d, M \text{ and } r$$

6.5 Corollary 1

Welfare comparison. Note that the social welfare obtained in the general case, SW_{GC}, is always a component of the social welfare when a policy is implemented by the government. We obtained from our previous results that,

$$SW_t^* = SW_{\tilde{t}}^* = SW_s^* = SW_{GC} + \frac{M}{36r}(-2c_1 + 2c_2 + 2g\lambda + 3d)^2 > SW_{GC}$$

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