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By

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Abstract

This paper examines some of the economic tradeoffs in the joint production of biochar and bio-oil from cellulosic biomass. The pyrolysis process can be performed with different final temperatures, and with different heating rates. While most carbonization technologies operating at low heating rates result in higher yields of charcoal, fast pyrolysis is the technology of choice to produce bio-oils. Varying operational and design parameters can change the relative quantity and quality of biochar and bio-oil produced for a given feedstock. These changes in quantity and quality of both products affect the potential revenue from their production and sale. We estimate quadratic production functions for biochar and bio-oil. The results are then used to calculate a product transformation curve that characterizes the yields of bio-oil and biochar that can be produced for a given amount of feedstock, movement along the curve corresponds to changes in temperatures, and it can be used to infer optimal pyrolysis temperature settings for a given ratio of biochar and bio-oil prices.

Keywords: biochar, bio-oil, pyrolysis, biomass conversion, economic tradeoff

1. Introduction

The pyrolysis of biomass produces a combination of gases, biochar, and bio-oil, each of which has potential economic value for various uses. Biochar can be used as a soil amendment, for further combustion for energy, and other uses. Some studies have found
increased crop productivity from adding biochar to the soil by itself [1],[2] or by adding it jointly with fertilizer [3],[4]. Much of the recent interest in biochar, however, is its potential for as a soil amendment for carbon sequestration, due to its high carbon stability [5]. Bio-oil can be used as a substitute for heating oil, and can be refined into biogasoline, ethanol and/or other chemical compounds [6],[7]. The gas is generally used as an energy source to sustain the pyrolysis process. Pyrolysis can be performed with different final temperatures, and with different heating rates. Varying these and other factors can change the relative quantity and quality of biochar and bio-oil produced for a given feedstock. These changes in quantity and quality of both products affect the potential revenue from their production and sale. Depending on the heating rate achieved pyrolysis technologies can be classified into slow (less than 10°C/s) and fast (more than 10°C/s)[8]. Because of the low thermal conductivity of biomass (0.1 W/m K along the grain and 0.05 cross grain) [9] only particles with diameter below 2 mm can reach fast pyrolysis regimes.

In general, yield of solid (char) decreases and the yield of gases increase at higher temperatures. A maximum yield of bio-oil is attained at approximately 500°C. Furthermore, for a given feedstock type, the quality of both bio-oil and biochar depends in part on temperature. If producers can receive high prices for bio-oil but receive low prices for biochar, then they can increase sales revenue if they produce bio-oil at the expense of biochar by choosing a temperature and a heating rate that best suits these market price conditions.
In this paper, we develop a conceptual model for maximizing revenue with two products, bio-oil and biochar. Using this model we develop a decision rule that provides the optimal temperature for a given set of biochar and bio-oil market prices. We also consider the case in which market prices for a given quantity of bio-oil and biochar depend on the quality of these products. In order to apply the model, the production relationships between temperature and both biochar and bio-oil must be estimated. That is, an estimate is required of how much bio-oil and biochar are produced for a given final pyrolysis temperature. Data collected from a number of published studies were used in conjunction with primary data reported in [10]. Standard statistical regression methods were used to estimate these production relationships.

The estimated production relationships are then used to simulate examples of optimal pyrolysis temperature under different economic conditions. For example, if a market supports the production of biochar at a break-even price of $600 per ton ($0.60 per kg) and if we assume $1.00 per gallon ($0.22 per kg) of bio-oil, this provides a ratio of bio-oil price to biochar price of 0.37, suggesting heavy emphasis on low temperature slow pyrolysis (at or around a minimum acceptable final temperature of 350°C). In contrast, if biochar is of relatively little value, say $50 per ton, then the price ratio is 4.4, suggesting that fast pyrolysis at higher temperatures (about 522°C) is optimal. Biochar yield declines from about 40% of the original dry feedstock mass at 350°C to 26% at 540°C for slow pyrolysis. For fast pyrolysis, it declines from 32% to 18% for the same temperature range. We
further examine the case in which bio-oil and biochar quality changes at temperature changes, and provide simulations based on the energy content value of each product.

The next section develops an economic model of revenue maximization. Section 3 develops an econometric model of the effect of temperature and other factors on the output of the two products, bio-oil and biochar. Section 4 discusses the data and section 5 presents estimation and simulation results corresponding to fixed price and quality-dependent price scenarios, respectively. Section 6 concludes.

2. Economic foundations

Although the quality of the biochar and bio-oil will change according to the final pyrolysis temperature, as well as the heating rate, data in published studies generally do not include quality characteristics. However, a few studies include data on the heating value of biochar and bio-oil as a function of temperature. So we perform two types of analyses: we first provide an analysis in which quality characteristics are not accounted for, and then extend the analysis to allow for prices to change according to changing product quality (called endogenous prices), which in turn depends on temperature.

Given that temperature is a primary variable of choice in the pyrolysis process, we focus on choosing the optimal temperature for a given set of prices, conditional on the type
Below we develop a two product objective, to maximize the sum of the revenues from the two outputs (bio-oil and biochar) minus the input costs, by choosing temperature:

\[
\max_T V = P_C C(T, Z) + P_L L(T, Z) - K ,
\]

where \( V \) is net value of production, \( T \) is final temperature, \( P_C \) and \( P_L \) are biochar price and bio-oil price respectively, \( Z \) is a vector (a set) of other factors affecting yield, \( C(\cdot) \) and \( L(\cdot) \) are production functions relating \( T \) and \( Z \) to biochar yield \( (C) \) and bio-oil yield \( (L) \) respectively, and \( K \) is a fixed cost of production per unit of feedstock (feedstock cost), which may include market purchase of supplemental energy to sustain the pyrolysis process, as well as other costs. The fixed cost \( K \) has little relevance for the main focus of this paper, but is relevant when assessing profit levels for a given revenue choice.

As temperature increases, biochar quantity decreases, but bio-oil increases up to a point, then declines. Thus, an increase in temperature increases bio-oil revenues at the expense of biochar revenues. To maximize revenue, temperature should be increased to the point at which the increase in revenue from bio-oil no longer outweighs the revenue losses from biochar with an increase in temperature. In other words, the temperature that

\[1\] For a given temperature, the bio-oil and biochar yields as a percentage of biomass will tend to be independent of the scale of production. This characteristic allows the production relationships estimated in this study to be applicable to larger pyrolysis units. Although it does not mean that the pyrolysis process is scale independent in terms of optimal total production output and profitability, it does imply that the optimal pyrolysis temperature is independent of scale.
maximizes $V$ in equation (1) is that which equates the marginal revenue gains from bio-oil to the revenue losses from biochar.\(^2\)

This condition is shown graphically in Figure 1. The curved line is the *product transformation curve* (PTC) [11]. Any point on this line represents the output of biochar (on the vertical axis) and bio-oil (on the horizontal axis) that will be produced from pyrolysis at a given temperature for a given feedstock quantity and type. An increase in temperature will lead to a movement along this line down and to the right, providing more bio-oil and less biochar. The straight line from axis to axis in Figure 1 is an *isorevenue line*. This is the set of combinations of biochar and bio-oil that provides a given total revenue for a given pair of biochar and bio-oil prices.\(^3\) A combination that provides the highest possible revenue for a unit of feedstock is provided by the specific combination where the product transformation curve is tangent with the isorevenue line. In Figure 1, $C_1^*$ and $L_1^*$ is the optimal combination of yields for the price ratio shown. The dotted (partial) isorevenue line implies a higher bio-oil price relative to biochar price, and is associated with a higher optimal bio-oil yield ($L_2^*$) and lower biochar yield ($C_2^*$).

In the discussion above, bio-oil and biochar prices do not depend on temperature. This is economically equivalent to assuming that the quality does not vary in terms of economic value over the feasible temperature range, or at least, that market prices would not vary

\(^2\)The mathematical treatment of this condition is provided in Appendix A.

\(^3\)If revenue is $R = P_C C + P_L L$, rearranging this with $C$ alone on the left hand side provides the line $C = \frac{R}{P_C} - \frac{P_L}{P_C} L$. This is the straight line from axis to axis in Fig. 5.1, with intercept $R/P_C$ and slope of $-P_L/P_C$. 

6
according to quality. This may be a misleading assumption in some cases. For example if energy content is the basis for economic value, both bio-oil and biochar increase in energy value as pyrolysis temperature increases over the relevant temperature range. In general, if the price increases with temperature, there is an additional revenue gain from each incremental temperature increase. This means that for otherwise similar circumstances, the incremental price increase will tend to push the optimal temperature higher than if prices were fixed and constant. In the production setting under consideration here, the combination of \( C \) and \( L \) are chosen indirectly by choosing the temperature at which pyrolysis is performed, and the pair of production relationships \( C(T) \) and \( L(T) \) imply a product transformation curve between \( C \) and \( L \). At this point, specific equations representing the relationships between process factors (temperature, feedstock, heating rate, etc.) and the two products (biochar and bio-oil), are necessary to proceed. Regression analysis is applied to the data we collected from published research on pyrolysis.

3. **Econometric model and yield function estimation**

A large literature exists for estimating production functions for application to economic problems. This literature provides a wide range of production relationships and estimation methods, ranging from restrictive to highly flexible functions to represent (global or local) production relationships, and restrictive to highly flexible estimation

\[\text{Equation}^4\]

The mathematical treatment of this problem is provided in Appendix A.
methods. After exploratory analysis, given the limited published data on biochar and bio-oil, and to provide a practical foundation for interpretation, this analysis relies on a relatively simple quadratic production relationship between the input (temperature) and the outputs (biochar and bio-oil).

The regression model used can be represented in general as:

\[
\begin{align*}
C(T_i) &= \alpha_0 Z_i + \alpha_1 T_i + \alpha_2 T_i^2 + \varepsilon_i \\
L(T_i) &= \beta_0 Z_i + \beta_1 T_i + \beta_2 T_i^2 + \nu_i
\end{align*}
\]

(2a) (2b)

where \(i\) is an observation index, \([\alpha_0 \alpha_1 \alpha_2]\) and \([\beta_0 \beta_1 \beta_2]\) are parameters to be estimated (bold represents a vector containing several parameters); \(C(T_i)\) and \(L(T_i)\) are biochar and bio-oil yield for observation \(i\), respectively; \(T_i\) and \(Z_i\) are factors that affect yield; and \(\varepsilon_i\) and \(\nu_i\) are random disturbance terms. Using \(T_i\) and the square of \(T_i\) as regressors allows for temperature to have a nonlinear effect on yields, which is important for solving for the optimal pyrolysis temperatures. The derivation of the optimal temperature and the product transformation curve is given in Appendix A.

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5 General theoretical foundations are provided in [9] and [10]. Among many seminal papers, an example of foundations and estimation of flexible production frontiers is given in [11]. More recently, some studies look at the productivity and utilization of scarce natural resources such as: fisheries (e.g., using a transformation production function [12]; short-run translog cost function [13]; Generalized Leontief production function [14]); forest resources (e.g., natural disturbance (forest fires, invasive species) production functions [15]; forest collection production functions of different household labor categories [16]). There are also studies that examine issues related to agriculture such as: technical efficiency of enterprises or farmers (e.g., stochastic frontier production function [18]); and risks in production of using pest control and GM crop technologies (e.g., 'flexible risk' production function models [19]; micro-data based agricultural production functions [20]).

6 This simplicity economizes on degrees of freedom in estimation, and allows for ease of interpretation within the sampling range (and economically meaningful range) of the available data, and provides a second-order approximation to more flexible forms for the relationship between temperature and yields. One issue arises because the dependent variables in these regressions are percentages, and do not range outside of the [0,100] interval. This in principle can cause complications for Ordinary Least Squares (OLS) if there are numerous observations near or on the range limit. This is generally not the case with the data used here, so we ignore this issue.
The data for this analysis are taken from published studies that are based on fast pyrolysis and/or slow pyrolysis, and numerous different types of feedstocks. The variables in the matrix $Z$ include factors other than temperature that affect product yield per unit of feedstock mass, such as feedstock type and heating rate (fast or slow). In the regression results presented below, we include as explanatory variables in $Z$ an indicator variable to distinguish between fast ($fast=1$) and slow ($fast=0$), as well as indicator variables for four different feedstock categories: agricultural crop residues, other agricultural feedstocks, forest products, and other feedstocks.\(^7\)

If we allow for price response to temperature differences, price response functions $P_C(T)$ and $P_L(T)$ must be specified. Our data do not allow direct estimation of these functions. However, assuming linear price response functions as an approximation, they can be characterized as

\begin{align}
    P_C(T) &= \delta_0 + \delta_1 T \tag{3a} \\
    P_L(T) &= \gamma_0 + \gamma_1 T, \tag{3b}
\end{align}

If prices increase with temperature in the relevant temperature range, then all coefficients will be positive. The consequence is that there is an additional incremental benefit from increasing temperature: not only do you have an incremental increase in bio-oil yield, but prices tend to increase with temperature, so that the optimal temperature will tend to be higher than when price is independent of temperature. The optimal temperature

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\(^7\)These categories include several different specific feedstock types, but are aggregated to larger categories in order to economize on degrees of freedom in estimation.
given these price functions is derived in Appendix A. Further, an explanation of how a practitioner can customize these calculations is provided in Appendix A as well.

4. Data

Data on biochar and bio-oil yields from different feedstocks were collected from various studies on pyrolysis and are classified as follows.\(^8\)

- **Agricultural field residue** — includes tobacco stalk, rice straw, cotton stalk, corn stover, and wheat straw;
- **Agricultural feedstock (other)** — includes hazelnut shell, sugarcane bagasse, coconut shell, sorghum bagasse, sunflower hulls, flax shives, corn cob, and olive waste (from oil production);
- **Forest products** — includes pine chips/wood/bark, pine sawdust, beech, poplar-aspen cellulose, maple bark, softwood bark, poplar sawdust, spruce sawdust, birch wood;
- **Other feedstock** — bamboo, tea factory waste, newsprint, fine paper, pulp mill waste, peat moss.

Figure 2 provides raw scatter plots of the data used in our analysis for the above set of feedstock types.

Biochar and bio-oil are joint products of a pyrolysis process, categorized into two types: fast pyrolysis and slow pyrolysis. The main difference in the two technologies in terms of revenue is how fast the materials are heated (i.e., the rate of increase in

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\(^8\) Data are obtained from studies listed in Appendix B.
temperature per minute up to the final process temperature). Faster heating rates in fast pyrolysis favor reactions leading to the formation of higher yields of oil and lower yields of biochar.

Slow pyrolysis, on the other hand, results in higher yields of biochar compared to fast pyrolysis, and in the formation of two liquid phases – a much lower amount of oil relative to the fast pyrolysis process, and an aqueous phase (water plus a variety of organo-oxygen compounds of low molecular weight). This phenomenon is due an intensification of dehydration and polycondensation reactions leading to the formation of extra-water and more charcoal. Different rates of temperature increase, final pyrolysis temperatures, and feedstock type alter the quality characteristics of bio-oil and biochar. This in turn will affect their economic value for different uses. The temperatures in our dataset correspond to the final heating temperature. Although our data differentiate between fast and slow pyrolysis applications, information is not available to control for differences in heating rates and other characteristics within these two categories. Further, data are not sufficiently available to allow modeling bio-oil and biochar quality differences. We therefore use a simple binary categorical distinction between fast and slow pyrolysis in the regression estimation.

Note that the above discussion relates to outputs and revenues specifically. There are also likely to be cost differences between fast and slow pyrolysis as well. In particular, fast pyrolysis requires the use of small (usually <2 mm), pre-processed feedstock particles [8], whereas slow pyrolysis does not require such preprocessing to be effective. Therefore, the
variable $K$ in equation (1) may be different for slow and fast pyrolysis, and the higher revenues for fast pyrolysis may not imply higher net revenues (revenues minus costs).

Table 1 provides undifferentiated summary statistics for the data used in the analysis. Seventy-one percent of our sample data from the published studies relied on were generated using fast pyrolysis, and the majority of the sample data (53%) are based on various (woody) forest product feedstocks. A fundamental choice variable in the pyrolysis process is temperature. Temperatures in our sample data range from 250°C to 1000°C.

Figure 2 is a series of descriptive scatter plots showing the relationship between bio-oil yield and biochar yield, by feedstock and pyrolysis type. The estimated regressions performed below are designed to be able to, among other things, define a relationship between biochar and bio-oil yields as a function of temperature, and holding constant other factors such as feedstock type. These quadratic fits are imprecise because they do not control for feedstock type.

5. Results and discussion

In this section we summarize the regression results, generate estimates of product transformation curves and optimal temperature for a given set of prices, and discuss revenue optimization.

Tables 2 and 3 provide the regression results for biochar yield and bio-oil yield, estimated for slow and fast pyrolysis separately. Indicator variables are included for
feedstock categories, although no observations exist for other feedstocks under slow pyrolysis. The coefficients on the other indicator variables represent the intercepts of the regression lines for each feedstock type. A constant was omitted from each regression to avoid perfect collinearity of the indicator variables. The difference in these parameter estimates then represents the difference in yield among feedstocks for any given temperature. For example, in the first (left hand) regression in Table 2 slow pyrolysis, the feedstocks included in the Agricultural field residue category tend to provide approximately 0.04 percent less biochar than the feedstocks included in the Forest Products category (103.43 - 103.39 = 0.04). The R-square measures in Tables 2 and 3 show that the regression explains approximately ninety percent of the variation in each regression. Estimation is carried out equation by equation using Ordinary Least Squares, and White’s Heteroskedastic-consistent standard errors are reported.

These regressions provide a foundation for understanding the revenue tradeoffs implicit in the choice of temperatures, feedstock types, and pyrolysis types. They also provide the information necessary to develop product transformation curves and construct the relationship between output prices and optimal temperature settings. For example, the estimates for the temperature (Temp(C)) parameters in the slow pyrolysis biochar and bio-oil regressions provide the values \( a_1 = -0.2253 \) and \( \beta_1 = 0.1371 \), respectively, in equations (2a), (2b), and appendix equations (A.3a), (A.3b), (A.4), (A.9) and (A.10). The

\[9\] There are only two observations that correspond to slow pyrolysis applied to agricultural field residues. Most of the explanatory power relating to agricultural field residues comes from the 35 observations of fast pyrolysis applied to field residues.
parameters associated with temperature squared ($Temp(C)\text{ sq.}$) are $\alpha_2 = 1.5E - 04 = 0.00015$ (Table 2) and $\beta_2 = -1.2E - 04 = -0.00012$ (Table 3).

Figure 3 plots the yields of biochar and bio-oil from woody forest products as a function of temperature, for slow and fast pyrolysis, respectively. The two graphs have common scales for clear comparison. First consider the differences between the fast and slow pyrolysis functions. Together, the graphs show that for any given temperature, slow pyrolysis is estimated to provide more biochar and less bio-oil for a given amount of feedstock than fast pyrolysis does. This implies that the relative economic efficacy of slow and fast pyrolysis will depend, in part, on the relative prices of the two outputs.

Second, the graphs show that for our temperature range, bio-oil yield increases with temperature for low temperatures, but then begins to decline. For slow pyrolysis, the temperature that provides maximum bio-oil is about $T=549^\circ C$ (at the vertical dotted line in the first panel of Figure 3). For fast pyrolysis, the temperature that provides maximum bio-oil is $524.92^\circ C$. In contrast, biochar yields decline over the entire range. Given the specific shapes of these two functions, the economic region of temperature must be lower than that which provides the maximum bio-oil yield, i.e., the area to the left of the vertical dotted lines.

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\(^{10}\) Note that in Figure 3 and some subsequent figures, the horizontal axis includes temperatures below our minimum in-sample temperature of $250^\circ C$ for exposition of the quadratic functions. This area of these graphs should be interpreted with some skepticism, both because they represent extrapolations beyond the sample region, and also because pyrolysis is usually not effective below $250^\circ C$. 

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At low temperatures, there is a tradeoff; biochar yield declines but bio-oil increases as temperature increases. The optimal temperature depends on the relative prices of the two outputs. At higher temperatures, both yields decline, so in this region, a further increase in temperature necessarily reduces revenue (and reducing temperature necessarily increases revenue), thus making production at and above the temperature where bio-oil yields begin to decline uneconomical regardless of relative prices. Figure 4 shows the estimated product transformation curves (PTC) under slow and fast pyrolysis. These two curves cross, such that slow pyrolysis PTC is above that for fast pyrolysis on the left, but below on the right. This is of some interest because it implies that slow pyrolysis may tend to provide higher revenues per unit of feedstock than fast pyrolysis under some price conditions, and vice versa. As Figure 1 shows, revenue maximization for a given quantity of feedstock entails choosing the combination of biochar and bio-oil (indirectly through pyrolysis temperature) subject to the prices of these two outputs.

5.1 Optimal temperature for fixed prices

Figure 5 shows the optimal temperature for a given price ratio under fast and slow pyrolysis. These curves are derived by substituting the appropriate regression coefficients from the fast and slow pyrolysis regressions, respectively, into appendix equation (A.4). Because the units of measure for the two outputs are the same, the units of measure should be equivalent for each price when interpreting the results. For concreteness, we examine

11 Costs are not accounted for here. Even if the product transformation curve for fast pyrolysis lies above that of slow pyrolysis at the optimal temperature, the production costs can still affect profitability.
specific cases below based on price per kilogram (kg) for each output. At relatively low bio-oil prices, the optimal temperature under fast pyrolysis is higher than for slow pyrolysis, but the opposite is true for higher ratios of bio-oil and biochar. Notice that the upper bounds on these curves (to the right on the graph) approach the economic maxima of temperature discussed earlier.

The minimum temperature in our data sample is 250°C. However, little pyrolysis occurs below about 300°C, and the valuable characteristics of both biochar and bio-oil deteriorate below about 350°C. This (latter) temperature is optimal under slow pyrolysis for a price ratio of \( P_L/P_C = 2.40 \). For fast pyrolysis, the price ratio implying an optimal temperature of 350°C is 1.36. Anything below this price ratio would call for more emphasis on biochar. On the other end of the economic spectrum, profits necessarily decline (regardless of prices) at about 549.3°C (slow) and 524.9°C (fast), so this will be an upper bound economically valid temperature, and would apply if, for example, the price of biochar is very small so that the price ratio \( P_L/P_C \) becomes very large.

It is useful to consider a set of feasible or possible prices and their outcomes as an example. Because the data on biochar and bio-oil yield (and therefore the estimated parameters and the analysis developed above) are based on percent of feedstock mass, the units of measure for biochar and bio-oil, as well as the prices for each, must be in the same unit of measure. This analysis below uses kilograms as units, and dollars per kilogram as the price unit. The range of estimates for bio-oil is from about $0.60 to about $1.06 per gallon [22]. If we assume $1.00 per gallon of bio-oil, this translates to $0.22 per kg of bio-
oil. The price of biochar could vary significantly depending on the market. Suppose a market supports biochar production at a break-even price of $600 per ton, this translates to $0.60 per kg. The result is a price ratio of $P_L/P_C = 0.37$, suggesting heavy emphasis on low temperature, slow pyrolysis. In contrast, if biochar is of relatively little value, say $50 per ton, then the price ratio is 4.4, suggesting that fast pyrolysis at higher temperatures (about 522°C) is optimal.

5.2 Optimal temperature for endogenous prices

When price is a function of quality, which in turn is a function of temperature, the price ratio itself is determined in part by the chosen temperature. Therefore, there is no fixed price ratio determining the optimal temperature. Instead, optimal temperature and price are simultaneously determined. A specific set of price parameters is developed here and used to solve for the optimal temperature. The data for this relationship between bio-oil quality (and therefore price) and temperature are very limited. The following analysis relies on linear extrapolation of the price-temperature relationship, putting some of the calculated results outside the range of the original data. This example begins with the assumption that both biochar and bio-oil will be used as an energy source, with price related to energy content of the product. Further, we assume that bio-oil is a substitute in use for fossil crude oil, and biochar is a substitute for coal. The energy content of biochar tends to be higher than coal, and the energy content of bio-oil tends to be lower than that of coal.

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12 This break-even price was based on one of several scenarios developed for an economic analysis presented in [10].
fossil crude oil. However, as an approximation, we assume that the *price per unit of energy* of bio-oil is equal to that of fossil oil, and that the price per unit energy of biochar is equal to that of coal.\(^\text{(13)}\)

Let the high heating calorific value of crude oil and bio-oil be 45.7 MJ/kg and 18 MJ/kg, respectively. If the price of crude oil is $52/barrel, the price is also equivalent to $0.398/kg or $0.008709/MJ. Figure 10 in [23] provides an estimated regression that relates fast pyrolysis temperature to high heating value (dry) based on temperatures between 350°C and 575°C. Substituting these numbers into equation (3b), we approximate the energy content function for bio-oil to be
\[
\text{Calories (MJ/kg)} = 17.85 + 0.0075 T^\circ \text{C},
\]
so that price in dollars per kilogram is
\[
P_L ($/kg) = 0.15545514 + 0.0000658 T^\circ \text{C}.
\]
This provides a price of $0.188 per kilogram of bio-oil at T=500°C.

A price of $68.10/metric ton of coal provides a price of $0.068/kg or $0.002528/MJ from coal [i.e., ($0.068/kg) / (26.9 MJ/kg)]. Substituting these numbers into equation (3a) provides an estimated relationship between calorific content of biochar as a function of pyrolysis temperature:

\[\text{-----------------}\]

\(^{13}\) This equal price reflects a market outcome driven by the assumed (perfect) substitutability of the two related goods (e.g. biochar and coal). If production cost of one of them is higher than the other — that is, if the market supply curves differ, then the quantities produced of the two goods will differ. If the two goods are not perfect substitutes in consumption, then prices will tend to differ also. In particular, if one has some disadvantages in terms of refinement (an intermediate demand), then it will tend to fetch a lower price per MJ.
*Calories (MJ/kg) = 16.2 + 0.02678571 T°C*, so the price of biochar would be

\[
P_c (\$/kg) = 0.002528 \times (16.2 + 0.02678571 T°C)
\]

\[
= 0.0409536 + 0.00006771 T°C.
\]

(5)

This provides a price of $0.0748 per kilogram of biochar at T=500°C.

Using equations (4) and (5) and estimated counterparts of equations (2a) and (2b) for slow pyrolysis applied to forest feedstocks to specify the revenue function (appendix equation A.7 omitting K) provides revenue as a function of temperature. This revenue function is shown in Figure 6. Using the derivations in appendix A beginning with equations (A.7), equation (A.10) allows the calculation of an optimal temperature of 536.4°C for the case with endogenous prices.

Making use of the yield functions and price functions, for slow pyrolysis the optimal estimated yield and price for biochar is 26.037% and $0.077/kg, and for bio-oil optimal yield and price are 38.19% and $0.192/kg. Maximum revenue for slow pyrolysis is $0.09296/kg of forest-based feedstock. The implied price ratio for slow pyrolysis is 2.4686. If the market prices were fixed and constant across the temperature range as in the previous section, a price ratio of 2.4686 would lead to optimal temperatures of 360.14°C for slow pyrolysis, which is much lower than for the case of endogenous prices.

For fast pyrolysis, the estimated optimal temperature is 521.9°C. Biochar yield is 19.8%, and bio-oil yield is 54.46%. Biochar price is $0.076/kg, and bio-oil price $0.19/kg. Maximum revenue is $0.11848/kg. So, based on energy content, the fast pyrolysis provides higher revenues by $0.0255, an increase of 27.4%. Thus, if energy content provides the
highest value for both products, fast pyrolysis provides higher revenues. Again, Appendix A contains details and a summary of parameter values to facilitate the calculation of optimal temperatures for a given setting.

6. Conclusion

This paper presents a model and estimates for maximizing the sum of revenues from the joint production of bio-oil and biochar. The primary control variables in this process are the type of pyrolysis used (either fast or slow), and the final pyrolysis temperature, but feedstock type is important as well. We provide a method for choosing the revenue maximizing pyrolysis temperature in two cases: first, when market prices are fixed and do not vary with temperature, and second, when output quality and therefore price changes as pyrolysis temperature is altered. The dataset is limited, and relies on several different types of feedstocks to estimate the yield parameters, using a relatively restrictive functional form. Further, the price response functions in the final example above are limited to only one, albeit fundamental, use of biochar and bio-oil, so these results are relatively narrow in scope. Nonetheless, the results are generally plausible and provide a foundation for refinement.

Maximizing revenues by choosing process temperature can be economically important. However, even if the optimal combination of bio-oil and bio-char is produced, it may not provide an economically viable (profitable) enterprise. This ultimately depends on whether the revenues from production outweigh the costs. The data used in this
analysis of the tradeoffs between biochar and bio-oil production do not allow a cost analysis.

Acknowledgments

This work is part of the project, *Use of Biochar from the Pyrolysis of Waste Organic Material as a Soil Amendment* [10]. We would like to thank the Washington State Department of Ecology for funding the project.
References


Figure 1. Optimal combination of biochar (C1*) and bio-oil (L1*) yield for prices PL and PC. Higher oil price PL relative to char price Pc (dotted isorevenue line) leads to higher optimal yields of bio-oil relative to biochar.
Figure 2. Scatter plots of biochar versus bio-oil yield, by feedstock category and pyrolysis type. This visual representation of the data from various studies roughly illustrates the trade-off in the production of bio-oil and biochar from various feedstock categories.
Figure 3. Biochar and bio-oil yields under slow and fast pyrolysis.
Figure 4 Product transformation curves under slow and fast pyrolysis.
Figure 5. Estimated optimal temperature for bio-oil and biochar price ratios.
**Figure 6.** Revenue function for slow pyrolysis with endogenous prices. Vertical dotted line shows optimal temperature of 536.4°C.
### Table 1. Summary statistics of variables used in estimation and analysis (n=206).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Biochar, % of feedstock mass</td>
<td>26.37</td>
<td>13.50</td>
<td>3</td>
<td>77</td>
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<tr>
<td>Bio-oil, % of feedstock mass</td>
<td>43.56</td>
<td>16.51</td>
<td>3.2</td>
<td>81.9</td>
</tr>
<tr>
<td>Indicator for fast pyrolysis</td>
<td>0.71</td>
<td>0.46</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Pyrolysis temperature, Celsius</td>
<td>553.76</td>
<td>155.97</td>
<td>250</td>
<td>1000</td>
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<tr>
<td>Feedstock: Agricultural field residue indicator</td>
<td>0.18</td>
<td>0.38</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Feedstock: Agricultural – other</td>
<td>0.23</td>
<td>0.42</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Feedstock: Forest products</td>
<td>0.53</td>
<td>0.50</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Feedstock: Other</td>
<td>0.06</td>
<td>0.24</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>
Table 2. Regression results. Dependent variable: Biochar as a percent feedstock mass.

<table>
<thead>
<tr>
<th>Indep. variable</th>
<th>Est.</th>
<th>Std. Err.</th>
<th>95% conf. int.</th>
<th>Est.</th>
<th>Std. Err.</th>
<th>95% conf. int.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temp (C) sq.¹</td>
<td>1.5E-04</td>
<td>5.8E-05</td>
<td>3.5E-05</td>
<td>2.7E-04</td>
<td>9.4E-05</td>
<td>4.5E-05</td>
</tr>
<tr>
<td>Temp (C)</td>
<td>-0.2253</td>
<td>0.0623</td>
<td>-0.3501</td>
<td>-0.1005</td>
<td>-0.1655</td>
<td>-0.2291</td>
</tr>
<tr>
<td>Forest prod.²</td>
<td>103.43</td>
<td>15.60</td>
<td>72.17</td>
<td>134.70</td>
<td>80.67</td>
<td>60.96</td>
</tr>
<tr>
<td>Ag residue</td>
<td>103.39</td>
<td>15.12</td>
<td>73.09</td>
<td>133.69</td>
<td>91.27</td>
<td>71.26</td>
</tr>
<tr>
<td>Other Ag</td>
<td>106.14</td>
<td>16.39</td>
<td>73.29</td>
<td>138.99</td>
<td>89.67</td>
<td>69.96</td>
</tr>
<tr>
<td>Other³</td>
<td>88.63</td>
<td>10.40</td>
<td>68.07</td>
<td>109.38</td>
<td>91.27</td>
<td>71.26</td>
</tr>
</tbody>
</table>

1The estimates for the temperature (Temp(C)) parameters in the biochar regressions correspond to $\alpha_1$ in equations 2a and 2b, and appendix equations A3a, A3b, A4, A7 and A8. The parameters associated with temperature squared (Temp sq.) correspond to $\alpha_2$.

2Coefficients associated with feedstock types (the four last rows of coefficients) represent the intercept for slow pyrolysis applied to each respective feedstock type. The constant is omitted to avoid perfect multicollinearity.

3 No observations are available for slow pyrolysis applied to Other Feedstock.

Table 3. Regression results. Dependent variable: Bio-oil as a percent feedstock mass.

<table>
<thead>
<tr>
<th>Indep. variable</th>
<th>Est.</th>
<th>Std. Err.</th>
<th>95% conf. int.</th>
<th>Est.</th>
<th>Std. Err.</th>
<th>95% conf. int.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temp (C) sq.¹</td>
<td>-1.2E-04</td>
<td>8.3E-05</td>
<td>-2.9E-04</td>
<td>4.2E-05</td>
<td>-2.1E-04</td>
<td>-2.8E-04</td>
</tr>
<tr>
<td>Temp (C)</td>
<td>0.1371</td>
<td>0.0894</td>
<td>-0.0421</td>
<td>0.3163</td>
<td>0.2205</td>
<td>0.1328</td>
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<tr>
<td>Forest prod.²</td>
<td>0.556</td>
<td>22.396</td>
<td>-44.327</td>
<td>45.438</td>
<td>-3.420</td>
<td>-30.610</td>
</tr>
<tr>
<td>Other Ag</td>
<td>14.552</td>
<td>23.533</td>
<td>-32.609</td>
<td>61.714</td>
<td>-9.531</td>
<td>-36.730</td>
</tr>
<tr>
<td>Other³</td>
<td>-7.446</td>
<td>14.351</td>
<td>-35.819</td>
<td>20.927</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1The estimates for the temperature (Temp(C)) parameters in the bio-oil regressions correspond to $\beta_1$ in equations 2a and 2b, and appendix equations A3a, A3b, A4, A7 and A8. The parameters associated with temperature squared (Temp sq.) correspond to $\beta_2$.

2Coefficients associated with feedstock types (the four last rows of coefficients) represent the intercept for slow pyrolysis applied to each respective feedstock type. The constant is omitted to avoid perfect multicollinearity.

3 No observations are available for slow pyrolysis applied to Other Feedstock.
Appendix A

This appendix provides (a) more detail than the text regarding the underlying optimization theory used to derive the optimal pyrolysis temperatures, (b) a compilation of the parameter values used in our analysis, and (c) a concise description of how a practitioner can utilize their own parameter estimates (for their own pyrolysis process and market conditions) to calculate optimal temperatures.

Optimization problem and optimal temperature for fixed prices

The objective is to maximize the sum of the revenues from the two outputs minus the input costs, by choosing temperature:

$$\max_T V = P_c C(T, Z) + P_l L(T, Z) - K,$$  \hspace{1cm} (A.1)

Assuming that $C$ and $L$ are increasing at a decreasing rate in $T$, the temperature that maximizes $V$ satisfies the following condition:

$$\frac{P_L}{P_c} = \frac{C'(T)}{L'(T)},$$  \hspace{1cm} (A.2)

where $C'(T)$ and $L'(T)$ is the marginal productivity of $T$ for the production of $C$ and $L$, respectively. The left side of equation (A.2) represents the ratio of changes in $L$ and $C$ in response to changes in $T$, and the equation shows that this value is equal to the price ratio at the optimal $T$. Given the quadratic forms shown in equations (2a) and (2b) in the main text, the estimated marginal productivity measures in equations used in (A.2) and (A.6) are

$$C'(T_i) = \alpha_1 + 2\alpha_2 T_i,$$  \hspace{1cm} (A.3a)

$$L'(T_i) = \beta_1 + 2\beta_2 T_i.$$  \hspace{1cm} (A.3b)
The optimal temperature can be solved from these equations. Using equations A.2, A3a, and A.3b, the optimal temperature for fixed prices $P_c$ and $P_L$ is

$$T^*(P_c, P_L) = -\frac{\alpha_1 P_c + \beta_1 P_L}{2(\alpha_2 P_c + \beta_2 P_L)}.$$  

(A.4)

This temperature will maximize revenues from the sale of both bio-oil and biochar given the prices.

**Derivation of product transformation curves for fixed prices**

From equations (2a) and (2b) in the main text, note that for any observation $i$, $\alpha'_0 Z_i$ and $\beta'_0 Z_i$ are scalars (constants not dependent on temperature) and can be thought of as data dependent intercepts in the quadratic relationship between temperature and biochar or bio-oil, respectively. The symbols $\alpha_0$ and $\beta_0$ will be used below to represent $\alpha'_0 Z_i$ and $\beta'_0 Z_i$ respectively, or a subset of these elements. All variables in $Z$ are indicator variables in our regressions, each of which taking the value 1 in an observation if the category applies, and zero otherwise. Therefore, the parameters $\alpha_0$ and $\beta_0$ depend on what feedstock and pyrolysis type is of interest for the calculations. For example, if forest products and fast pyrolysis is of interest, then $\alpha_0$ equals the parameter associated with forest products, in the fast pyrolysis equation, which is $\alpha_0 = 80.67 \alpha_0 = 103.43$ (Table 2 in the main text).

The product transformation curve $C(L)$ can be derived by first solving for the inverse of $L(T)$; that is, solving equation (2b) in the main text for $T$ in terms of $L$. This is a quadratic in $T$, so there are two solutions based on the quadratic formula. However, only one of these solutions is consistent with profit maximization. Given the empirical results provided
below, the economically valid solution for temperature in terms of bio-oil quantity (that is, the inverse of \(L(T)\)) is

\[
T(L) = \frac{-b_1 + \sqrt{b_1^2 - 4b_2(b_2 - L)}}{2b_2}
\]

(A.5)

The right hand side of (A.7) is then substituted into equation (2a) in the main text, which provides the product transformation curve:

\[
C(L) = \alpha_0 + \alpha_1 T(L) + \alpha_2 T(L)^2 + \alpha' Z,
\]

(A.6)

where \(T(L)\) is given by equation (A.5).

Optimization and optimal temperature for temperature-dependent prices

Allowing for price to vary as a function of temperature, the maximization problem in equation (A.1) can be recast as

\[
\max_T V = P_C(T)C(T, Z) + P_L(T)L(T, Z) - K,
\]

(A.7)

and the optimality conditions analogous to equation (A.2) are

\[
\left( P'_C(T)C(T) + P_C(T)C'(T) \right) = -\left( P'_L(T)L(T) + P_L(T)L'(T) \right).
\]

(A.8)

Because prices are now a function of temperature, there is no constant price ratio to compare relative productivity to as in equation (A.2). However, the interpretation of this optimality condition is similar. The right side represents the revenue received (or lost) from biochar from a unit increase in temperature, and the left hand side represents the revenue lost (or received) from bio-oil from a unit increase in temperature. The marginal revenue gains from one of the products equals the marginal revenue losses from the other at the optimal temperature.
Substituting (A.3a), (A.3b) and $P_c'(T) = \delta_1$ and $P_L'(T) = \gamma_1$ (derived from equations (3a) and (3b) in the main text, with numbers given in equations (4) and (5)) into optimality condition (A.6) provides

$$
(\delta_1(\alpha_0 + \alpha_1 T + \alpha_2 T^2) + (\delta_0 + \delta_1 T)(\alpha_1 + \alpha_2 2T)) = -(\gamma_1(\beta_0 + \beta_1 T + \beta_2 T^2) + (\gamma_0 + \gamma_1 T)(\beta_1 + \beta_2 2T)).
$$

(A.9)

This can be rewritten as $AT^2 + BT + C = 0$, where

$$
A = 3(\delta_1 \alpha_2 + \gamma_1 \beta_2), B = 2(\delta_0 \alpha_2 + \delta_1 \alpha_1 + \gamma_0 \beta_2 + \gamma_1 \beta_1), C = (\delta_1 \alpha_0 + \delta_0 \alpha_1 + \gamma_1 \beta_0 + \gamma_0 \beta_1).
$$

Using A, B, and C above, the optimal temperature within the economic temperature range (based on the quadratic formula) is

$$
T^{**} = (2A)^{-1}(-B + \sqrt{B^2 - 4AC}),
$$

(A.10)

Guidance for practitioners: custom calculation of optimal temperatures

We begin by collecting and summarizing all of the information presented in this article that is required to calculate optimal temperatures. We then describe the substitutions necessary for generating and using customized parameters estimates and market prices to calculate optimal temperatures.

The numbers for each of these parameters that we used for our examples can be found in the text based on the regressions in tables 2 and 3, and price equations 4 and 3, but are collected in Table A1 below. These can be used in conjunction with equations (A.4) and (A.10) in this appendix to calculate the optimal temperatures. That is, if the numbers in Table A1 are substituted for the associated symbols, the optimal temperature will result.
Table A.1. Parameter values from text and regressions.

<table>
<thead>
<tr>
<th>Parameter symbol</th>
<th>Number used for calculations</th>
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<tbody>
<tr>
<td>$\alpha_1$</td>
<td>-0.2253</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>0.0015</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.1371</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>0.00012</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>0.00006771</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>0.0000658</td>
</tr>
</tbody>
</table>

These parameters in Table A.1 come from three sources: (1) regressions that relate production to temperature and other characteristics (Tables 2 and 3), (2) market prices, and (3) when quality affects price, the temperature, price relationship (in our case, equations 4 and 5). Each and all of these parameters can be customized for a given feedstock, pyrolysis process, and market conditions. To calculate a customized optimal temperature, analogous custom data are needed. First, data for output percentages from a range of temperatures, and regressions with a quadratic relationship between temperature and bio-oil and biochar output percentages. Second, if market prices are assumed to be fixed and not dependent on output quality (the simplest case), market prices for per kilogram of each output are needed. If there is a known price-quality relationship that can be measured as a price-temperature relationship, then custom estimates analogous to the linear relationships given in equations (3a) and (3b) (and (4) and (5)) are needed. With these data in hand, it is a matter of substituting the appropriate values in for the parameter symbols in Table A.1, and equations (A.4) or (A.10).
### Appendix B

#### Study Feedstock biomass

<table>
<thead>
<tr>
<th>A. Fast Pyrolysis</th>
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Appendix B (continued)

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<th>Study</th>
<th>Feedstock biomass</th>
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<tbody>
<tr>
<td><strong>A. Fast Pyrolysis (continued)</strong></td>
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<tr>
<td>Scott, Donald S., Piotr Majerski, Jan Piskorz, and Desmond Radlein.</td>
<td>Poplar sawdust, spruce sawdust, sugarcane bagasse, sorghum bagasse, wheat chaff, sunflower hulls, wheat straw, flax shives, newsprint, fine paper, pulp mill waste, peat moss</td>
</tr>
<tr>
<td>Straw, Sugarcane Bagasse and Coconut Shell in an Induction-Heating</td>
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<tr>
<td>Wang, Xiaouquan, Sascha R. A. Kersten, Wolter Prins, and Wim P. M.</td>
<td>Pine, beech, bamboo</td>
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<tr>
<td>2: Experimental Validation of Model Results.” *Industrial and</td>
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<tr>
<td>Engineering Chemistry Research*, 44(23):8,786-8,795.</td>
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<tr>
<td>Pyrolysis of Agricultural Residues at High Temperature.” *Biomass</td>
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<tr>
<td><strong>B. Slow Pyrolysis</strong></td>
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<tr>
<td>Sultan, and M.R. Alam. 2007. “Production of Bio-oil from Fixed Bed</td>
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<tr>
<td>Pyrolysis of Bagasse.” <em>Fuel</em>, 86:2,514-2,520.</td>
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<tr>
<td>Experimental Investigation of Biomass Waste (Rice Straw, Cotton</td>
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<td>Stalk, and Pine Sawdust) Pyrolysis Characteristics.” <em>Energy Sources</em></td>
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<td>, 25:331–337.</td>
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<tr>
<td>García-Perez, Manuel, Thomas T. Adams, John W. Goodrum, Daniel P.</td>
<td>Pine chips, pine pellets</td>
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</table>
### Appendix B (continued)

#### B. Slow Pyrolysis (continued)

<table>
<thead>
<tr>
<th>Study</th>
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