# Common Pool Resources with Equity Shares and Cost Externalities


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**Abstract**

We consider a common pool resource (CPR) where every firm holds an equity share on its rivals' profits (cross-ownership), and its exploitation of the resource produces an externality on its rivals' costs. We identify equilibrium appropriation in this setting, and compare it against the social optimum. Our results show that equity shares can be welfare improving only under certain conditions, but lead to a socially insufficient exploitation of the CPR if shares are large enough. As a result, we demonstrate that emission fees are decreasing in equity shares, potentially becoming subsidies if these shares are sufficiently large, as in mergers between firms. We also find that, as the number of firms exploiting the resource increases, socially excessive exploitation occurs under larger parameter combinations. Our findings suggest the use of a simple policy tool in certain CPRs: setting maximum equity shares.



Keywords: Common pool resources; Equity shares; Social optimum; Emission fees.

JEL classification: D21, D62, Q5.



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# 1 Introduction

The "tragedy of the commons," described originally by Hardin (1968), states that every individual firm exploiting a common property resource (CPR), such as a fishing ground, a forest, or an acquifer, ignores the external effect that its appropriation causes on other firms exploiting the commons; ultimately leading to an overexploitation of the resource relative to the social optimum. Several policy tools are often used to ameliorate this external effect, such as quotas and fees. However, these tools assume that every firm does not hold equity shares on other firms exploiting the same CPR. Our paper shows that, when equity shares are present, fees become less stringent and, under some parameter conditions, they can become negative (subsidies), thus inducing firms to exploit the resource more intensively than they do in the unregulated equilibrium.

Our model considers a CPR where every firm produces a cost externality on its rival. For simplicity, we first consider a setting with two firms and subsequently allow for $N$ firms; showing that our main results are qualitatively unaffected. As opposed to previous studies, we allow firms to hold equity shares on each other's profits (known as "cross-ownership" in the literature).[1] When these shares are absent, every firm considers only its own profit when choosing its appropriation level; and when equity shares are present, every firm considers both companies' profits while choosing appropriation, which reduces its exploitation of the resource.[2] When equity shares are maximal, firms equally share profits, and their behavior coincides with that under a merger (or a multi-plant monopoly) where firms coordinate their appropriation levels. A close example of the latter are "corporate-cooperative management" systems in the US, where firms exploiting the resource coordinate their appropriation decisions as a single entity. Examples include the Northeast Tilefish fishery, Kitts et al. (2007); the Alaskan Chignik Salmon fishery, Deacon et al. (2008); the Pacific Whiting fishery, Sullivan (2001); and the Bearing Sea Pollock fishery, Kitts and Edwards (2003).[3,4]

We then identify firms' equilibrium appropriation, and compare it against the socially optimal exploitation of the resource. We find under which parameter combinations a socially excessive exploitation of the resource emerges (overexploitation, as in standard CPRs problems); for which

[1] Partial cross-ownership has been observed in the automobile industry, where Renault currently holds 44.3% equity shares in Nissan, while Nissan holds 15% in Renault; see Barcena-Ruiz and Campo (2012) and www.nissan-global.com. Other examples include only one firm holding equity shares on their rival's profits, such as Gillette, which owns 22.9% of the non-voting stock and 13.6% of the debt of Wilkison Sword, Gilo et al. (2006); Ford, which purchased 25% of Mazda's shares in 1979; and General Motors, which acquired 20% of Subaru's stock 1999, Ono et al. (2004).

[2] This result has been empirically confirmed in several industries where cross-ownership reduces output and increases prices, such as telecommunications, Parker and Roller (1997); Italian banks, Trivieri (2007); and energy industry in Northern Europe, Amundsen and Bergman (2002).

[3] In "corporate management systems," however, firms transfer their appropriation decisions to a separate corporation, which centrally determines the appropriation levels for each member. While these systems have been fully implemented yet, some fisheries adopted variants of its approach since 1995, such as New Zealand's Bluff Oyster and Challenger Scallop fisheries Yang et al. (2014); and Australia's Exmouth Gulf Prawn fishery, Rogers (2009).

[4] A cooperative of individuals or firms exploiting a CPR, such as a cooperative of fishermen, would fit our model. Catch share programs, however, do not. In catch shares, such as those supported by NOAA, a portion of the catch for a species of fish is allocated to individual fishermen. Some programs allow every fisherman to purchase a larger catch share from other fishermen, which lets the fisherman increase his individual appropriation. The catch share program, however, do not provide the fisherman with a proportion of other fishermen profits.

parameters, in contrast, a socially insufficient appropriation arises (underexploitation); and for which parameters firms' exploitation exactly coincides with the social optimum.

Intuitively, when every firm shares a sufficiently high proportion of its profits with other companies, its profit-maximization problem resembles that of a merged firm, inducing each firm to reduce its appropriation since it now internalizes the cost externality that its exploitation imposes on its rival. We demonstrate that, if such a reduction in output is relatively small, the presence of equity shares can help approach equilibrium appropriation towards its socially optimal level. In other words, equity shares can ameliorate the over-exploitation of the stock. If the decrease in appropriation is severe, however, firms exploit the resource below what the social planner would recommend. In this case, firms' equilibrium behavior changes from an overexploitation of the stock (under no equity shares) to an underexploitation (when equity shares are significant). In extreme settings where firms equally share profits (such as in a merger), our results find that underexploitation can be sustained under large parameter conditions.

Specifically, we show that the presence of equity shares produces a small reduction in output, thus helping ameliorate overexploitation, when the following conditions hold: (1) firms sell most of their appropriation overseas; (2) the resource is not extremely abundant; and (3) firms' exploitation of the resource does not generate environmental externalities (e.g., biodiversity loss and pollution). In these cases, the social planner's problem becomes similar to that of firms, as consumer surplus and environmental damage are both minor. In contrast, when one or more conditions (1)-(3) do not hold, firms decrease their equilibrium appropriation below what a social planner would recommend, ultimately yielding socially insufficient output levels (under-exploitation).

Our findings suggest that, in CPRs exhibiting conditions (1)-(3), equity shares are welfare improving. In this type of CPRs, regulatory authorities could set a lower bound on firms' equity shares, or provide tax breaks for the purchase of equity shares in other companies exploiting the same CPR. Firms would respond reduce appropriation, approaching socially optimal levels. Several CPRs, however, do not satisfy one or more of the above three conditions. In these contexts, our results suggest that large equity shares are not welfare improving but, instead, can reduce social welfare. Regulators could set, in these cases, an upper bound on firms' equity shares (maximum participation on other firms exploiting the same resource). This cap would induce every firm to increase its equilibrium appropriation, ameliorating underproduction, and ultimately increasing social welfare.

**Related literature.** Several studies have analyzed the overexploitation of the commons; for a detailed review of the literature see Ostrom (1990), Ostrom et al. (1994) and Faysee (2005). In order to reduce this excessive appropriation, Ostrom (1999) suggested that CPRs can be managed by local governance structures, Kirkley et al. (2003) examined the importance of preserving CPRs for the long term, especially in developing countries where there is excess capacity, Hackett et al. (1994) analyzed equal appropriation rules in irrigation in India, and Coward (1979) discusses water assignments as a function of land held in the Phillipines.[5]

[5] Other articles consider uncertainty in the resource's stock, and how such uncertainty affect individual appropria-

In our paper, we consider about an alternate policy tool, and evaluate its effectiveness in helping to avoid overexploitation. In particular, we allow firms to hold an equity share (also referred to as equity swaps) on each others' profits. Ellis (2001) presents a similar model, but he assumes that welfare coincides with the sum of firms' profits, thus ignoring the role of consumer surplus and the environmental damage that the exploitation of the CPR may cause.[6] We show that our model can reproduce Ellis' results in the special case where all appropriation is sold overseas (no consumer surplus) and exploitation does not cause any environmental damage. In that setting, the optimal equity share inducing firms to behave optimally is $\frac{1}{2}$, i.e., evenly shared equities. However, when consumer surplus, environmental damage, and cost externalities, such finding no longer applies. Instead, firms should be optimally hold smaller equity shares, even converging to zero in some specific cases.

Our paper connects with the literature analyzing the effect of equity shares in industrial organization. In particular, Reynolds and Snapp (1986) examines a standard Cournot model when firms hold shares in each other's profits, showing that equilibrium quantities decrease as equity shares increase, regardless of which company shares increases.[7] While our paper considers a similar model, it extends their setting along two dimensions: it allows for cost externalities, thus helping understand how their results apply to CPRs, and considers a polluting industry and its optimal environmental regulation. Dietzenbacher et al. (1999) use data from the Dutch financial sector, empirically confirming that output is lower when firms hold shares on each other than otherwise.[8]

Finally, Barcena-Ruiz and Campo (2012) investigate a country's optimal emission fee on a polluting firm when this company holds equity shares on another polluting firm located at a different country. The article then compares emission fees in a non-cooperative setting (independent environmental policies across countries) and a cooperative context (coordination of environmental policies). However, it assumes a single firm in each jurisdiction, symmetric equity shares across firms, and that firms do not impose cost externalities on one another (as in CPRs). We relax all three assumptions.

The following section describes the model. Section 3 identifies equilibrium appropriation, Section 4 finds socially optimal appropriation, and compares it against equilibrium values. At the end

tion levels in the commons, approaching them to socially optimal levels; see Suleiman and Rapoport (1988), Suleiman et al. (1996), and Apesteguia (2006).

[6] His model was extended in Ellis and Nouweland (2006) where they consider that every individual firm exploits the resource in the first stage, and invests in equity shares during the second stage; earning profits only at the end of the game. Anticipating the equilibrium profile of shares at the second stage, every firm's appropriation during the first stage approaches the cooperative solution.

[7] Farrell and Shapiro (1990) consider a Cournot oligopoly in which firms buy new capital either by acquiring it from a rival, from a third party (not a rival), or buying shares from a rival. Fanti (2015) modifies this setting, by considering that one firm owns all its shares and holds a participation on its rival's, while the other firm only holds those shares not participated by the former company. In addition, the paper allows for asymmetries in production costs, showing that the output reduction arising from cross-ownership can be welfare improving if the firm owned by a single shareholder is less efficient than its rival.

[8] Malueg (1992) considers a setting in which firms hold symmetric shares on each others profits, showing that collusive behavior becomes more difficult to sustain than when firms do not own equity shares. Gilo and Spiegel (2006) extend this model to a context in which firms are allowed to hold asymmetric equity shares, demonstrating that collusion can become easier to sustain under certain equity profiles.

of the section, we examine under which parameter conditions overexploitation or underexploitation can more likely arise. Section 5 discusses our results and offers policy implications.

## 2 Model

Consider two firms exploiting a CPR with size $\theta \in (0,1]$. Every firm $i = \{1,2\}$ simultaneously and independently chooses its appropriation level $q_i$, facing inverse demand function $p(q_i,q_j) = a - b(q_i+q_j)$, where $a,b>0$ and $j \neq i$. Firm $i$'s cost function is

$$C(q_i,q_j) = \frac{q_i(q_i+\lambda q_j)}{\theta},$$

entailing that total and marginal costs, $\frac{\partial C(q_i,q_j)}{\partial q_i} = \frac{2q_i+\lambda q_j}{\theta}$, increase in the firm's appropriation, $q_i$, and in its rival's appropriation, $q_j$, but decrease in the available stock, $\theta$. Intuitively, the resource becomes more difficult to capture when either firm $i$ or $j$ increase their appropriation, but easier to capture when the stock is more abundant.[9] Parameter $\lambda \in [0,1]$ indicates how firm $j$'s appropriation affects firm $i$'s costs, i.e., cost externalities. When $\lambda = 0$, every firm's costs are unaffected by its rival's appropriation; whereas when $\lambda = 1$, every unit of firm $j$'s appropriation increases firm $i$'s marginal costs by one dollar.[10]

## 3 Equilibrium Analysis

Every firm $i$ solves the following profit-maximization problem

$$\max_{q_i \geq 0} \; V_i = (1-\alpha_j)\pi_i + \alpha_i \pi_j \tag{1}$$

where $\pi_i \equiv [a - b(q_i+q_j)]\, q_i - \frac{q_i(q_i+\lambda q_j)}{\theta}$ denotes firm $i$'s profit, and $j \neq i$. Therefore, each firm $i$ has two components in its objective function: (1) the share that firm $i$ keeps in its own profit $\pi_i$, after subtracting the share that firm $j$ holds, $\alpha_j \in [0,1/2]$; and (2) a share $\alpha_i \in [0,1/2]$ on the other firm's profits $\pi_j$. Intuitively, parameter $\alpha_i$ can be understood as an equity share on its rival's profits. Specifically, when $\alpha_i = \alpha_j = 0$, the above objective function collapses to $\pi_i$, indicating that every firm only considers its own profit when choosing its individual appropriation level $q_i$; as in standard CPR models. When $\alpha_i = \alpha_j = 1/2$, firms equally share their profits (as in a merger). In that setting, the above objective function simplifies to $\pi_i + \pi_j$, leading every firm $i$ to fully consider

[9] Firm $i$'s marginal cost is $\frac{2q_i+\lambda q_j}{\theta}$, while a marginal increase in firm $j$'s appropriation causes an increase of $\frac{\lambda q_i}{\theta}$, which is smaller than the marginal cost for all admissible parameter values. Hence, a marginal increase in its own appropriation produces a larger increase in firm $i$'s costs than a marginal increase in its rival's appropriation, i.e., $\frac{\partial C(q_i,q_j)}{\partial q_i} > \frac{\partial C(q_i,q_j)}{\partial q_j}$.

[10] Even in the case that $\lambda = 1$, an increase in $q_i$ produces a larger increase in firm $i$'s total and marginal costs than an increase in firm $j$'s appropriation.

the profits of its rival in its individual appropriation decisions.[11] Finally, when $\alpha_i > 0$ but $\alpha_j = 0$, firm $i$ fully retains its profits and receives a share $\alpha_i$ of firm $j$'s profits.[12]

**Lemma 1.** *Every firm i's best response function is given by*

$$q_i(q_j) = \begin{cases} \frac{a\theta}{2(1+b\theta)} - \frac{(\lambda+b\theta)(1+\alpha_i-\alpha_j)}{2(1+b\theta)(1-\alpha_j)}q_j & \text{if } \ q_j \leq \frac{a\theta(1-\alpha_j)}{(\lambda+b\theta)(1+\alpha_i-\alpha_j)} \\ 0 & \text{otherwise.} \end{cases}$$

Therefore, the vertical intercept of the best response function, $\frac{a\theta}{2(1+b\theta)}$, is only affected by the CPR's stock, $\theta$, and demand parameters $a$ and $b$; as depicted in Figure 1. For presentation purposes, we next examine the best response function in different settings.

*Case 1: Cournot models (no cost externalities and no equity shares).* When firms compete in a standard Cournot model (where they do not share equity, $\alpha_i = \alpha_j = 0$, and do not impose cost externalities on each other, $\lambda = 0$), their best response function collapses to

$$q_i(q_j) = \frac{a\theta}{2(1+b\theta)} - \frac{b\theta}{2(1+b\theta)}q_j, \tag{BRF$_1$}$$

which is positive if $q_j \leq \frac{a}{b}$. Our results help us understand firms' exploitation of the resource when they sell all their appropriation in a perfectly competitive market,[13] i.e., firms take the competitive price as given. In particular, our model captures this setting by evaluating demand parameters at $a = 1$ and $b = 0$, which normalizes the given price to 1. In a perfectly competitive market, the above best response function further simplifies to $q_i(q_j) = \frac{\theta}{2}$, entailing that every firm produces a constant output, which becomes unaffected by its rival's appropriation.

*Case 2: CPR models without equity shares.* If, instead, cost externalities are present but firm $i$ does not yet hold equity shares, $\alpha_i = 0$, its best response function collapses to

$$q_i(q_j) = \frac{a\theta}{2(1+b\theta)} - \frac{\lambda+b\theta}{2(1+b\theta)}q_j \tag{BRF$_2$}$$

which is positive for all $q_j \leq \frac{a\theta}{\lambda+b\theta}$; as in standard CPR models. Relative to BRF$_1$ in Cournot models, cost externalities pivot the best response function inwards, thus reducing firm $i$'s appropriation.

[11] Allowing for equity shares above 1/2 would entail that a firm holds more equity on its rival than the rival holds in its own company; as in an acquisition. For simplicity, we do not consider these cases in our analysis.

[12] Alternatively, parameter $\alpha_i$ could represent firms' altruistic concerns. In that case, $\alpha_i = 0$ reflects a selfish agent who only cares about its own payoff, whereas $\alpha_i = 1/2$ indicates a fully altruistic agent. See Velez et al. (2009) for controlled experiments in artisanal fisheries in Colombia, reporting that individuals who exploit a fishery display altruism and other socially favorable behaviors. Our subsequent analysis applies, nonetheless, to both interpretations.

[13] This can be the case if firms sell all their fish captures in an international market for that fish variety, where they compete against thousands of other fishermen, all of them representing a negligible proportion of aggregate sales.

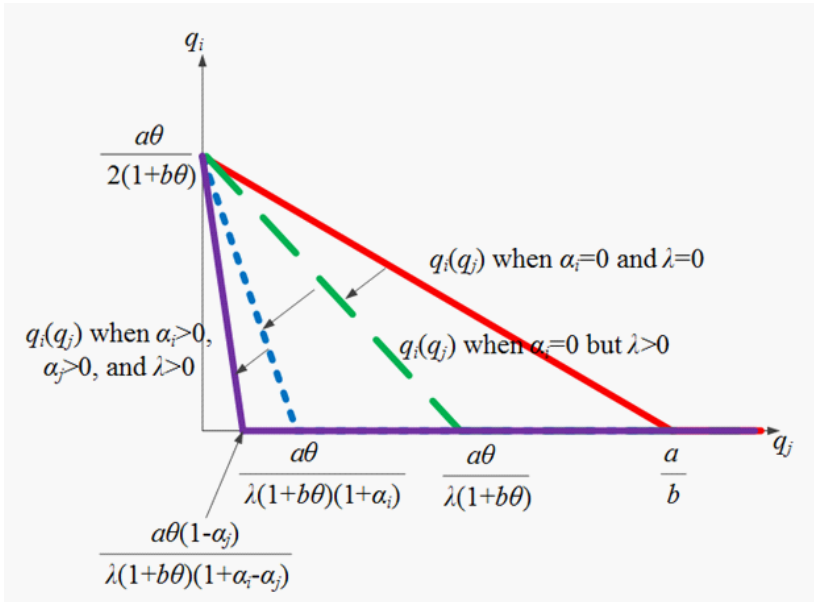


Figure 1. Firm $i$'s best response function.

*Case 3: CPR models with equity shares.* When firm $i$ is the only company holding a positive equity on its rival's profits, $\alpha_i > 0$ but $\alpha_j = 0$, its best response function becomes

$$q_i(q_j) = \frac{a\theta}{2(1+b\theta)} - \frac{(\lambda + b\theta)(1+\alpha_i)}{2(1+b\theta)} q_j, \tag{BRF$_3$}$$

thus pivoting inwards relative to $BRF_2$, as depicted in Figure 1.[14] Intuitively, firm $i$ internalizes a share of the external effect that its appropriation causes on its rival's profit, and thus reduces its own exploitation of the resource. Intuitively, firms' appropriation becomes strategic substitutes to a greater extent. If both companies hold equity shares, $\alpha_i, \alpha_j > 0$, we obtain the best response function in Lemma 1, which experiences a further pivoting effect inwards relative to the case in which $\alpha_i > 0$ but $\alpha_j = 0$; as illustrated by the function closer to the vertical axis in Figure 1. In this case, exploitation also decreases, which is due now to the fact that firm $i$ retains a smaller share of its own profits when $\alpha_j > 0$.

*More competition in the product market.* Furthermore, best response function $q_i(q_j)$ rotates outwards as $b$ decreases, and becomes

$$q_i(q_j) = \frac{a\theta}{2} - \frac{\lambda(1+\alpha_i - \alpha_j)}{2(1-\alpha_j)} q_j \tag{BRF$_4$}$$

when $b = 0$. Since parameter $b$ captures how close the product market is to perfect competition,

[14] In this setting, the horizontal intercept of the best response function is $q_j = \frac{\theta}{\lambda(1+\alpha_i)}$, where $\frac{\theta}{\lambda(1+\alpha_i)} < \frac{\theta}{\lambda}$ since $\alpha_i > 0$.

the above result indicates that, as firms' appropriation yields smaller price reductions (lower $b$) they tend to increase their appropriation of the resource. Even when the market is competitive, our findings suggest that firm $i$'s best response function is decreasing in its rival's appropriation $q_j$ if and only if cost externalities are present, $\lambda > 0$. Intuitively, firm $i$ deems appropriation levels as strategic substitutes when cost externalities exist. Otherwise, firm $i$'s appropriation is constant in its rival's, $q_i(q_j) = \frac{a\theta}{2}$, i.e., graphically, best response function in Figure 1 would be a flat line at $\frac{a\theta}{2}$.

The following Proposition analyzes equilibrium appropriation levels.

**Proposition 1.** *Every firm i's equilibrium appropriation is*

$$q_i^* = \frac{a\theta(1-\alpha_i)\left[2-\alpha_j(2-\lambda)-\lambda(1+\alpha_i)+(1-\alpha_i-\alpha_j)b\theta\right]}{\alpha_i^2(\lambda+b\theta)^2+(1-\alpha_j)\left\{4-(1+\alpha_j)\lambda^2-2(1+\alpha_j)b\lambda\theta+b\theta\left[8+(3-\alpha_j)b\theta\right]\right\}+A}$$

*where* $A \equiv 2\alpha_i\left\{2(1+b\theta)^2 - a2\left[2-\lambda^2-2b\lambda\theta+b\theta(4+b\theta)\right]\right\}$.

Comparative statics of equilibrium appropriation $q_i^*$ with respect to parameters $\alpha_i$, $\alpha_j$, and $\lambda$ yield intractable expressions. Our discussion of firms' best response function in Lemma 1 nonetheless showed that appropriation decreases in these three parameters, emphasizing the strategic substitutability between the firms' appropriation levels.

The following corollary evaluates equilibrium appropriation at special cases.

**Corollary 1.** *Equilibrium appropriation $q_i^*$ becomes*

1. *Standard Cournot model:* $q_i^* = \frac{a\theta}{2+3b\theta}$*, i.e.,* $\alpha_i = 0$ *for every firm i, and* $\lambda = 0$*;*
2. *CPR model without equity shares:* $q_i^* = \frac{a\theta}{2+\lambda+3b\theta}$*, i.e.,* $\alpha_i = 0$ *for every firm i;*
3. *CPR model with symmetric equity shares:* $q_i^* = \frac{a(1-\alpha)\theta}{2+\lambda+3b\theta-2\alpha(1+b\theta)}$*, i.e.,* $\alpha_i = \alpha_j = \alpha$*;*
4. *CPR model with equally shared equity:* $q_i^* = \frac{a\theta}{2+2\lambda+4b\theta}$ *when firms equally share equity, i.e.,* $\alpha_i = \alpha_j = \frac{1}{2}$.

In the standard Cournot model (without equity shares and cost externalities), output is $q_i^* = \frac{a\theta}{2+3b\theta}$; which decreases to $q_i^* = \frac{a\theta}{2+\lambda+3b\theta}$ when cost externalities are present; and further decreases to $q_i^* = \frac{a(1-\alpha)\theta}{2+\lambda+3b\theta-2\alpha(1+b\theta)}$ when both cost externalities and equity shares exist. In the case of symmetric equity shares, Corollary 1 confirms the results in Proposition 1, namely, that appropriation decreases as firms sustain a larger equity share on their rivals' profits, $\alpha$, and in the size of the cost externality, $\lambda$.[15] Essentially, appropriation ranges from its upper bound at $\frac{a(1-\alpha)\theta}{2+3b\theta-2\alpha(1+b\theta)}$, which occurs when every firm's appropriation does not affect its rival's costs, i.e., $\lambda = 0$; to its lower bound at $\frac{a\theta}{4(1+b\theta)}$, which happens when firms equally share equity and the cost externality is maximal, i.e., $\alpha = 1/2$ and $\lambda = 1$.

[15] When firms do not share equity, $\alpha = 0$, appropriation is decreasing in $\lambda$ since it increases firm $i$'s cost function.

For simplicity, our subsequent analysis focuses on the case in which firms face perfect competition in the product market, where price is normalized to one, as evaluated in the next corollary.

**Corollary 2.** *Equilibrium appropriation in the CPR model when firms face perfect competition in the product market becomes*

$$q_i^* = \frac{\theta(1-\alpha_j)\left[2-\lambda-2\alpha_i\right]}{4\left[1-(\alpha_i-\alpha_i\alpha_j+\alpha_j)\right]-\lambda^2},$$

*which is weakly positive for all admissible parameter values. In addition,* $q_i^* \geq q_j^*$ *if and only if* $\alpha_i \leq \alpha_j$.

Like in Corollary 1, when firms hold no equity shares and suffer no cost externalities ($\alpha_i = \alpha_j = 0$ and $\lambda = 0$, as in standard Cournot models) equilibrium appropriation is the largest, at $q_i^* = \frac{\theta}{2}$. This output decreases to $q_i^* = \frac{\theta}{2+\lambda}$ when cost externalities alone are present (as in CPR models with no equity shares); and further reduces to $q_i^* = \frac{(1-\alpha)\theta}{2(1-\alpha)+\lambda}$ when both cost externalities and symmetric equity shares exist. Finally, firm $i$ exploits the resource more intensively than its rival if it holds a smaller share of equity, $\alpha_i \leq \alpha_j$.

# 4 Welfare analysis

We next compare the equilibrium results against the social optimum. The social planner solves

$$\max_{q_i,q_j} W = \gamma CS(q_i,q_j) + PS(q_i,q_j) - Env(q_i,q_j). \tag{2}$$

where consumer surplus is $CS(q_i,q_j) = \frac{(a-1)(q_i+q_j)}{2}$, $a > 1$, producer surplus $PS(q_i,q_j) = V_i + V_j$ sums the objective functions of both firms in problem (1), and $Env(q_i,q_j) = d(q_i+q_j)^2$ denotes the environmental damage which is convex in aggregate appropriation $(q_i+q_j)$, and $d \in [0,1]$. Note that producer surplus $PS(q_i,q_j)$ collapses to $\pi_i + \pi_j$.[16] Therefore, the social planner's problem in (2) does not contain equity shares.

The next proposition identifies social optimum appropriation levels.

**Proposition 2.** *The socially optimal appropriation for every firm* $i$ *is*

$$q_i^{SO} = \frac{\theta\left[2+(a-1)\gamma\right]}{4(1+\lambda+2d\theta)}$$

In the case that firms sell their appropriation in a perfectly competitive market, $a = 1$ and $b = 0$, socially optimal output becomes $q_i^{SO} = \frac{\theta}{2[1+\lambda+2\theta d]}$. This output increases to $q_i^{SO} = \frac{\theta}{2[1+\lambda]}$ when

[16] In particular, producer surplus becomes $V_i + V_j = [(1-\alpha_j)\pi_i + \alpha_i\pi_j] + [(1-\alpha_i)\pi_j + \alpha_j\pi_i]$, which simplifies to $\pi_i + \pi_j$.

appropriation does not generate environmental damage, $d = 0$; and further increases to $q_i^{SO} = \frac{\theta}{2}$ when cost externalities are absent, $\lambda = 0$.

We next compare socially optimal appropriation and the exploitation levels that firms choose in equilibrium. For simplicity, we hereafter focus on the case of symmetric equity shares $\alpha_i = \alpha_j = \alpha$, but the proof of Corollary 2 reports our results in the general case in which equity shares are allowed to be asymmetric.

**Corollary 3.** *Socially optimal output $q_i^{SO}$ satisfies $q_i^{SO} \leq q_i^*$ if and only if $\alpha \leq \overline{\alpha}$, where*

$$\overline{\alpha} \equiv 1 + \frac{\lambda\left[2 + \gamma(a-1)\right]}{2(a-1)\gamma - 4(\lambda + 2d\theta)}.$$

*Cutoff $\overline{\alpha} \geq 0$ if and only if $a \leq a_1 \equiv 1 + \frac{2(\lambda+4d\theta)}{\gamma(2+\lambda)}$, and $\overline{\alpha} \leq \frac{1}{2}$ if and only if $a \geq a_2 \equiv 1 + \frac{4d\theta}{\gamma(1+\lambda)}$, where $a_1 > a_2$.*

Corollary 3 states that equilibrium appropriation is socially excessive, $q_i^{SO} \leq q_i^*$, if firms share relatively low equities, i.e., $\alpha \leq \overline{\alpha}$, which includes the setting in which they ignore each other's profits, $\alpha = 0$, as a special case.[17] In addition, it is negative when demand is relatively high, $a > a_1$, entailing that condition $\alpha \leq \overline{\alpha}$ cannot hold for any admissible $\alpha \in [0, 1/2]$. Intuitively, when world demand for the product is strong, the exploitation of the resource that the social planner would recommend exceeds that arising in equilibrium (which is unaffected by demand as firms take world price of the product as given). As a consequence, equilibrium appropriation becomes socially insufficient for all values of the remaining parameters ($\lambda$, $d$, $\theta$, $\gamma$).

In contrast, cutoff $\overline{\alpha}$ lies above its upper bound, 1/2, when demand is relatively weak, $a < a_2$. In that context, condition $\alpha \leq \overline{\alpha}$ holds for all admissible $\alpha$'s. In words, socially optimal output lies below the equilibrium appropriation due to the weak demand, yielding a socially excessive exploitation of the resource for all values of the remaining parameters ($\lambda$, $d$, $\theta$, $\gamma$). Overall, condition $\alpha \leq \overline{\alpha}$ becomes binding when cutoff $\overline{\alpha}$ satisfies $\overline{\alpha} \in [0, 1/2]$, which holds if demand is intermediate, i.e., $a \in [a_2, a_1]$.

Our results can be applied to the special cases analyzed in the previous section. When firms compete in a standard Cournot model (without equity shares or cost externalities, as in Case 1), cutoff $\overline{\alpha}$ becomes $\overline{\alpha} \equiv 1 + \frac{\lambda[2+\gamma(a-1)]}{2(a-1)\gamma - 8d\theta}$. Since $\alpha = 0$ in this setting, condition $\alpha \leq \overline{\alpha}$ simplifies to $0 \leq \overline{\alpha}$, which holds for all $a \leq 1 + \frac{4d\theta}{\gamma}$, i.e., demand cannot be too strong. When firms exploit a CPR without holding equity shares ($\lambda > 0$ but $\alpha = 0$, as in Case 2), cutoff $\overline{\alpha}$ is not simplified relative to the expression in Corollary 3, but condition $\alpha \leq \overline{\alpha}$ collapses to $0 \leq \overline{\alpha}$ in this context, which is satisfied for all $a \leq a_1$; as described in Corollary 3.[18]

[17] The proof of Corollary 3 in the appendix reports the value of $\alpha_i$ for which $q_i^{SO} = q_i^*$ when allowing for asymmetric equity shares $\alpha_i \neq \alpha_j$. The expression, however, does not allow for interpretation or comparative statics, and thus we focus on the symmetric case here.

[18] Comparing the condition we found in Cases 1 and 2, it is straightforward to show that $a_1 > 1 + \frac{4d\theta}{\gamma}$ occurs when $d < \frac{\theta}{2}$. Therefore, condition $a \leq a_1$ holds for a larger range of $a$'s than condition $a \leq 1 + \frac{4d\theta}{\gamma}$, which intuitively implies that the presence of cost externalities allow for overexploitation to be sustained under larger parameter conditions if environmental damage is relatively low, $d < \frac{\theta}{2}$.

We next evaluate the comparative statics of cutoff $\overline{\alpha}$.

**Corollary 4.** *Cutoff* $\overline{\alpha}$ *is*

1. *decreasing in* $\lambda$ *if and only if* $a < 1 + \frac{4d\theta}{\gamma}$,
2. *decreasing in* $\gamma$ *and* $a$ *for all parameter values,*
3. *increasing in* $d$ *and* $\theta$ *for all parameter values.*
4. *When environmental externalities are absent,* $d = 0$, *cutoff* $\overline{\alpha}$ *reduces to* $\overline{\alpha} \equiv 1 + \frac{\lambda[2+\gamma(a-1)]}{2(a-1)\gamma-4\lambda}$; *which further simplifies to* $\overline{\alpha} \equiv \frac{1}{2}$ *when* $\gamma = 0$.

Therefore, cutoff $\overline{\alpha}$ decreases in the cost externality, $\lambda$, when demand is relatively weak, but increases in $\lambda$ when demand is strong. Figure 2 illustrates cutoff $\overline{\alpha}$ as a function of $\lambda$.[19] The region of $(\alpha, \lambda)$-pairs below cutoff $\overline{\alpha}$ indicate a socially excessive exploitation of the resource, i.e., $q_i^{SO} \leq q_i^*$, while a socially insufficient exploitation occurs at points above the cutoff. For instance, when firms do not impose a cost externality on each other, $\lambda = 0$, cutoff $\overline{\alpha}$ becomes $\overline{\alpha} = 1$, thus implying that appropriation is socially excessive under all parameter values. When firms impose a cost externalities on each other, however, cutoff $\overline{\alpha}$ decreases, since firms decrease their equilibrium output, thus giving rise to a region of $(\alpha, \lambda)$-pairs for which insufficient exploitation can be sustained.

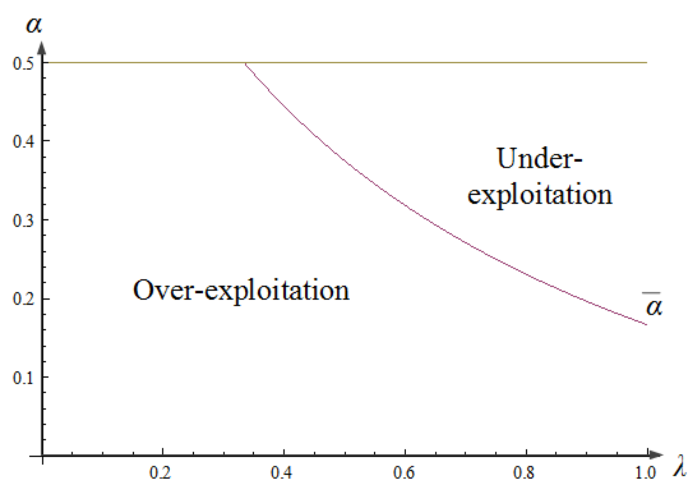


Figure 2. Cuoff $\overline{\alpha}$ as a function of $\lambda$.

From Corollary 4, cutoff $\overline{\alpha}$ shifts downwards as $a$ increase. In words, the resource increases consumer surplus thus pushing the social planners optimum up, while leaving the firm's optimal

[19] Figure 1 assumes, for simplicity, $a = 7$, $d = \theta = 1$, and $\gamma = 1/2$. Other parameters produce similar results, and can be provided by the authors upon request.

exploitation unaffected, and thus reducing the region of $(\alpha, \lambda)$-pairs for which exploitation is socially excessive. a similar argument applies for $\gamma$, which also shifts cutoff $\overline{\alpha}$ downwards. Intuitively, the social planner would seek a larger output as a larger share is sold domestically, whereas individual firms maintain their exploitation unaffected. As a result, the region of $(\alpha, \lambda)$-pairs for which the resource is insufficiently exploited expands.

Cutoff $\overline{\alpha}$ shifts upwards when the environmental damage $d$ increases. In this case, optimal appropriation would be lower, while equilibrium appropriation is unaffected by $d$, entailing that the region of $(\alpha, \lambda)$-pairs for which socially excessive appropriation arises expands. Similarly, cutoff $\overline{\alpha}$ shifts upwards when the size of the stock $\theta$ increases, indicating that socially excessive exploitation becomes more likely.[20]

Last, when environmental externalities are absent and firms do not sell products domestically (i.e., $d = \gamma = 0$), the above cutoff reduces to $\overline{\alpha} = 1/2$. Intuitively, equilibrium appropriation is only socially optimal when firms equally share their profits; as shown in Ellis (2001, Proposition 1). Otherwise, the resource is overexploited.

**Social planner ignoring equity shares.** Our results in Proposition 2 and Corollary 3-4 extend to settings in which the social planner ignores firms' equity shares, i.e., he incorrectly assumes that $\alpha_i = \alpha_j = 0$. This could occur if his information about equity shares is inaccurate.[21] In this context, the social planner's problem coincides with that in (2), where recall that producer surplus simplifies to $PS(q_i, q_j) = \pi_i + \pi_j$. Therefore, the planner selects the same socially optimal appropriation as in Proposition 2, $q_i^{SO} = q_j^{SO} = \frac{\theta[2+\gamma(a-1)]}{4(1+\lambda+2d\theta)}$. Comparing $q_i^{SO}$ against the equilibrium output $q_i^*$ yields that $q_i^{SO} \leq q_i^*$ holds under the same conditions, i.e., $\alpha \leq \overline{\alpha}$; entailing the same set of comparative statics properties for cutoff $\overline{\alpha}$ as in Corollary 4. In words, socially excessive exploitation arises under the same conditions when the planner considers equity shares, or when he does not.

### 4.1 Policy tools with equity shares

In this section, we analyze two commonly used policy tools, quotas and emission fees, and how they are affected by equity shares. For simplicity, we maintain perfect competition in the product market and the symmetric setting from previous sections.

When the regulator uses a quota to induce socially optimal appropriation, he only needs to set $q_i^{SO}$ for every firm $i$. However, when he sets an emission fee $t$ per unit of appropriation, we obtain the results in the next proposition.

**Proposition 3.** *The optimal emission fee when firms hold equity shares is*

$$t^* = \frac{2(1-2\alpha)\lambda - (a-1)\gamma\left[2(1-\alpha)+\lambda\right] + 8(1-\alpha)d\theta}{(1-\alpha)(1+\lambda+2d\theta)},$$

[20] In particular, a given increase in $\theta$ produces a larger increase in equilibrium appropriation $q_i^*$ than in the socially optimal appropriation $q_i^{SO}$, since the social planner internalizes the effect of the externality.

[21] Alternatively, when parameter $\alpha_i$ is interpreted as firm $i$'s altruism towards firm $j$, the planner may not be able to reliably observe every firm's other-regarding considerations.

*which is positive if and only if $\alpha$ satisfies $\alpha \leq \overline{\alpha}$. In addition, emission fee $t^*$ decreases in the equity share that firms hold, $\alpha$.*

In words, when equity shares are sufficiently low, the regulator sets an emission fee to induce firms reduce their exploitation of the resource. This encompasses the standard CPR models, where $\alpha = 0$, as a special case. In contrast, if firms hold a substantial proportion of equity shares, $\alpha > \overline{\alpha}$, regulatory agencies find that equilibrium appropriation is socially insufficient, driving them to set a negative emission fee $t^* < 0$ (that is, a subsidy per unit of appropriation) which induces firms to increase their exploitation of the resource until reaching the socially optimal level. When firms hold the maximum amount of equity shares ($\alpha = 1/2$, as in a merger), condition $\alpha > \overline{\alpha}$ is likely satisfied, leading regulators to offer production subsidies rather than taxes.[22]

A similar argument explains why emission fee $t^*$ is decreasing in the equity share that firms hold, $\alpha$, i.e., $\frac{\partial t^*}{\partial \alpha} < 0$. Intuitively, as firms hold a larger equity on their rival's profits, they reduce their equilibrium appropriation, approaching it to the social optimal. As a result, the regulator does not need to set stringent fees to induce firms exploit the resource at the socially optimal level.

## 5 Extension to $N$ firms

For completeness, Appendix 1 extends our model to a setting with $N$ firms, showing that our above results are qualitatively unaffected. As expected, we demonstrate that equilibrium appropriation and socially optimal appropriation are both decreasing in the number of firms, $N$. In addition, we evaluate cutoff $\overline{\alpha}(N)$ as a function of the number of firms, to examine how less concentrated markets affect the region of $(\alpha, \lambda)$-pairs for which socially excessive exploitation can be sustained. Figure 3 depicts cutoff $\overline{\alpha}(N)$ assuming the same parameter values as Figure 1, at $N = 2$, $N = 4$ and $N = 10$ firms.

[22] In the standard Cournot model where firms hold no equity shares and cost externalities are absent ($\lambda = \alpha = 0$, as in Case 1), the above emission fee collapses to $t^* = \frac{2[1-(a-1)\gamma]+8d\theta}{(1+2d\theta)}$; which further simplifies to $2[1-(a-1)\gamma]$ when exploitation of the resource does not generate environmental damage, $d = 0$. This fee can become a subsidy if $2[1-(a-1)\gamma] < 0$, or $\gamma > \frac{1}{a-1}$, i.e., when a sufficiently large share of output is sold domestically.

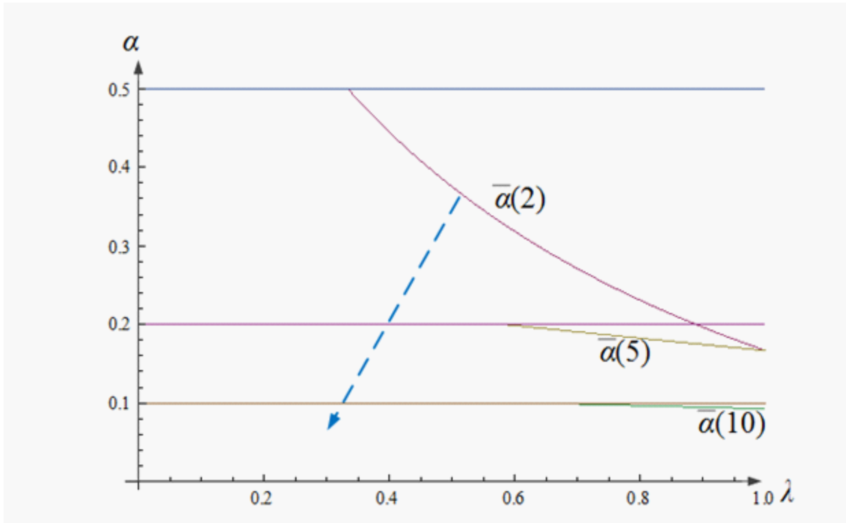


Figure 3. Cutoff $\overline{\alpha}(N)$ for different values of $N$.

Cutoff $\overline{\alpha}(N)$ is decreasing in the number of firms. However, the set of parameter values for which socially excessive exploitation holds, $\alpha \leq \overline{\alpha}(N)$, needs to be compared against the upper bound on $\alpha$, $1/N$. When comparing them, it is clear that the region below cutoff $\overline{\alpha}(N)$ spans over a larger share of the area below $1/N$ as more firms compete for the resource.[23] Intuitively, as more firms enter into the CPR, their appropriation becomes socially excessive under most parameter conditions. Indeed, while individual appropriation $q_i^*$ decreases in $N$, aggregate appropriation $Q^* = Nq_i^*$ increases, which explains socially excessive exploitation arises under larger conditions or, alternatively, why firms' exploitation is only socially optimal if equity shares are close to $1/2$.

# 6 Discussion

Our results identify the exact equity share that firms exploiting a CPR must hold for their equilibrium appropriation to coincide with the social optimum. Starting from the most basic setting, where all appropriation is sold overseas and exploiting the resource does not generate environmental externalities (i.e., $\gamma = d = 0$), the socially optimal equity share (as captured by cutoff $\overline{\alpha}$) lies at its highest: firms must equally share profits for them to produce socially optimal amounts.

[23] Formally, we first need to find the crossing point between the horizontal line $1/N$ and cutoff $\overline{\alpha}(N)$. (Considering the parameter values in Figures 2 and 3, this crossing point occurs at $\widehat{\lambda} = \frac{3-2N}{3-3N}$.) We then evaluate: (a) the area to the left-hand side of $\widehat{\lambda}$, a rectangle with height $1/N$ and base $\widehat{\lambda}$; and (b) the area below cutoff $\overline{\alpha}(N)$ to the right-hand side of crossing point $\widehat{\lambda}$, i.e., between $\widehat{\lambda}$ and 1, given by the integral $\int_{\widehat{\lambda}}^{1} \overline{\alpha}(N)d\lambda$. Adding up the areas in (a) and (b), we obtain an expression that increases in the number of firms, $N$. In particular, we can compute the percentage that areas (a) and (b) represent out of the rectangle with height $1/N$ and base 1, $SER \equiv \frac{(a)+(b)}{1/N}$, where $SER$ denotes the percentage of admissible $(\alpha, \lambda)$-pairs for which socially excessive exploitation emerges in equilibrium. For our parameter values in Figures 2 and 3, we find that $SER = 0.73$ in the case of $N = 2$ firms, increases to $SER = 0.96$ in the case of $N = 5$ firms, and to $SER = 0.99$ in the case of $N = 10$ firms.

*Recommending lower equity shares.* However, when a positive share of output is sold domestically, $\gamma > 0$, cutoff $\overline{\alpha}$ decreases. In words, equilibrium appropriation can be socially optimal, even if firms do not equally share their profits. Interestingly, their exploitation of the resource can become socially insufficient if they sustain large equity shares. Intuitively, in that setting firms' output would approach that of a merged company, while the social planner would prefer a higher production given its positive effect on consumer surplus. A similar argument applies when demand becomes stronger (higher $a$), since the planner seeks a larger appropriation level under larger parameter conditions. In settings where both $\gamma$ and $a$ are sufficiently high, it may become optimal for firms to not sustain equity shares on each other's profits; as otherwise their output would be socially insufficient. This argument also extends to contexts in which firms impose a larger cost externality on each other's profits (higher $\lambda$), whereby firm voluntarily decrease their equilibrium output, thus enlarging the range of parameter values for which the resource is insufficiently exploited.

*Recommending higher equity shares.* The opposite effect emerges when environmental damage is positive, $d > 0$, shifting cutoff $\overline{\alpha}$ upwards. In this case, equilibrium appropriation becomes socially excessive under larger parameter combinations. In this setting, a policy recommendation would be for firms to hold a larger equity share on their rival's profits, which would reduce their equilibrium appropriation, ultimately curbing their associated environmental damage (e.g., pollution and biodiversity loss). A similar argument applies when the stock becomes more abundant (higher $\theta$), which also shifts cutoff $\overline{\alpha}$ upward, yielding socially excessive exploitation under larger conditions. In extreme settings where the stock is abundant, but its exploitation generates substantial environmental damages, firms would only have incentives to produce socially optimal levels if they equally share profits.

*Welfare improving bounds.* Our findings can be used by regulatory agencies to set bounds (both upper and lower) on the equity shares that firms exploiting CPRs can hold. As discussed above, when a large proportion of the appropriation is sold domestically and demand is strong, such upper bound would be relatively low, even banning CPR firms to hold equity shares. In contrast, when the resource is abundant but appropriation generates large environmental damages, regulators could optimally set relatively high lower bounds, inducing firms to sustain significant equity shares. Overall, the implementation of these bounds could help environmental protection agencies induce firms to exploit the resource at the socially optimal level, without the need to use more direct policy tools, such as quotas on appropriation or emission fees.

*Further research.* Our model can be extended to consider settings in which firms exhibit asymmetric cost externalities, $\lambda_i \neq \lambda_j$; to contexts where firm $i$ exploits CPR $A$ while firm $j$ exploits $B$, rather than both appropriating from the same commons; and to industries in which firms do not perfectly observe each other's cost externality parameter. Alternatively, the model can also be applied to a repeated game, where the resource depletes as time progresses. Additionally, our results can be empirically estimated, and tested in field experiments to test if observed appropriation approaches our theoretical predictions.

# 7 Appendix

## 7.1 Appendix 1 - Extension to $N$ firms

In this appendix, we extend our model to a setting with $N$ firms. For compactness in our equilibrium results, we assume symmetric equity shares $\alpha_i = \alpha_j = \alpha$ for every two firms $i$ and $j$, and perfect competition in the product market. Following the same approach as in the main text, we first identify equilibrium appropriation in this context, then the socially optimal appropriation, and finally compare these two findings.

*Equilibrium appropriation.* Every firm $i$ solves

$$\max_{q_i \geq 0} V_i = [1 - (N-1)\,\alpha]\,\pi_i + \sum_{j \neq i} \alpha \pi_j$$

where $\pi_i \equiv q_i - \frac{q_i(q_i + \lambda Q_{-i})}{\theta}$ denotes firm $i$'s profit, $Q_{-i} \equiv \sum_{j \neq i} q_j$ represents the aggregate appropriation from all firms other than $i$, and $\pi_j \equiv q_j - \frac{q_j(q_j + \lambda Q_{-j})}{\theta}$ denotes firm $j$'s profit. Differentiating with respect to $q_i$ yields best response function

$$q_i(Q_{-i}) = \frac{\theta}{2} - \frac{\lambda\,[(N-2)\alpha - 1]}{2\,[(N-1)\alpha - 1]} Q_{-i}$$

Relative to the best response function identified in Lemma 1 for two firms, $q_i(Q_{-i})$ has the same vertical intercept, $\frac{\theta}{2}$, and decreases in its rivals' appropriation, $Q_{-i}$; but has a different slope, $\frac{\lambda[(N-2)\alpha-1]}{2[(N-1)\alpha-1]}$. In particular, the slope becomes more negative as the number of firms $N$ increases. Intuitively, competition becomes tougther, and every individual firm reduces its own appropriation more significantly, i.e., firms' exploitation of the resource are strategic substitutes to a greater extent. Graphically, the best response function rotates inwards.

Invoking symmetry in equilibrium, $q_i^* = q_j^* = q^*$, we obtain that $Q_{-i}^* = (N-1)q^*$, which helps us simplify the above expression to

$$q^* = \frac{\theta}{2} - \frac{\lambda\,[(N-2)\alpha - 1]}{2\,[(N-1)\alpha - 1]}(N-1)q^*$$

Solving for $q^*$, yields the equilibrium appropriation in this $N$-firm setting

$$q_i^*(N) = \frac{\theta\,[\alpha(N-1) - 1]}{\alpha\,[2 + \lambda(N-2)]\,(N-1) - \lambda(N-1) - 2}$$

We next examine special cases, as in Corollary 1 in the main text, showing that equilibrium appropriation $q_i^*$ becomes:

1. *Standard Cournot model*: $q_i^* = \frac{\theta}{2}$, i.e., $\alpha_i = 0$ for every firm $i$ and $\lambda = 0$;

2. *CPR model without equity shares*: $q_i^* = \frac{\theta}{\lambda(N-1)+2}$, i.e., $\alpha_i = 0$ for every firm $i$;

3. *CPR model with equally shared equity*: $q_i^* = \frac{\theta\left[\frac{1}{2}(N-1)-1\right]}{\frac{1}{2}[2+\lambda(N-2)](N-1)-\lambda(N-1)-2}$ when firms equally share equity, i.e., $\alpha_i = \alpha_j = \frac{1}{2}$.

When only two firms operate in the commons, $N = 2$, equilibrium appropriation simplifies to $q_i^*(2) = \frac{\theta(1-\alpha)}{2(1-\alpha)+\lambda}$, which coincides with that in Corollary 1 when equity shares are equal across firms.

*Social optimum.* The social planner solves

$$\begin{aligned} \max_{q_1,,,,q_N} \; W &= \gamma CS(Q) + PS(Q) - dQ^2 \\ &= \gamma \frac{(a-1)}{2} Q + \sum_{i=1}^{N} \pi_i - dQ^2 \end{aligned}$$

where $Q \equiv \sum_{i=1}^{N} q_i$ denotes aggregate output. Note that, since equity shares are symmetric in this setting, they cancel out from the producer surplus, i.e., $PS(Q) = \sum_{i=1}^{N} V_i = \sum_{i=1}^{N} \pi_i$. Differentiating with respect to $q_i$, we obtain

$$q_i(Q_{-i}, Q) = \frac{[2 + (a-1)\gamma - 4dQ]\,\theta}{4} - \lambda Q_{-i}$$

Invoking symmetric appropriation, we find that the socially optimal exploitation level becomes

$$q_i^{SO}(N) = \frac{\theta\,[2 + (a-1)\gamma]}{4\,[1 + \lambda(N-1) + Nd\theta]}$$

which collapses to $q_i^{SO}(2) = \frac{\theta(2+(a-1)\gamma)}{4(1+\lambda+2d\theta)}$ when only two firms operate, $N = 2$, thus coinciding with our result in Proposition 2. Socially optimal output $q_i^{SO}(N)$ decreases in the number of firms, $N$ since $\frac{\partial q_i^{SO}(N)}{\partial N} = -\frac{\theta(2+(a-1)\gamma)(\lambda+d\theta)}{4(1+\lambda(N-1)+Nd\theta)^2} \leq 0$. Finally, equilibrium appropriation is socially excessive, $q_i^{SO}(N) \leq q_i^*(N)$, if and only if

$$\alpha \leq \overline{\alpha}(N) \equiv \frac{2\lambda + (a-1)\gamma\,[2 + \lambda(N-1)] - 2N(\lambda + 2d\theta)}{(N-1)\,[(a-1)\gamma\,[2 + \lambda(N-2)] - 2N(\lambda + 2d\theta)]}.$$

In the case that only two firms compete in the commons, $N = 2$, cutoff $\overline{\alpha}(N)$ collapses to $\overline{\alpha}(2) = 1 + \frac{\lambda(2+(a-1)\gamma)}{2\gamma(a-1)-4(\lambda+2d\theta)}$, thus coinciding with cutoff $\overline{\alpha}$ in Corollary 2.

## 7.2 *Proof for Lemma 1*

Firm $i$ solves problem (1). Differentiating with respect to $q_i$, we find $q_i(q_j) = \frac{a\theta}{2(1+b\theta)} - \frac{(\lambda+b\theta)(1+\alpha_i-\alpha_j)}{2(1-\alpha_j)(1+b\theta)} q_j$. Since firms appropriate weakly positive amounts, $q_i(q_j) = \frac{a\theta}{2(1+b\theta)} - \frac{(\lambda+b\theta)(1+\alpha_i-\alpha_j)}{2(1-\alpha_j)(1+b\theta)} q_j \geq 0$ or, solving for $q_j$, $q_j \leq \frac{a\theta(1-\alpha_j)}{(\lambda+b\theta)(1+\alpha_i-\alpha_j)}$. Therefore, firm $i$'s best response function is

$$q_i(q_j) = \begin{cases} \frac{a\theta}{2(1+b\theta)} - \frac{(\lambda+b\theta)(1+\alpha_i-\alpha_j)}{2(1-\alpha_j)(1+b\theta)} q_j & \text{if } \; q_j \leq \frac{a\theta(1-\alpha_j)}{(\lambda+b\theta)(1+\alpha_i-\alpha_j)} \\ 0 & \text{otherwise.} \end{cases}$$

### 7.3 *Proof for Proposition 1*

From Lemma 1, we found the best response function for firms $i$ and $j$. Simultaneously solving for $q_i$ and $q_j$ in $q_i(q_j)$ and $q_j(q_i)$, we obtain the optimal appropriation for every firm $i$ as

$$q_i^* = \frac{a\theta(1-\alpha_i)\left[2-\alpha_j(2-\lambda)-\lambda(1+\alpha_i)+(1-\alpha_i-\alpha_j)b\theta\right]}{\alpha_i^2(\lambda+b\theta)^2+(1-\alpha_j)\left\{4-(1+\alpha_j)\lambda^2-2(1+\alpha_j)b\lambda\theta+b\theta\left[8+(3-\alpha_j)b\theta\right]\right\}+A}$$

where $A \equiv 2\alpha_i\left\{2(1+b\theta)^2 - a2\left[2-\lambda^2-2b\lambda\theta+b\theta(4+b\theta)\right]\right\}$.

### 7.4 *Proof of Corollary 2*

Under perfect competition in the product market, every firm $i$ solves problem (1) where now $\pi_i = q_i - \frac{q_i(q_i+\lambda q_j)}{\theta}$. Differentiating with respect to $q_i$ and rearranging yields

$$q_i^* = \frac{\theta(1-\alpha_i)\left[2-\alpha_j(2-\lambda)-\lambda(1+\alpha_i)\right]}{4(1-\alpha_i)(1-\alpha_j)-\left[1-(\alpha_i-\alpha_j)\right]\lambda^2}$$

To show this appropriation is unambiguously positive, we consider the numerator and denominator separately. Term $\theta(1-\alpha_i)$ is always positive, while $[2-\alpha_j(2-\lambda)-\lambda(1+\alpha_i)]$ is non-negative. To see this, note that the term is strictly decreasing in $\alpha_i$ and $\alpha_j$. Therefore, evaluating this term at the upper bound of both $\alpha_i$ and $\alpha_j$, i.e., $\alpha_i = \alpha_j = \frac{1}{2}$, we obtain that we need $\lambda > 1$ for the term to be negative, which is impossible by definition. For the denominator, we follow similar steps to show that is is positive. Specifically, term $4(1-\alpha_i)(1-\alpha_j)$ is positive, and its lower bound is 1, which occurs when $\alpha_i = \alpha_j = \frac{1}{2}$. Term $[1-(\alpha_i-\alpha_j)]\lambda^2$ reaches its upper bound when $\alpha_i = \alpha_j$ and $\lambda = 1$. Therefore, the denominator is weakly positive.

Finally, subtracting equilibrium terms of $q_j^*$ from $q_i^*$, we obtain

$$q_i^* - q_j^* = \frac{(\alpha_j-\alpha_i)(1-\alpha_i-\alpha_j)\lambda\theta}{4(1-\alpha_i)(1-\alpha_j)-\left[1-(\alpha_i-\alpha_j)^2\right]\lambda^2}$$

which, solving for $\alpha_i$, is positive if only if $\alpha_i \leq \alpha_j$.

### 7.5 *Proof for Proposition 2*

The social planner solves (2). Differentiating with respect to $q_i$ yields

$$q_i(q_j) = \frac{\left[2+(a-1)\gamma\right]\theta - 4(\lambda+d\theta)q_j}{4(1+d\theta)}$$

and a symmetric expression for $q_j(q_i)$. Simultaneously solving for $q_i(q_j)$ and $q_j(q_i)$, we find that the social optimum is

$$q_i^{SO} = \frac{\theta\left[2+(a-1)\gamma\right]}{4(1+\lambda+2d\theta)}$$

### 7.6 *Proof of Corollary 3*

Evaluating the difference between the socially optimal appropriation and the equilibrium appropriation, $q_i^{SO} - q_i^*$, we obtain that such difference is negative (i.e., $q_i^{SO} \leq q_i^*$) if and only if $\alpha_i$ satisfies $\alpha \leq \overline{\alpha_i}$, where cutoff $\overline{\alpha_i}$ is given by

$$\overline{\alpha}_i \equiv \frac{1}{B}\left[(2(\alpha_j - 2)\lambda + (a-1)\gamma\left[2+\alpha_j(\lambda^2-2)\right] - 4d\left[2+\alpha_j(\lambda-2)\right]\theta + \sqrt{D}\right]$$

where $B \equiv \lambda\left[((a-1)\gamma - 2)\lambda - 8d\theta - 4\right]$ and

$$\begin{aligned} D \equiv \; & (2(\alpha_j - 2)\lambda + (a-1)\gamma\left[2+\alpha_j(\lambda^2-2)\right] - 4d\left[2+\alpha_j(-2+\lambda))\theta\right]^2 \\ & +(\alpha_j - 1)\lambda\left[4+(2+\gamma-a\gamma)\lambda+8d\theta\right]\left[2\lambda\left[2+(\alpha_j-1)\lambda\right]+(a-1)\gamma\left[(1+\alpha_j)\lambda^2-4\right]-8d(\lambda-2)\theta\right] \end{aligned}$$

This cutoff does not allow for tractable comparative statics results in subsequent sections. For simplicity, we then focus in the case of symmetric equity shares, $\alpha_i = \alpha_j = \alpha$, which collapses the above cutoff to

$$\overline{\alpha} \equiv 1 + \frac{\lambda\left[2+\gamma(a-1)\right]}{2(a-1)\gamma - 4(\lambda+2d\theta)}.$$

Since $\alpha \in \left[0, \frac{1}{2}\right]$ by definition, we need first need that cuotff $\overline{\alpha}$ satisfies $\overline{\alpha} \geq 0$ which, solving for $a$, yields $a \leq 1 + \frac{2\lambda+8d\theta}{2\gamma+\gamma\lambda}$. Second, we need that cutoff $\overline{\alpha}$ satisfies $\overline{\alpha} \leq \frac{1}{2}$ which, solving for $a$, entails $a \geq 1 + \frac{4d\theta}{\gamma(1+\lambda)}$. In addition,

$$1 + \frac{2\lambda+8d\theta}{2\gamma+\gamma\lambda} \geq 1 + \frac{4d\theta}{\gamma(1+\lambda)}$$

holds for all admissible parameter values. Therefore, cutoff $\overline{\alpha}$ lies in $\overline{\alpha} \in \left[0, \frac{1}{2}\right]$ if and only if $a \in \left[1 + \frac{4d\theta}{\gamma(1+\lambda)}, 1 + \frac{2\lambda+8d\theta}{2\gamma+\gamma\lambda}\right]$.

### 7.7 *Proof of Corollary 4*

We next differentiate cutoff $\overline{\alpha}$ with respect to parameters. First,

$$\frac{\partial\overline{\alpha}}{\partial a} = -\frac{\gamma\lambda(1+\lambda+2d\theta)}{(\gamma - a\gamma + 2\lambda + 4d\theta)^2}$$

which is negative for all parameter values, implying that cutoff $\overline{\alpha}$ is decreasing in $a$. Second,

$$\frac{\partial\overline{\alpha}}{\partial\lambda} = \frac{(2+(a-1)\gamma)((a-1)\gamma - 4d\theta)}{2(\gamma - a\gamma + 2\lambda + 4d\theta)^2}$$

which is negative if $a < 1 + \frac{4d\theta}{\gamma}$. In such a case, cutoff $\overline{\alpha}$ is decreasing in $\lambda$. Third,

$$\frac{\partial\overline{\alpha}}{\partial\gamma} = -\frac{(a-1)\lambda(1+\lambda+2d\theta)}{\left[(1-a)\gamma + 2(\lambda+2d\theta)\right]^2}$$

which is negative for all parameter values, entailing that cutoff $\overline{\alpha}$ is decreasing in $\gamma$. Fourth,

$$\frac{\partial\overline{\alpha}}{\partial\theta} = \frac{2d(2+(a-1)\gamma)\lambda}{[(1-a)\gamma+2(\lambda+2d\theta)]^2}$$

which is positive for all parameter values. Therefore, cutoff $\overline{\alpha}$ is increasing in $\theta$. Fifth,

$$\frac{\partial\overline{\alpha}}{\partial d} = \frac{2(2+(a-1)\gamma)\lambda\theta}{[(1-a)\gamma+2(\lambda+2d\theta)]^2}$$

which is positive for all parameter values. Therefore, cutoff $\overline{\alpha}$ is increasing in $d$.

### 7.8 *Proof of Proposition 3*

When firm $i$ faces an emission fee $t$, it solves problem (1) where now its profit is given by $\pi_i = q_i - \frac{q_i(q_i+\lambda q_j)}{\theta} - tq_i$. Differentiating with respect to $q_i$, and simultaneously solving, yields

$$q_i^*(t) = \frac{\theta(1-\alpha)\left[2(1-\alpha)-\lambda\right](1-t)}{4-\lambda^2},$$

which coincides with the equilibrium appropriation $q_i^*$ when emission fees are absent, $t=0$. When fees are positive, $t>0$, equilibrium appropriation becomes lower.

In order to set the optimal emission fee, the regulator finds the fee $t$ that solves $q_i^*(t) = q_i^{SO}$, that is

$$t^* = \frac{(a-1)\gamma\left[2(1-\alpha)+\lambda\right]-2(1-2\alpha)\lambda-8(1-\alpha)d\theta}{(\alpha-1)(1+\lambda+2d\theta)},$$

which is positive as long as $\alpha$ satisfies $\alpha \leq \overline{\alpha}$. Finally, differentiating emission fee $t^*$ with respect to $\alpha$ yields

$$\frac{\partial t^*}{\partial\alpha} = -\frac{\left[2+\gamma(a-1)\right]\lambda}{4(1-\alpha)^2(1+\lambda+2d\theta)} < 0.$$

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