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Social Comparisons as a device for cooperation in simultaneous-move games

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Social comparisons as a device for cooperation in simultaneous-move games*

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Abstract

This paper analyzes the effects of players' relative comparisons in complete information simultaneous-move games. In particular, every individual is assumed to evaluate the kindness she infers from other players' choices by comparing these choices with respect to a given reference level. Specifically, this paper identifies under what conditions the introduction of relative comparisons leads players to be more cooperative than in standard game-theoretic models. I show that this result holds under certain conditions on the specific reference point that players use in their relative comparisons, and on whether players' relative comparisons leads them to regard each others' actions as more strategic complementary or substitutable. The model is then applied to different examples in public good games which enhance the intuition behind the results. Finally, I show that some existing models in the literature of intentions-based reciprocity and social status acquisition can be rationalized as special cases.

Keywords: Relative comparisons, Reference points, Simultaneous-move games, Kindness, Strategic complementarities.

JEL classification: C72, C78, C91

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1 Introduction

During the last decade several elements have been separately suggested to explain agents' behavior in experimental settings: from individuals' inequity aversion, as in Fehr and Schmidt (1999) and Bolton and Ockenfels (2000), to agents' preference for social status, as in Hopkins and Kornienko (2004) and Duffy and Kornienko (2005). Despite their ability to rationalize human conduct in specific economic environments, there is a substantial controversy about what particular facet most generally drives individuals' behavior in unrestricted environments. Or in other words, about the possibility to identify a *common element* connecting most of these experimental observations.

In this paper, I examine a model describing individual behavior that embeds many of these approaches as special cases of a broader explanation of human conduct in strategic settings. Specifically, this model is based on the common observation that people's choices are usually affected by the "kindness" they infer from the actions of the individuals they interact with, such as their neighbors, friends and relatives. Of course, the particular measure of "kindness" that each of us uses to evaluate other individuals' actions might be different. For instance, some people compare other agents' choices with respect to their own. Other individuals may instead evaluate other agents' actions with respect to some specific action they deem as "kind." Indeed many other examples abound; yet, they share a common pattern: in all of them individuals evaluate other agents' *choices* with respect to a particular reference action, which they use as a reference point for comparison.

Using this general definition of kindness, this paper examines the effects of social comparisons on strategic interaction. In particular, this study identifies under what conditions one can predict that individuals playing simultaneous-move games become more *cooperative* when they assign a positive importance to kindness, relative to when they do not. Particularly, this result holds under certain conditions on the reference point they use for comparison —which determines when a particular action by other agent is considered to be relatively kind or unkind— and on whether these considerations about kindness lead players to regard each others' actions as more strategically substitutable or complementary.

Specifically, I show that when players consider other players' choices as relatively kind and players' actions become more strategically complementary, both players increase their equilibrium strategies beyond the equilibrium level in standard models. Similarly, this result is also applicable to the case in which players consider other agents' strategies as relatively unkind but actions become more strategically substitutable. Finally, I demonstrate that these results are not only valid for games where players' actions are regarded as strategic complements, but also for those in which these actions are strategic substitutes. Hence, this paper identifies under what conditions players' relative comparisons (evaluating other players' kindness) act as a device for cooperation that triggers higher strategy choices by both players.

Therefore, this paper's main contributions can be divided into two. *First*, from a general perspective, this paper shows that, under certain conditions, agents' consideration of relative comparisons may lead them to become more cooperative than in standard models. Importantly, this result applies even when players are not concerned about other players' material payoffs. Indeed, unlike models with inequity averse individuals where players do care about other individuals' payoffs (social preferences), this paper analyzes conditions under which agents cooperate more than in standard models without the need to assume that they care about other players' payoffs, i.e., even when agents' preferences can be regarded as "strictly individualistic." *Second*, I show that the model this paper describes embeds as special cases existing behavioral models: from models on intentions-based reciprocity to those analyzing social status acquisition.

The paper is organized as follows. In the next section, I introduce the measure of kindness that players use and as how it enters into individuals' preferences. Sections three and four analyze players' equilibrium strategies when either both or only one of the parties assigns a positive weight to kindness in these simultaneous-move games. Then, in section five, I apply this model to different examples of public good games in which donors simultaneously contribute to a charity. Section six summarizes the main contributions of the paper.

2 Model

Let us consider complete information simultaneous-move games in which every player i chooses an action from her strategy space $S_i \in \left[\underline{s_i}, \overline{s_i}\right] \subset \mathbb{R}_+$. This strategy may represent, for example, player i's voluntary contribution to a public good, or in the context of oligopoly games, its production decision in a Cournot model. In particular, let us use $U_i^{NC} \equiv U_i^{NC}(s_i, s_j)$ to refer to player i's utility function when she is not concerned about relative comparisons. Since this utility function does coincide with those in the standard game-theoretic models, I alternatively refer to U_i^{NC} as player i's *material payoff*, where the superscript NC denotes the fact that player i is "not concerned" about relative comparisons. On the other hand, let $U_i^C(s_i, s_j)$ be player i's utility function when she is "concerned" about relative comparisons. In the following subsection, I describe how players make their comparisons, and in subsection 3.2 how every player introduces the result of this comparison into her utility function.

2.1 How players measure kindness

Let us now describe how players evaluate the kindness behind other players' actions. In particular, we assume that player i measures kindness through the following distance function, $D_i(s_i, s_j)$, and that he infers kindness when the outcome of this distance function is positive, and unkindness otherwise (see assumption 1 below).

$$D_i(s_i, s_j) = \alpha_i \left[s_j - s_j^{R_i}(s_i, s_j)\right]$$

for any $\alpha_i \in \mathbb{R}$. Thus, player i evaluates player j's kindness by comparing the difference between the action that player j's chooses in equilibrium, s_j, and a particular reference action that player i uses for comparison, $s_j^{R_i}(s_i, s_j) \in S_j$, among player j's available choices, as defined below.[1] I believe that this reference-dependent measure is a natural way for player i to assess player j's actions, which is yet general enough to embed different behavioral models as special cases. In particular, this distance function is similar to that in the literature on reference-dependent preferences, such as Köszegi and Rabin (2006). However, their model analyzes individual decision making, unlike this paper where we examine strategic effects. On the other hand, the distance function suggested in this paper differs from that in Rabin (1993) for simultaneous-move games and that in Dufwenberg and Kirchsteiger (2004) for sequential-move games. Indeed, these studies assume that player i compares his actual payoff with respect to the "equitable" payoff (his equitable share in the Pareto-efficient payoffs). In contrast, I allow player i to compare the action that player j's chooses in equilibrium with respect to *any* feasible action, $s_j^{R_i}(s_i, s_j) \in S_j$, leading to equitable or non-equitable payoffs. Let us next define the concept of reference action, $s_j^{R_i}(s_i, s_j)$, which player i uses as a reference point in order to evaluate the kindness that he perceives from player j's chosen action, s_j.

Definition 1. *Player i's reference point function $s_j^{R_i} : S_i \times S_j \to S_j$, maps the pair (s_i, s_j) of both players' chosen actions, into a reference action $s_j^{R_i} \in S_j$ from player j's set of available choices. In addition, $s_j^{R_i}(s_i, s_j)$ is weakly increasing in s_i and s_j, and twice continuously differentiable in s_i and s_j.*

Hence, player i can use any of player j's available actions in S_j as a reference point.[2] That is, $s_j^{R_i}(s_i, s_j)$ is allowed to be above/below/equal to player j's chosen action, s_j, which leads to negative/positive/null distances, respectively. Obviously, the particular sign of such distance affects player i's utility function, $U_i^C(s_i, s_j)$, as we describe below. Additionally, note that when both players' strategy spaces are identical, $S_i = S_j = S$, player i's reference point function becomes $s_j^{R_i} : S^2 \to S$. In this context, the reference point function can be, for instance, $s_j^{R_i}(s_i, s_j) = s_i$ for all s_j. In such case, the distance function becomes $D_i(s_i, s_j) = \alpha_i [s_j - s_i]$, and player i compares the action that player j chooses in equilibrium, s_j, with respect to her own action, s_i.

In particular, note two specific examples of this distance function. First, when $\alpha_i > 0$, it may represent the case in which players' equilibrium actions satisfy $s_j > s_i$, and player i interprets kindness from player j's choices (e.g., her commitment to contribute high donations to the public good), whereas $s_j < s_i$ is evaluated by player i as a sign of unkindness by her opponent (e.g.,

[1] For simplicity, this distance function was chosen to be linear. Nonetheless, from a more general perspective, player i's distance function could be nonlinear, as long as it *increases* in player j's actually chosen strategy, s_j, and *decreases* in the reference action that player i uses for comparison. Note that in such setting, Bolton and Ockenfels' (2000) model (whereby agents' utility increases in their share of total income) could be embedded as a special case. For the sake of clarity, however, I henceforth use the above linear distance function.

[2] For simplicity, I restrict the range of reference points to player j's *available* choices, S_j. More generally, $s_j^{R_i}(s_i, s_j)$ could take values outside S_j. I believe, however, that it is more natural to assume that player i compares player j's actions with respect to her foregone options than to actions which were not even available to her.

free-riding). The second example is related to players' concerns for status acquisition. Particularly, when $\alpha_i < 0$, player i makes the same comparison, but introduces the outcome of $D_i(s_i, s_j)$ into her utility function *negatively*, i.e. $D_i(s_i, s_j) = -\alpha_i [s_j - s_i] = \alpha_i [s_i - s_j]$ In these cases, player i may evaluate $s_j > s_i$ negatively because the action space might represent the consumption of a given positional good that enhances social status.

Furthermore, we allow player i to modify the reference action he uses to compare player j's chosen action, i.e., $s_j^{R_i}(s_i, s_j)$ is not restricted to be constant for all s_j. In particular, we assume that, for a given increase in player j's action, s_j, the reference point that player i uses, $s_j^{R_i}(s_i, s_j)$, does not increase as fast as player j's action, i.e., $1 \geq \partial s_j^{R_i}(s_i, s_j)/\partial s_j$. Intuitively, this condition makes higher values of player j's action meaningful for player i, since they increase the outcome of his distance function, i.e., $\partial D_i(s_i, s_j)/\partial s_j = 1 - \partial s_j^{R_i}(s_i, s_j)/\partial s_j$. And as we describe below, positive distances ultimately raise player i's utility level.

2.2 How kindness enters into players' preferences

After examining how players evaluate other players' actions through the construction of a distance D_i, let us next analyze how this distance enters into players' utility function. First, I consider how a player prefers, for a given pair of chosen actions s_i and s_j, those pairs (s_i, s_j) associated to positive rather than negative distances.

Assumption 1. *Kindness.* For any actions $s_i \in S_i$ and $s_j \in S_j$,

$$\begin{array}{rcll} U_i^C(s_i, s_j) & \geq & U_i^{NC}(s_i, s_j) & \text{for all } D_i(s_i, s_j) \geq 0 \\ U_i^C(s_i, s_j) & < & U_i^{NC}(s_i, s_j) & \text{for all } D_i(s_i, s_j) < 0 \end{array}$$

Therefore, this assumption determines that player i interprets kindness from player j's chosen actions when the outcome of her distance function is positive, and infers unkindness otherwise. That is, when player i is concerned about social comparisons and she interprets kindness from player j's actions, $D_i(s_i, s_j) \geq 0$, her utility level is higher than when she is not concerned about these comparisons; and it is lower when she infers unkindness. Let us finally define when a player's relative comparisons are considered as relatively "demanding" with respect to other players' actions, and when they can be regarded as "not-demanding".

Definition 2. *Player i's relative comparisons are defined as "demanding" if and only if she infers unkindness (negative distance) from player j's equilibrium action when players are not concerned about social comparisons, s_j^{NC}. That is, $D_i^{NC} \equiv \alpha_i \left[s_j^{NC} - s_j^R\right] < 0$. Otherwise, player i's relative comparisons are denoted as "not-demanding."*

Intuitively, player i would be regarded as "demanding," $D_i^{NC} < 0$, if the reference level she uses to compare player j's actions is above s_j^{NC}, i.e., she sets a high standard to assess player j's actions (demanding). On the contrary, player i would be regarded as "not-demanding," $D_i^{NC} > 0$, if the reference level she uses to compare player j's actions is below s_j^{NC}, setting a low standard to evaluate player j's choices.

3 Best response function

The previous section described the structure behind players' preferences, how they use the distance function to evaluate other players' actions, and how this distance enters into players' utility function. In this section, I characterize players' best response function in this class of simultaneous-move games.

Let $s_i^C(s_j) \in \arg\max_{s_i} U_i^C(s_i, s_j)$ denote player i's best response function when she assigns a positive importance to relative comparisons, and let $s_i^{NC}(s_j) \in \arg\max_{s_i} U_i^{NC}(s_i, s_j)$ represent her best response function when she does not assign any weight to such comparisons. For simplicity, both $U_i^{NC}(s_i, s_j)$ and $U_i^C(s_i, s_j)$ are assumed to be strictly concave in every player i's own strategy, s_i, which guarantees that best response functions are uniquely defined. Additionally, in order to have a unique equilibrium in pure strategies, we consider the usual sufficient condition for best response functions to intersect only once.

Assumption 2. For any given strategy pair (s_i, s_j), every player i's best response function satisfies $\left|\frac{\partial s_i^K(s_j)}{\partial s_i}\right| < 1$ where $K = \{C, NC\}$, i.e., $\left|\frac{\partial^2 U_i^K}{\partial s_i^2}\right| < \left|\frac{\partial^2 U_i^K}{\partial s_i \partial s_j}\right|$, for all $i \neq j$.

That is, for players with positive concerns about relative comparisons, $s_i^C(s_j)$ crosses $s_j^C(s_i)$ from below, and similarly for players without concerns about comparisons. Let us henceforth denote by single (double) subscripts in the utility and distance functions their first (and second) order derivatives. Next, I start by specifying some properties about the *level* of the best response function, whereas lemma 2 determines properties about its *slope*. Thereafter, all proofs can be found in the appendix.

Lemma 1. *Player i's best response function when she assigns a value to relative comparisons is above that when she does not, $s_i^C(s_j) \geq s_i^{NC}(s_j)$, for all s_j, if and only if the distance function that player i uses to evaluate kindness is increasing in her own strategy, s_i, for all s_i and s_j, i.e., $D_{s_i} \geq 0$ for all s_i and s_j.*

Therefore, lemma 1 determines a necessary and sufficient condition ($D_{s_i} \geq 0$) which guarantees that player i's best response function when she is concerned about relative comparisons is above that when she is not, $s_i^C(s_j) \geq s_i^{NC}(s_j)$, for any actions of player j. Graphically, lemma 1 can be interpreted as an upward shift in player i's best response function, as figure 1 illustrates below.

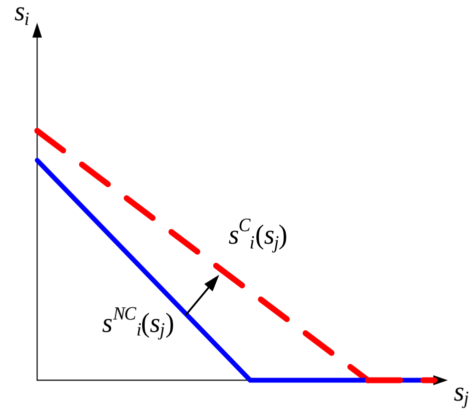


Figure 1

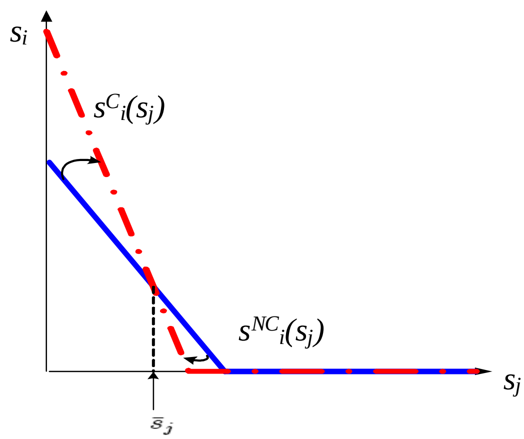
Intuitively, if an increase in player i's strategy raises the outcome of her distance function (i.e., if $D_{s_i} \geq 0$ for all s_j) then player i's best response function when she assigns a positive importance to relative comparisons is above that when she does not, i.e., $s_i^C(s_j) \geq s_i^{NC}(s_j)$ for all player j's strategies. Interestingly, the case that lemma 1 describes is applicable, for instance, to games where players are concerned about status acquisition. Specifically, note that the distance function players use as a measure of the status they acquire, $D_i(s_i, s_j) \equiv -\alpha_i(s_j - s_i) = \alpha_i(s_i - s_j)$, should clearly satisfy $D_{s_i} > 0$.

Finally, let $U^{NC}_{s_i s_j}$ represent the cross-derivative between player i and j's strategies when players does not assign a value to social comparisons, and $U^C_{s_i s_j}$ be that when they do. Intuitively, an increase in this cross-derivative when players become concerned about social comparisons, from $U^{NC}_{s_i s_j}$ to $U^C_{s_i s_j}$, implies that players' actions become more strategic substitutable. In contrast, a decrease in this cross-derivative means that players' actions become more complementary to each other.

Lemma 2. *If* $\Delta_i = \frac{U^C_{s_i s_j}}{U^C_{s_i s_i}} - \frac{U^{NC}_{s_i s_j}}{U^{NC}_{s_i s_i}} \geq 0$, *then the slope of player i's best response function increases when she assigns a value to social comparisons relative to when she does not; and decreases otherwise. That is,*

$$\textit{If } \Delta_i \geq (<)\, 0 \textit{ then } \frac{\partial s_i^C(s_j)}{\partial s_j} \geq (<) \frac{\partial s_i^{NC}(s_j)}{\partial s_j} \textit{ for all } s_j$$

Thus, lemma 2 specifies that, when player i's utility function satisfies condition $\Delta_i > 0$, her best response function experiences a *anticlockwise* rotation from $s_i^{NC}(s_j)$ to $s_i^C(s_j)$; whereas this rotation is *clockwise* in the case that $\Delta_i < 0$, as the figures below illustrate.



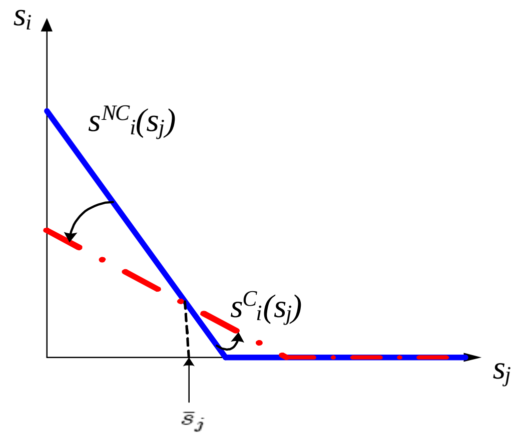


Figure 2(a). $\Delta_i < 0$, Clockwise rotation, "Compensating" type of player.

Figure 2(b). $\Delta_i > 0$, Anticlockwise rotation, "Reciprocating" type of player.

Graphically, when player i's best response function is negatively sloped, these results imply that $s_i^C(s_j)$ is *steeper* than $s_i^{NC}(s_j)$ when $\Delta_i < 0$, as figure 2(a) illustrates; while it determines the opposite when $\Delta_i > 0$ as figure 2(b) indicates. (In contrast, when player i's best response function is positively sloped, lemma 2 specifies that $s_i^C(s_j)$ is flatter than $s_i^{NC}(s_j)$ when $\Delta_i < 0$ is satisfied; and steeper otherwise.) In the above figures, note that $\bar{s}_j \in S_j$ represents the level of player j's strategy for which $s_i^C(s_j) = s_i^{NC}(s_j)$.[3]

Intuitively, a clockwise rotation can be understood in terms of a greater necessity to *compensate* player j's actions as figure 2(a) illustrates: when $s_j < \bar{s}_j$ player i chooses equilibrium levels of s_i above those in the game without concerns for relative comparisons, whereas when $s_j > \bar{s}_j$ player i chooses lower levels of s_i in equilibrium. This is the case of the public good games presented in the example of section five, where player i considers her contributions to the charity more "necessary" when player j does not reach a minimum level, $\bar{s}_j$, but her contributions are less necessary when player j exceeds this level.

An opposite argument is applicable to anticlockwise rotations of player i's best response functions (i.e., when $\Delta_i < 0$) where player i can be interpreted to *reciprocate* player j's actions. Indeed, player i reduces her strategy choice below that in standard models when player j does not reach threshold $\bar{s}_j$. In contrast, when $s_j > \bar{s}_j$ player i "rewards" player j for exceeding such level. Because of this underlying intuitive reasoning, I define the reciprocating and compensating types of players as follows.[4]

[3] Note that in the case of a clockwise rotation, if $\bar{s}_j$ takes a sufficiently *high* value, then $s_i^C(s_j) \geq s_i^{NC}(s_j)$ for all s_j, leading to a similar result to that of lemma 1, illustrated in figure 1. Similarly, in the case of an anticlockwise rotation, if $\bar{s}_j$ takes a sufficiently *low* value, then $s_i^C(s_j) \geq s_i^{NC}(s_j)$ for all s_j.

[4] These intuitions also hold when players' best response functions are positively sloped. Indeed, when $\Delta_i < 0$ one

Definition 3. *Player i's behavior is defined as "compensating" if and only if her best response function rotates clockwise (i.e., $\Delta_i < 0$ holds). Otherwise, her behavior is "reciprocating".*

4 Equilibrium analysis

From our previous analysis, one can anticipate that player i's equilibrium strategies in this model, s_i^C, are higher than in models without concerns about distances, s_i^{NC}, when $s_i^C(s_j) > s_i^{NC}(s_j)$ for all s_j, i.e., when $D_{s_i} \geq 0$ is satisfied as specified in lemma 1. Indeed, in such cases the consideration of distances shifts upwards player i's best response function along *all* player j's strategies, what leads player i to choose higher equilibrium strategy levels. The following proposition confirms this result.

Proposition 1. *If condition $D_{s_i} \geq 0$ holds for all $s_i \in S_i$ and $s_j \in S_j$, then $s_i^C \geq s_i^{NC}$, for any reciprocating or compensating behavior of players i and j.*

Hence, proposition 1 determines that player i's equilibrium strategy when she is concerned about relative comparisons is weakly higher that when she is not, if $D_{s_i} \geq 0$ holds. In that case, player i's Nash equilibrium strategy increases for any type of player (compensating or reciprocating), and for any distance function players might use (demanding or not-demanding). This is indeed a useful result, since it allows for a prediction about the ranking between equilibrium strategies s_i^C and s_i^{NC} just by checking whether condition $D_{s_i} \geq 0$ holds. As commented above, condition $D_{s_i} > 0$ is specially relevant in the case of those players who are concerned about status acquisition. Indeed, as the example of section five illustrates, $s_i^C \geq s_i^{NC}$ is satisfied for any parameter values when players assign a positive importance to status, confirming the above result of proposition 1.

One may ask, however, if the above result still holds when condition $D_{s_i} \geq 0$ is not satisfied for all s_j, i.e., when the best response function $s_i^C(s_j)$ is above $s_i^{NC}(s_j)$ for some values of s_j but below for others. Indeed, $D_{s_i} \geq 0$ is a relatively strong condition, which we henceforth relax. (In particular, we assume that $D_{s_i} \geq 0$ holds only for some values of s_i, whereas $D_{s_i} < 0$ is satisfied for others, which leads to best response function $s_i^C(s_j)$ to be above $s_i^{NC}(s_j)$ for some values of s_j but below for others). For expositional clarity, let us first analyze the case in which both players are concerned about relative comparisons. Then, section 4.2 examines the case where player i is the only individual who assigns a value to these comparisons.

4.1 Both players are concerned about comparisons

In this section I examine how the above ranking of equilibrium strategy choices varies when both players assign a positive importance to the outcome of their distance function. For simplicity, let

can interpret $s_i^C(s_j)$ being flatter than $s_i^{NC}(s_j)$ as that player i "compensates" player j's actions. On the contrary, when condition $\Delta_i > 0$ holds, and $s_i^C(s_j)$ becomes steeper than $s_i^{NC}(s_j)$, one can infer that player i "reciprocates" player j's strategy.

us assume that both players' relative comparisons are symmetric: $\Delta_i \times \Delta_j > 0$, i.e., both players are relative reciprocators or compensators, although the "intensity" of these effects does not need to coincide $\Delta_i \neq \Delta_j$.

Proposition 2. *Every player i's equilibrium strategy satisfies $s_i^C \geq s_i^{NC}$ if player i is either:*

1. *a compensator using a demanding distance function; or*
2. *a reciprocator using a not-demanding distance function.*

In addition, this result holds both for strategic substitutes and strategic complements.

The figures below illustrate the results behind proposition 2 analyzing the ranking of players' equilibrium strategies. In particular, the type of player is represented in rows and the kind of distance function she uses is in columns. Specifically, figure 3(a) describes the results for negatively sloped best response functions (strategic substitutes), while 3(b) summarizes proposition 2 for the case that players' best response functions have a positive slope (strategic complements).

	Not demanding $D^{NC}>0$	*Demanding* $D^{NC}<0$
Compensators $\Delta<0$	$s_i^C<s_i^{NC}$ $s_j^C<s_j^{NC}$	$s_i^C>s_i^{NC}$ $s_j^C>s_j^{NC}$
Reciprocators $\Delta>0$	$s_i^C>s_i^{NC}$ $s_j^C>s_j^{NC}$	$s_i^C<s_i^{NC}$ $s_j^C<s_j^{NC}$

Figure 3(a). Strategic substitutes.

	Not demanding $D^{NC}>0$	*Demanding* $D^{NC}<0$
Compensators $\Delta<0$	$s_i^C<s_i^{NC}$ $s_j^C<s_j^{NC}$	$s_i^C>s_i^{NC}$ $s_j^C>s_j^{NC}$
Reciprocators $\Delta>0$	$s_i^C>s_i^{NC}$ $s_j^C>s_j^{NC}$	$s_i^C<s_i^{NC}$ $s_j^C<s_j^{NC}$

Figure 3(b). Strategic complements.

Interestingly, for the case of strategic complements but also for strategic substitutes, $s_i^C > s_i^{NC}$ and $s_j^C > s_j^{NC}$ are satisfied either when: (1) players are compensators with relatively demanding distance functions; or (2) when players are reciprocators with not-demanding distance functions. Intuitively, in the first case player i evaluates player j's actions as relatively low given that she uses a demanding distance function. Additionally, since she is a compensating type of player, she increases her equilibrium strategy. In contrast, in the second case, player i evaluates player j's actions as relatively high, given that she uses a not-demanding distance function. Since, in addition, she is a reciprocating type of player, she raises her strategy in equilibrium.

Note an interesting implication of these results. In particular, if players compare each others' actions with respect to the *highest* choice available to each other (i.e., both players are extremely "demanding"), then further cooperation among the players can only be predicted when individuals are regarded as compensators, e.g., they compensate each others' lack of contributions to the public good. In contrast, if players compare each others' actions with respect to the *lowest* available choice of the other player (and players can then be regarded as "not-demanding"), stronger cooperation occurs only when players are reciprocators.

4.2 Only player i is concerned about comparisons

Let us now analyze the case in which player i is the only individual concerned about the outcome of her distance function, i.e. $\Delta_i \neq 0$ and $\Delta_j = 0$.

Proposition 3. *Consider that $\Delta_i \neq 0$ and $\Delta_j = 0$ for all $j \neq i$, then*

1. *Player i's equilibrium strategy satisfies $s_i^C \geq s_i^{NC}$ if and only if he is either: (1) a compensator using a demanding distance function; or (2) a reciprocator using a not-demanding distance function. This result holds both for strategic substitutes and complements.*

2. *Player j's equilibrium strategy satisfies $s_j^C \geq s_j^{NC}$ if and only if $s_i^C < s_i^{NC}$ in the case of strategic substitutes, and if $s_i^C > s_i^{NC}$ in the case of strategic complements.*

$\Delta_j = 0$	*Not demanding* $D^{NC}_i > 0$	*Demanding* $D^{NC}_i < 0$
Compensator $\Delta_i < 0$	$s_i^C < s_i^{NC}$ $s_j^C > s_j^{NC}$	$s_i^C > s_i^{NC}$ $s_j^C < s_j^{NC}$
Reciprocator $\Delta_i > 0$	$s_i^C > s_i^{NC}$ $s_j^C < s_j^{NC}$	$s_i^C < s_i^{NC}$ $s_j^C > s_j^{NC}$

Figure 4(a). Strategic substitutes.

$\Delta_j = 0$	*Not demanding* $D^{NC}_i > 0$	*Demanding* $D^{NC}_i < 0$
Compensator $\Delta_i < 0$	$s_i^C < s_i^{NC}$ $s_j^C < s_j^{NC}$	$s_i^C > s_i^{NC}$ $s_j^C > s_j^{NC}$
Reciprocator $\Delta_i > 0$	$s_i^C > s_i^{NC}$ $s_j^C > s_j^{NC}$	$s_i^C < s_i^{NC}$ $s_j^C < s_j^{NC}$

Figure 4(b). Strategic complements.

The above two figures describe the results of proposition 3, emphasizing the ranking of player i and j's equilibrium strategies when only player i is concerned about relative comparisons. In particular, note that the ranking of equilibrium strategy choices for the concerned individual (player i) coincides with that when both players assign a positive value to relative comparisons. That is,

$s_i^C \geq s_i^{NC}$ holds in the same contexts regarding player i for figures 3(a) and 4(a) in the case of strategic substitutes, and for figures 3(b) and 4(b) in the case of strategic complements.

On the other hand, player j's equilibrium strategy moves in the *opposite* direction of player i's when actions are strategic substitutes, whereas it moves in the *same* direction when they are strategic complements. Intuitively, when players' actions are strategic substitutes, player j decreases her equilibrium strategy when she knows that player i increases hers, as figure 4(a) indicates. In contrast, when players' actions work as strategic complements (as in figure 4b), player j raises her strategy choice when she predicts that player i increases hers in equilibrium.[5]

We can extract two main conclusions from the above results. First, a single individual with positive concerns about social comparisons suffices for higher strategy choices in equilibrium $s_i^C \geq s_i^{NC}$ (at least for that player) under certain contexts; and it is valid for both players if their actions are strategic complements. Second, when both individuals assign a positive importance to social comparisons, players' equilibrium strategies move in the same direction, i.e., they experience a "coordinating effect." Importantly, this result is not only valid when players' actions are strategic complements, but also when they are strategic substitutes.

4.3 Connection with the literature

In this section, I analyze how the model presented in this paper encompasses certain models on social preferences and intentions-based reciprocity as special cases, as the following proposition shows.

Proposition 4. *Assume $s_j^{R_i}(s_i, s_j) = s_i$ for all s_j. Then, the player i's preferences over player j's actions can be represented by*

$$U_i^C(s_i, s_j) = \gamma_i U_i^{NC}(s_i, s_j) + \gamma_j U_j^{NC}(s_i, s_j) \quad \text{where } \gamma_i, \gamma_j \in \mathbb{R}$$

In particular, the above proposition specifies that when player i compares player j's chosen action, s_j, with that chosen by her, s_i, her utility function $U_i^C(s_i, s_j)$ can be represented as a weighted average of her material payoffs and those of player j. Therefore, in such context our model captures players' concerns for inequity aversion (or altruism) as a special case, such as in Fehr and Schmidt (1999) and in Bolton and Ockenfels (2000). In addition, this model also captures certain concerns about intentions-based reciprocity as a special case. For example, the

[5] Finally, note that these results can be easily generalized to simultaneous-move games with N players. In such settings, however, every player measures the kindness he infers from the actually chosen strategies of each of the other $N-1$ players. The outcome of each of these individual comparisons can then be added up (or even scaled in a weighted average), in order to evaluate player i's distance function. Despite the greater generality of such model, nonetheless, its results and intuition are already captured by the two-player setting I consider in this paper.

above utility representation embodies Charness and Rabin's (2002) model[6] for the case that player i infers misbehavior from player j's actions, and for $\gamma_i = 1-\theta$ and $\gamma_j = -\theta$. That is,

$$\begin{aligned} U_i^C(s_i, s_j) &= (1-\theta)\, U_i^{NC}(s_i, s_j) - \theta U_j^{NC}(s_j, s_i) \\ &= U_i^{NC}(s_i, s_j) + \theta\left[U_i^{NC}(s_i, s_j) - U_j^{NC}(s_j, s_i)\right] \end{aligned}$$

Finally, note that the model presented in this paper also encompasses contexts in which players care about social status. Indeed, as commented in section 3, this occurs when players compare others' actions with respect to her own and they introduce the outcome of this comparison negatively into her utility function. In particular, the distance function becomes $D_i(s_i, s_j) \equiv -\alpha_i(s_j - s_i) = \alpha_i(s_i - s_j)$, where player i's utility increases when $s_i > s_j$ and decreases otherwise.

5 Application to public good games

In this section, I construct a simple example in which the above general model is applied to a public good game (PGG). Specifically, let us first assume that player i's utility function coincides with those in standard public good games,

$$U_i^{NC}(s_i, s_j) = [w - s_i]^{0.5} + [m(s_i + s_j)]^{0.5}$$

where w represents the amount of money available for contributions to the public good, $s_i \in \mathbb{R}_+$. Hence, $w - s_i$ denotes the remaining units of money which have not been contributed and that can be used for consumption of private goods. Finally, let $m \in \mathbb{R}_+$ be the (constant) return from the total contributions to the public good, $s_i + s_j$. Let us now introduce players' concerns about relative comparisons. In order to be consistent with the above model, let us first construct an example of a distance function that increases in player i's strategy, i.e., $D_{s_i} > 0$ for all s_j, as in the case in which players care about status acquisition. Second, I analyze an example of a distance function that is not increasing for all player i's strategy, i.e., $D_{s_i} > 0$ does not hold for all s_j.

5.1 An example about status acquisition

Let us first consider that players increase their perception of social status when their contribution to the public good is above that of the other donor, i.e., when $s_i > s_j$. For simplicity, let us construct a linear distance function $D_i \equiv -\alpha_i(s_j - s_i) = \alpha_i(s_i - s_j)$, where player i compares her equilibrium contribution , s_i, with that of player j's , s_j. Therefore, player i's utility function becomes

$$U_i^C(s_i, s_j) = [w - s_i]^{0.5} + [m(s_i + s_j) + \alpha(s_i - s_j)]^{0.5}$$

[6] Clearly, this representation of player i's utility function does not capture Charness and Rabin's (2002) complete model, since they analyze other facets of individuals' behavior, such as inequity aversion, in addition to reciprocity. However, when restricted to intensions-based reciprocity alone, and when player i infers misbehavior from player j's actions, the above utility function coincides with that in Charness and Rabin (2002).

where $\alpha_i = \alpha_j = \alpha$ for simplicity. The next proposition describes player i's equilibrium contribution in this context, and below I compare it with respect to hers in the standard PGG.

Proposition 5. *In the simultaneous PGG game where players assign a value to status, every player $i = \{1, 2\}$ submits a Nash equilibrium contribution of $s_i^C = \frac{(\alpha+m)^2 w}{2m+(\alpha+m)^2}$.*

Specifically, the following corollary shows that, indeed, player i's equilibrium contribution in this model is strictly higher than when she is not concerned about status acquisition (and generally about distances such that $D_{s_i} > 0$ for all s_j).

Corollary 1. *Every player i's equilibrium contribution in the simultaneous PGG game, s_i^C, when all players assign value to status, $\alpha > 0$, is (strictly) higher than her contribution when they do not, $\alpha = 0$.*

Interestingly, this result could be anticipated by directly using proposition 1. Indeed, since player i can increase the outcome of the distance function by increasing her own strategy (i.e., $D_{s_i} > 0$ for all s_j as in this case) then the ranking result $s_i^C > s_i^{NC}$ could be predicted without the need to find reduced form solutions for the players' equilibrium contributions.

5.2 An example where comparisons are defined over s_j

Let us now construct a similar example in order to gain a clearer intuition about proposition 2's results. Particularly, let us assume that player i makes relative comparisons with a distance function that is *not* increasing in player i's own strategy choice, i.e., $D_{s_i} > 0$ does not hold for all s_j. For example, if player i wants to evaluate player j's commitment with the provision of the public good, she might use distance function $D_i \equiv \alpha_i(s_j - s_j^{ref})$, where s_j represents player j's equilibrium contribution, and $s_j^{ref} \in (0, 1)$ denotes a particular contribution to the public good that players may have agreed upon before the beginning of the game, and that player i uses as a reference point to compare s_j. Thus, player i's utility function in this model becomes,

$$U_i^C(s_i, s_j) = [w - s_i]^{0.5} + [m(s_i + s_j) + \alpha(s_j - s_j^{ref})]^{0.5}$$

Specifically, note that player i's utility level increases when player j contributes to the public good above her reference level $s_j > s_j^{ref}$ (for example, more than what she committed to), since player i might infer that player j's chosen strategy is a signal of a strong commitment with the provision of the public good. Let us next analyze player i's best response function.

Proposition 6. *In the simultaneous PGG game, where every player $i = \{1, 2\}$ assigns a value to the distance $s_j - s_j^{ref}$, player i's best response function, $s_i^C(s_j)$, is given by*

$$s_i^C(s_j) = \begin{cases} \frac{\alpha s_j^{ref} + m^2 w}{m(1+m)} - \frac{\alpha+m}{m(1+m)} s_j \text{ if } s_j \in \left[0, \frac{\alpha s_j^{ref} + m^2 w}{\alpha + m(2+m)}\right] \\ 0 \text{ if } s_j > \frac{\alpha s_j^{ref} + m^2 w}{\alpha + m(2+m)} \end{cases}$$

Comparing it with player i's best response function when she assigns no importance to distances,

$$s_i^{NC}(s_j) = \begin{cases} \frac{mw}{1+m} - \frac{1}{1+m}s_j \text{ if } s_j \in \left[0, \frac{mw}{2+m}\right] \\ 0 \text{ if } s_j > \frac{mw}{2+m} \end{cases}$$

one can clearly observe two main differences between these best response functions, from which we can conclude that player i is a "compensator". First, the vertical intercept of $s_i^C(s_j)$ is higher than that of $s_i^{NC}(s_j)$ for any $\alpha > 0$ and $s_j^{ref} > 0$, i.e., $\frac{\alpha s_j^{ref}+m^2w}{m(1+m)} > \frac{mw}{1+m}$. And second, $s_i^C(s_j)$ is steeper than $s_i^{NC}(s_j)$, i.e., $\frac{\alpha+m}{m(1+m)} > \frac{1}{1+m}$. Therefore, player i's best response function experiences a clockwise rotation from $s_i^{NC}(s_j)$ to $s_i^C(s_j)$ similar to that figure 2(a) illustrates. In contrast, when $\alpha < 0$ player i becomes a "reciprocator." Indeed, the vertical intercept of $s_i^C(s_j)$ is now lower than that of $s_i^{NC}(s_j)$ for any $\alpha < 0$; in addition, $s_i^C(s_j)$ is now flatter than $s_i^{NC}(s_j)$ since $\frac{m-\alpha}{m(1+m)} < \frac{1}{1+m}$. Hence, when $\alpha < 0$ player i's best response function experiences an anticlockwise rotation from $s_i^{NC}(s_j)$ to $s_i^C(s_j)$ similar to that illustrated in figure 2(b). Given the above results about player i's best response function, let us now determine player i's equilibrium contribution to the public good for any value of α.

Proposition 7. *In the simultaneous PGG game, every player i's contribution when both players assign a value to the distance $s_j - s_j^{ref}$ is given by $s_i^C = \frac{\alpha s_j^{ref}+m^2w}{\alpha+m(2+m)}$.*

Let us finally compare, alike in the previous example, every player i's donation in this model with respect to hers in the (standard) case when she assigns no value to distances.

Corollary 2. *In the simultaneous PGG game, every player i's Nash equilibrium contribution when she assigns a value to the distance $s_j - s_j^{ref}$, s_i^C, is strictly higher than hers when she assigns no weight to such distance, s_i^{NC}, if*

1. *players are "compensators" using a demanding distance function, i.e., conditions $\alpha > 0$ and $s_j^{NC} < s_j^{ref}$ hold; or*
2. *players are "reciprocators" using a not-demanding distance function, i.e., conditions $\alpha < 0$ and $s_j^{NC} > s_j^{ref}$ hold.*

This result confirms proposition 2 in the general description of the model. Indeed, it specifies an alternative procedure to check whether $s_i^C > s_i^{NC}$ without the need to find reduced form solutions for player i's equilibrium contribution level. In particular, one just needs to check the conditions it describes: when players can be regarded as "compensators," $s_i^C > s_i^{NC}$ holds if these players use demanding distance functions, $s_j^{NC} < s_j^{ref}$. Otherwise, when players are regarded as "reciprocators," $s_i^C > s_i^{NC}$ is satisfied only if players use not-demanding distance functions, $s_j^{NC} > s_j^{ref}$; as proposition 2 showed.

6 Conclusions

This paper analyzes the effect of players' relative comparisons on their equilibrium strategies in simultaneous-move games. In particular, I show that when players relative comparisons lead them to regard each others' actions as more strategically complementary (players are regarded as "reciprocators"), and when they are not-demanding on the actions that they expect from each other, predicted levels of cooperation among the players are higher when they care about these comparisons than when they do not. Similarly, when players' considerations for relative comparisons lead their actions to become more strategically substitutable (players are regarded as "compensators"), and they demand high actions from each other, players' cooperation is stronger than when they do not. Interestingly, these results are not only valid for games where players' actions are regarded as strategic complements, but also for those in which they are strategic substitutes. Therefore, this paper shows the role of social comparisons as devices of cooperation in a relatively general class of simultaneous-move games. Specifically, these results explain why individuals choose to cooperate even when they do not assign any value to each others' payoffs; a common assumption in the literature predicting cooperation, which this paper does not consider.

Furthermore, I demonstrate that the results of this paper embed some existing behavioral models: from intentions-based reciprocity and status acquisition. Hence, this paper furthers our understanding of the facets explaining players' observed cooperation in multiple experiments. Let us finally remark some of the several extensions to the model introduced in this paper. Particularly, note that the action space was exogenously determined before the beginning of the game. However, it would be interesting to allow players to strategically select their available choices (their action space) before the game starts, given that the kindness other players perceive from their own choices depends on which actions are *not* chosen. This strategic selection of available choices is observed in different contexts, where a player uses one of her unchosen alternatives as an excuse to support her actual behavior. Further research in the effect of relative comparisons in individuals' strategic interaction will indeed improve our understanding of economic behavior in a greater variety of settings.

7 Appendix

7.1 *Proof of Lemma 1*

I first show that player i's best response functions when she is concerned about distance $D_i(\cdot)$ and when she is not, respectively, $s_i^C(s_j) \in \arg\max_{s_i} U_i^C(s_i, s_j)$, and $s_i^{NC}(s_j) \in \arg\max_{s_i} U_i^{NC}(s_i, s_j)$, contain a single point. Then, I show the result stated in lemma 1.

Note that player i's utility function when she is concerned about distance $D_i(\cdot)$, $U_i^C(s_i, s_j)$, is strictly concave in s_i and it is defined over a strictly convex domain $S_i \times S_j$. This guarantees that player i's best response function

$$s_i^C(s_j) \in \arg\max_{s_i} U_i^C(s_i, s_j)$$

contains a single point. A similar argument is also applicable for player i's utility function when she does not assign any relevance to distance $D_i(\cdot)$, $U_i^{NC}(s_i, s_j)$, since it is also strictly concave in s_i and it is defined over a strictly convex domain $S_i \times S_j$. Hence,

$$s_i^{NC}(s_j) \in \arg\max_{s_i} U_i^{NC}(s_i, s_j)$$

also contains a single point.

Next, I want to show that $U_{s_i}^C(s_i, s_j) \geq U_{s_i}^{NC}(s_i, s_j)$ holds if and only if $s_i^C(s_j) \geq s_i^{NC}(s_j)$ for all s_j. First, suppose by contradiction, that $U_{s_i}^C(s_i, s_j) \geq U_{s_i}^{NC}(s_i, s_j)$ but $s_i^C(s_j) < s_i^{NC}(s_j)$ for all s_i and s_j. Let us then take a linear combination $\hat{s}_i(s_j)$ of these two best response functions, $s_i^C(s_j)$ and $s_i^{NC}(s_j)$, such that

$$\hat{s}_i(s_j) = \theta s_i^C(s_j) + (1-\theta)\, s_i^{NC}(s_j) \text{ for all } s_j, \text{ where } \theta \in (0,1)$$

When $U_{s_i}^{NC}(s_i, s_j)$ is evaluated at $\hat{s}_i(s_j)$, we must have $U_{s_i}^{NC}(\hat{s}_i(s_j), s_j) > 0$. However, if $s_i^C(s_j) < s_i^{NC}(s_j)$, then $U_{s_i}^C(\hat{s}_i(s_j), s_j) < 0$. Therefore,

$$U_{s_i}^C(\hat{s}_i(s_j), s_j) < U_{s_i}^{NC}(\hat{s}_i, s_j)\text{, which is a contradiction.}$$

Hence, if $U_{s_i}^C(s_i, s_j) \geq U_{s_i}^{NC}(s_i, s_j)$ for all s_i and s_j then $s_i^C(s_j) \geq s_i^{NC}(s_j)$ for all s_i and s_j. Let us next show that if $s_i^C(s_j) \geq s_i^{NC}(s_j)$ for all s_i and s_j, then $U_{s_i}^C(s_i, s_j) \geq U_{s_i}^{NC}(s_i, s_j)$ for all s_i and s_j. Suppose by contradiction that $s_i^C(s_j) \geq s_i^{NC}(s_j)$ for all s_j, but $U_{s_i}^C(s_i, s_j) < U_{s_i}^{NC}(s_i, s_j)$ for some s_i and s_j. Then, $s_i^C(s_j) < s_i^{NC}(s_j)$ would hold for some s_i and s_j, which is a contradiction. Thus, $U_{s_i}^C(s_i, s_j) \geq U_{s_i}^{NC}(s_i, s_j)$ holds if and only if $s_i^C(s_j) \geq s_i^{NC}(s_j)$ for all s_j.

Applying this condition to player i's utility function, we have $U_{s_i}^{NC}(s_i, s_j) = U_{s_i}$ and $U_{s_i}^C(s_i, s_j) = U_{s_i}^C\left(U_i^{NC}, D_i\right)$. Hence, $U_{s_i}^C(s_i, s_j) = U_{s_i} + U_{D_i} D_{s_i}$. Thus, $U_{s_i}^C(s_i, s_j) \geq U_{s_i}^{NC}(s_i, s_j)$ in this context means $U_{s_i} + U_{D_i} D_{s_i} \geq U_{s_i}$, which reduces to $U_{D_i} D_{s_i} \geq 0$. Finally, since $U_{D_i} \geq 0$ given that positive distances increase players' utility level (kindness assumption), condition $U_{D_i} D_{s_i} \geq 0$ can be reduced to $D_{s_i} \geq 0$. Hence, $s_i^C(s_j) \geq s_i^{NC}(s_j)$ is satisfied for all s_j if and only if condition $D_{s_i} \geq 0$ holds for all s_i and s_j. ■

7.2 *Proof of Lemma 2*

Let us first find the slope of player i's best response function in the standard game without concerns about distances. Applying the implicit function theorem, we have

$$\frac{\partial s_i^{NC}(s_j)}{\partial s_j} = -\frac{U_{s_i s_j}^{NC}}{U_{s_i s_i}^{NC}}$$

Let us now compare it with the slope of player i's best response function when player i is concerned about distances. Applying the implicit function theorem again,

$$\frac{\partial s_i^C(s_j)}{\partial s_j} = -\frac{U^C_{s_i s_j}}{U^C_{s_i s_i}}$$

Comparing the absolute value of both slopes, $\frac{\partial s_i^C(s_j)}{\partial s_j} > \frac{\partial s_i^{NC}(s_j)}{\partial s_j}$ holds if and only if $\Delta_i = \frac{U^C_{s_i s_j}}{U^C_{s_i s_i}} - \frac{U^{NC}_{s_i s_j}}{U^{NC}_{s_i s_i}} > 0$. ■

7.3 *Proof of Proposition 1*

From Lemma 1 we know that if $D_{s_i} \geq 0$ holds for all s_j, then $s_i^C(s_j) > s_i^{NC}(s_j)$ for all s_j.

Now we want to show that if $s_i^C(s_j) \geq s_i^{NC}(s_j)$ for all s_j, then $s_i^C \geq s_i^{NC}$. Suppose by contradiction that $s_i^C(s_j) \geq s_i^{NC}(s_j)$ for all s_j, but $s_i^C < s_i^{NC}$. Since this counterpositive statement must be true for any slopes of player i and j's best response functions, it must also be true when $s_i^C(s_j)$ and $s_i^{NC}(s_j)$ are both negatively sloped, and when player j is not concerned about distances, i.e., $s_j^C(s_j) = s_j^{NC}(s_j)$. In this case, if $s_i^C < s_i^{NC}$ then, either

1. $s_i^C(s_j) < s_i^{NC}(s_j)$ for all s_j, and $\left|\frac{\partial s_i^K(s_j)}{\partial s_j}\right| < 1$ for all $i \neq j$ and $K = \{C, NC\}$, or

2. $s_i^C(s_j) \geq s_i^{NC}(s_j)$ for all s_j, and $\left|\frac{\partial s_i^K(s_j)}{\partial s_j}\right| > 1$ for all $i \neq j$ and $K = \{C, NC\}$,

which are both a contradiction. Thus, if $s_i^C(s_j) \geq s_i^{NC}(s_j)$ for all s_j, then $s_i^C \geq s_i^{NC}$. ■

7.4 *Proof of Proposition 2*

Let us first find an useful result about player i's best response functions when evaluated at $s_j = s_j^{NC}$.

Lemma A. *If assumptions 1 and 2 are satisfied, then for every player* $i = \{1, 2\}$,

$$\left[\Delta_i \times D_i^{NC}\right] \times \left[s_i^C(s_j^{NC}) - s_i^{NC}(s_j^{NC})\right] > 0$$

Proof of Lemma A:

We want to show that if $\Delta_i \times D_i^{NC} > 0$ then $s_i^C(s_j^{NC}) > s_i^{NC}(s_j^{NC})$. Notice that:

1. If $\Delta_i < 0$ and $D_i^{NC} < 0$, then $s_i^C(s_j)$ rotates clockwise and $s_j^{NC} < \bar{s}_j$. Then, $s_i^C(s_j) > s_i^{NC}(s_j)$ for all $s_j < \bar{s}_j$, including s_j^{NC} (see figure 1a).

2. If $\Delta_i > 0$ and $D_i^{NC} > 0$, then $s_i^C(s_j)$ rotates anticlockwise and $s_j^{NC} > \bar{s}_j$, as in figure 1b Then, $s_i^C(s_j) > s_i^{NC}(s_j)$ for all $s_j > \bar{s}_j$, including s_j^{NC}.

Therefore, $\left[\Delta_i \times D_i^{NC}\right] \times \left[s_i^C(s_j^{NC}) - s_i^{NC}(s_j^{NC})\right] > 0$. ■

Thus, lemma A specifies a ranking for player i's best response functions when evaluated at $s_j = s_j^{NC}$. In particular, it determines that $s_i^C(s_j^{NC}) > s_i^{NC}(s_j^{NC})$ if either: (1) player i is a compensator using a relatively demanding distance function, $\Delta_i < 0$ and $D_i^{NC} < 0$; or if (2) player i is a reciprocator using a not-demanding distance function, $\Delta_i > 0$ and $D_i^{NC} > 0$. With this result, we can now prove Proposition 2.

First result

From the above Lemma A we know that

$$\Delta_i \times D_i^{NC} > 0 \Longrightarrow s_i^C(s_j^{NC}) > s_i^{NC}(s_j^{NC}) = s_i^{NC}$$

Let us now show that, for a given player j's best response function, $s_j^C(s_i) = s_j^{NC}(s_i)$,

$$s_i^C(s_j^{NC}) > s_i^{NC} \Longrightarrow s_i^C > s_i^{NC}$$

In order to show the above result, assume by contradiction that for a given $s_j^{NC}(s_i)$, $s_i^C(s_j^{NC}) > s_i^{NC}$ implies $s_i^C < s_i^{NC}$. First, take two negatively sloped best response functions, and assume $s_i^C < s_i^{NC}$. Then, when evaluated at $s_j = s_j^{NC}$, player i's best response function must satisfy $s_i^C(s_j^{NC}) < s_i^{NC}(s_j^{NC}) = s_i^{NC}$, which is a contradiction. Thus, if $s_i^C(s_j^{NC}) > s_i^{NC}$ then $s_i^C > s_i^{NC}$. Therefore, for a given $s_j^{NC}(s_i)$, and using Lemma A we have that

$$\Delta_i \times D_i^{NC} > 0 \Longrightarrow s_i^C(s_j^{NC}) > s_i^{NC} \Longrightarrow s_i^C > s_i^{NC}$$

Second result

From Lemma A we know that

$$\Delta_j \times D_j^{NC} < 0 \Longrightarrow s_j^C(s_i^{NC}) < s_j^{NC}(s_i^{NC})$$

Then, we now want to show that if $s_j^C(s_i^{NC}) < s_j^{NC}(s_i^{NC})$ is satisfied, then

$$s_i^C > s_i^{NC} \text{ when } s_i^{NC}(s_j) \text{ is negatively sloped}$$

and $s_i^C < s_i^{NC}$ otherwise. Then, assume that $s_j^C(s_i^{NC}) < s_j^{NC}(s_i^{NC})$ and that $s_j^C(s_i)$ is negatively sloped. Therefore, $s_j^C(s_i) < s_j^{NC}(s_i)$ for all $s_i > \bar{s}_i$, including s_i^{NC}. Since, in addition, $s_j^C(s_i)$ must cross $s_i^{NC}(s_j)$ from below by assumption 3, then $s_i^C > s_i^{NC}$. Similarly, when $\Delta_j \times D_j^{NC} > 0$ holds and players' best response functions have positive slope, $\frac{U^C_{s_i s_j}}{U^C_{s_i s_i}} < 0$, then we have an analog reasoning,

$$\Delta_j \times D_j^{NC} > 0 \Longrightarrow s_j^C(s_i^{NC}) > s_j^{NC}(s_i^{NC}) \text{ from Lemma A}$$

Therefore, $s_j^C(s_i^{NC}) > s_j^{NC}(s_i^{NC})$ for all $s_i < \bar{s}_i$, including s_i^{NC}. Finally, assumption 2 for the context of positively sloped best response functions implies that $s_j^C(s_i)$ must cross $s_i^{NC}(s_j)$ from above. Hence, $s_i^C > s_i^{NC}$. ■

7.5 *Proof of Proposition 3*

From Lemma A above we know that

$$\Delta \times D^{NC} > 0 \Longrightarrow s_i^C(s_j^{NC}) > s_i^{NC}(s_j^{NC}) \text{ for all } i \neq j$$

Now we want to show that,

$$s_i^C(s_j^{NC}) > s_i^{NC} \text{ for all } i \neq j \Longrightarrow s_i^C > s_i^{NC} \text{ for all } i = \{1, 2\}$$

In order to show the above claim, assume by contradiction that $s_i^C(s_j^{NC}) > s_i^{NC}(s_j^{NC})$ holds for all $i \neq j$ but $s_i^C < s_i^{NC}$ for at least some i. For simplicity, let us take two best response functions with negative slopes, and consider that $s_i^C < s_i^{NC}$. Then, when evaluated at $s_j = s_j^{NC}$, $s_i^C(s_j^{NC})$ must be below $s_i^{NC}(s_j^{NC})$. Applying the same reasoning to player j, we conclude that

$$s_i^C(s_j^{NC}) < s_i^{NC}(s_j^{NC}) \text{ and } s_j^C(s_i^{NC}) < s_j^{NC}(s_i^{NC})$$

which is a contradiction. Therefore, if $s_i^C(s_j^{NC}) > s_i^{NC}(s_j^{NC})$ for all $i \neq j$, then $s_i^C > s_i^{NC}$ for all $i = \{1, 2\}$. Thus, using Lemma A we have that

$$\Delta \times D^{NC} > 0 \Longrightarrow s_i^C(s_j^{NC}) > s_i^{NC} \Longrightarrow s_i^C > s_i^{NC} \quad ■$$

7.6 *Proof of Proposition 4*

Using Segal and Sobel (1999), we know that the second mover's preferences over the first mover's actions can be represented by

$$U_{s_i}^C(s_i, s_j) = \gamma_i U_i^{NC}(s_i, s_j) + \gamma_j U_j^{NC}(s_i, s_j) \quad \text{where } \gamma_i, \gamma_j \in \mathbb{R}$$

if preferences satisfy continuity and independence, as well as Segal and Sobel's (1999) condition (★) which states that

$$\text{if } U_i^{NC}\left(s_i', s_j\right) = U_i^{NC}(s_i, s_j), \text{ then } s_i' \sim_i s_i \tag{★}$$

which are all satisfied in our model. ■

7.7 *Proof of Proposition 5*

Both players are asked to simultaneously submit their voluntary contributions to the public good. Fixing subject j's contribution, s_j, we have that

$$s_i(s_j) = \begin{cases} \frac{\alpha+m}{1+\alpha+m} w \text{ if } s_j = 0 \\ \frac{\alpha+m}{1+\alpha+m} w + \frac{\alpha-m}{(\alpha+m)(1+\alpha+m)} s_j \text{ if } s_j \in \left(0, \frac{(\alpha+m)^2}{m-\alpha}\right] \\ 0 \text{ if } s_j \in \left(\frac{(\alpha+m)^2}{m-\alpha}, +\infty\right) \end{cases}$$

if $\alpha < m$. In contrast, when $\alpha > m$ $s_i(s_j)$ does not become zero or negative for any value of s_j. The corresponding best response function for player i in this case is

$$s_i(s_j) = \begin{cases} \frac{\alpha+m}{1+\alpha+m} w \text{ if } s_j = 0 \\ \frac{\alpha+m}{1+\alpha+m} w + \frac{\alpha-m}{(\alpha+m)(1+\alpha+m)} s_j \text{ if } s_j > 0 \end{cases}$$

Regarding the equilibrium contributions, note that symmetry eliminates corner solutions in this case. Hence, the only equilibrium contribution is that resulting from the crossing point of player i's and j's best response functions (interior solution). Solving for s_i and s_j in a system of two equations, we obtain $s_i^C = \frac{(\alpha+m)^2 w}{2m+(\alpha+m)^2}$, as the interior Nash equilibrium contribution level.

Finally, if both players are equally *not* concerned about status, $\alpha_i = \alpha_j = 0$, we obtain the interior solution in standard public good games, where every player i's Nash equilibrium contribution level is given by $s_i^{NC} = \frac{mw}{2+m}$. ■

7.8 *Proof of Corollary 1*

Recall that player i's equilibrium contribution in the model without status acquisition is $s_i^{NC} = \frac{mw}{2+m}$. Comparing it with the equilibrium contribution level in the model with status considerations, $s_i^C = \frac{(\alpha+m)^2 w}{2m+(\alpha+m)^2}$,

$$s_i^C - s_i^{NC} = \frac{(\alpha+m)^2 w}{2m+(\alpha+m)^2} - \frac{mw}{2+m} = \frac{2\alpha(\alpha+2m)w}{(2+m)\left[2m+(\alpha+m)^2\right]}$$

which is positive for any $\alpha > 0$, reflecting that $s_i^C > s_i^{NC}$. ■

7.9 *Proof of Proposition 6*

In this public good game, both players are asked to simultaneously submit their contributions. Fixing player j's contribution, s_j, player i's utility maximization problem becomes

$$\max_{s_i} \quad [w - s_i]^{0.5} + \left[m(s_i + s_j) + \alpha\left(s_j - s_j^{ref}\right)\right]^{0.5}$$

And the argument that maximizes this utility function gives us

$$s_i^C(s_j) = \begin{cases} \frac{\alpha s_j^{ref}+m^2w}{m(1+m)} \text{ if } s_j = 0 \\ \frac{\alpha s_j^{ref}+m^2w}{m(1+m)} - \frac{\alpha+m}{m(1+m)}s_j \text{ if } s_j \in \left(0, \frac{\alpha s_j^{ref}+m^2w}{\alpha+m(2+m)}\right] \\ 0 \text{ if } s_j > \frac{\alpha s_j^{ref}+m^2w}{\alpha+m(2+m)} \end{cases}$$

Since $\frac{\alpha s_j^{ref}+m^2w}{m(1+m)} - \frac{\alpha+m}{m(1+m)}s_j = 0$ at exactly $s_j = \frac{\alpha s_j^{ref}+m^2w}{\alpha+m(2+m)}$. Hence, $s_i^C(s_j)$ becomes

$$s_i^C(s_j) = \begin{cases} \frac{\alpha s_j^{ref}+m^2w}{m(1+m)} - \frac{\alpha+m}{m(1+m)}s_j \text{ if } s_j \in \left(0, \frac{\alpha s_j^{ref}+m^2w}{\alpha+m(2+m)}\right] \\ 0 \text{ if } s_j > \frac{\alpha s_j^{ref}+m^2w}{\alpha+m(2+m)} \end{cases}$$

7.10 *Proof of Proposition 7*

By symmetry, player i and j's best response functions can only cross each other at interior points. Therefore, there must be a unique and interior Nash equilibrium contribution level for every player, which we can obtain by plugging player j's best response function into player i's. In particular,

$$s_i^C = \frac{\alpha s_j^{ref} + m^2w}{m(1+m)} - \frac{\alpha+m}{m(1+m)}\left(\frac{\alpha s_j^{ref} + m^2w}{m(1+m)} - \frac{\alpha+m}{m(1+m)}s_i^C\right)$$

Solving for s_i^C, we have $s_i^C = \frac{\alpha s_j^{ref}+m^2w}{\alpha+m(2+m)}$. ■

7.11 *Proof of Corollary 2*

Recall that player i's equilibrium contribution in the model where players do not assign value to distances is $s_i^{NC} = \frac{mw}{2+m}$. On the other hand, by comparing the equilibrium contribution level when distances are considered, s_i^C, obtained in the above proposition 7 with respect to s_i^{NC},

$$s_i^C - s_i^{NC} = \frac{\alpha s_j^{ref} + m^2w}{\alpha + m(2+m)} - \frac{mw}{2+m} = \frac{\alpha\left[s_j^{ref}\left(2+m-mw\right)\right]}{(2+m)\left[\alpha+m\left(2+m\right)\right]}$$

and this difference is positive if and only if $s_j^{NC} = \frac{mw}{2+m} < s_j^{ref}$. Hence, $s_i^C > s_i^{NC}$ if and only if $D_i^{NC} \equiv \alpha\left(s_j^{NC} - s_j^{ref}\right) < 0$, which is satisfied for any s_j^{ref} such that $s_j^{NC} = \frac{mw}{2+m} < s_j^{ref}$

Otherwise, if $s_j^{NC} = \frac{mw}{2+m} > s_j^{ref}$ holds, then this difference is negative. However, if $\frac{mw}{2+m} > s_j^{ref}$ and $\alpha < 0$ are simultaneously satisfied, then this difference is positive, and $s_i^C > s_i^{NC}$. ■

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