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THE ROLE OF INDIVIDUAL IDENTITIES IN HOUSEHOLD DECISION-MAKING PROCESS

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This paper analyzes how individual identity—narrowly defined as the sense of a wife's role—affects married females' market work hours. We bring identity into a neoclassical model of household decision making. Our results show that when a husband's and wife's views of her time allocation do not coincide, exogenous income changes can have both income and substitution effects, and it is possible for the substitution effect to dominate. This can have significant importance for social policy; in-kind transfers designed to increase wives' labor market opportunities can have the opposite effect.

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I. Introduction

In psychology identity–or sense of self–is founded on the conceptions, qualities, beliefs, and other characteristics that make a person who she is. In economics, identity helps define expected behavior, and inconsistencies about identity between the parties involved may cause disruptions in a relationship. In this paper we focus on a wife's time allocation identity and explore how her view her role in the home production interacts with her husband's view to see how identity affects married females' labor market work.

Akerlof and Kranton (2000, 2008) were the first to use the concept of identity to explain the relationship between economic outcomes and expected behaviors. They explicitly consider how identity defines people into categories and thereby influences how they "should" behave. Deviations from expected behavior—that is, deviations from ones' sense of self—alter outcomes by imposing external effects on others and internal effects on oneself. In a later work (Akerlof and Kranton, 2010) they suggest that the expectations of traditional gender roles would affect woman's attachment to the labor market, and have side comments that gender identity may affect welfare distributions within a household as well.

Several empirical studies apply Akerlof and Kranton's taxonomy to the tests for the role of gender identity in women's labor market participation. Bertrand, Pan, and Kamenica (2013) referred to gender identity to assess how specific perception such as “a man should earn more than his wife” affects the division of home production and the gender gap in labor force participation. Fortin’s empirical studies (2005, 2015) also showed that there exist relationships between females’ identity and labor market participation. However, neither Akerlof and Kranton’s work or the empirical papers that followed have a household model that considers the partner’s view of his spouse’s identity, and thus explicitly explains how identity issues help determine a household’s final outcomes.

While the literature incorporating identity into household decision making is sparse, most early economic studies of marriage implicitly recognized the presence of expectations on a wife’s role. Becker (1976) provided the first explicit model of household behavior which acknowledged joint and individual production and time constraints to explain household outcomes. In a unitary decision-making framework he attributed the specialization within a household to the relative labor productivity of a couple under the implicit assumption that all household members have concordant expectations on a wife's role in a household's production (Becker, 1985).

Chiappori (1988, 1992) expands on Becker with a collective model of decision making, which allows for heterogeneity and possible dissonances in a couple’s expectations. In particular, he postulated two stages of joint and individual utility maximization problems—each household member has a separate utility and budget constraint from a joint household utility and budget constraint. A critical assumption is that household members have shared expectations of a women's role in a household as exogenously determined by culture or social norms. Critical to his model is that shares of non-labor are determined by an exogenous "sharing rule" while the individual members keep their own labor earnings. Within this construct the household always achieves Pareto efficient outcomes in the joint maximization problem.

Such a collective model has two shortcomings. First, Chiappori assumes individuals are either egoistic or altruistic, ignoring that households often allow for bargaining between its members. Husbands and wives are rarely completely egoistic or altruistic, instead recognizing the need for compromise. Second, the exogenous sharing rule cannot fully show the internal decision process in a household because the rule involves only in sharing non-labor income. In fact, the sharing rule in her models is based on members' respective wage levels, ignoring that household faces a joint budget constraint that allows the transfer of labor income between parties, even while accepting that joint utility consists of individual utilities.

Incorporating identity into a model of household decision making allows us to improve upon Chiappori's collective model in several ways. First, a sharing rule is no longer necessary. Households maximize a joint utility (a weighted sum of individual utilities) subject to a single budget constraint that includes all non-labor and labor income, whether it is brought to the marriage by the husband or wife. In addition, partners are not pigeon-holed into altruistic or selfish. Instead, in the quest to maximize household utility, individual members may trade-off some personal utility for improved spousal happiness. Most specifically, perceived identity—whether it is shared or not—allows household members to compromise their individualism to make their partner happy.

By explicitly accounting for potential differences in a couple's perception of the wife's time allocation we can explain the distribution of household welfare. In particular, by looking at a household's decision-making process as a transmission mechanism of expectations, we show that bringing in identity leads to different results from what is seen in both Becker's unitary model and Chiappori's collective model. Most specifically, when the couple does not share an identity for the wife, non-labor income may have a substitution effect allwoing income to be distributed to compensate a member's sacrifice due to identity deviation. This new approach explains the distribution of household welfare without individual budget constraints; instead the inclusion of identities changes the household's demand for leisure, home activity, and market work. A key result is that dissonance between a couple's perception of the wife's time use can lead to compromised values compared to the case where the partners agree, with most of the burden falling on the wife. The result, contributes to the continuation of gender inequality in the labor market. We thus can explain why labor force participation rates between married females and males differ so much from those of single people.

The paper is structured as follows. In the next section we introduce our model which brings identity to the household decision-making process. Section 3 uses a specific functional form to analyze the process, first when the couple shares an identical identity and then when the wife's own identity differs from what her husband wants of her. Section 4 offers conclusions and extensions.

II. ADDING IDENTITY TO A HOUSEHOLD MODEL

A household consists of two members $(i = 1, 2)$ of a different genders. For convenience we let 1 stand for the husband while 2 indicates the wife. Each member i is assumed to face a constraint of available time T, which is to devote for leisure l_i, market work m_i, or home activity h_i such that $T = l_i + m_i + h_i$. Under the time constraint, each member i has an individual utility:

$$U_i = U_i\left(c_i, l_i, h_i\right), \tag{1}$$

which is an increasing, diminishing function of private consumption c_i, leisure l_i, and home activity h_i. Home activity is defined as work time spent for their home production that is constant returns to scale. This paper addresses the home activity as a unique input to public goods for the household such as raising children, which can be paid for by time spent on home production or by hiring a service for the task. If an individual i chooses to spend time on home activity h_i, she or he has less time for leisure or market work.

We model identity in the context of home activity, defining a household member's identity as his or her desired time devoted to home production. For example, member i 's identity toward member j can be written as $h_j^{E_i}$, where $i, j = 1, 2$. If $i = 2$ and $j = 1$ it is the number of hours the wife wants her husband to contribute to their home production. If $i = 2$ and $j = 2$ then it is the number of hours the wife wants to contribute to home production.

II.A. Household Utility and Identity Deviation Weights

Chiappori (1988, 1992) modeled household utility as a weighted sum of individual member utilities

(2) $$H = U_1 + \mu U_2,$$

where μ is the implicit Pareto weight of the second member's utility in the collective decision-making process when the household maximizes the joint utility. We follow this construct and also her assumptions of a two-step decision-making process and collective rationality. A household first considers μ and then allocates their time to have their final choices as Pareto optimal. We bring identity into the process by modeling individual utility as U_i^* where $U_i^* = ID_i \times U_i$ and ID_i is what we call an identity deviation weight. $ID_i = 1$ if there is no deviation of behavior from identity. In the context of our model that means an individual's expectation of both household members' contribution to household production matches the behavior of the respective members. More concretely, we define member i 's identity deviation weight as a function of the difference between the member j 's actual choice of home activity and member i's expectations on j 's home activity $h_j - h_j^{E_i}$, where $i, j = 1, 2$. If member i's expectations are not met $0 \leq ID_i < 1$. More formally

(3) $$ID_i = ID_i\left(h_i - h_i^{E_i}, h_j - h_j^{E_i}\right),$$

where this function is concave, so that as the difference between actual choices and member i 's expectations gets larger, i's identity deviation weight decreases. Notice in our model the husband's individual utility would be diminished if his own goal is not met or his expectation of his spouse is not met, but his goals not being met would not affect his spouse's utility. This also holds for the wife. Household utility now becomes

(4) $$H = U_1^* + \mu U_2^* = U_1 ID_1 + \mu U_2 ID_2 .$$

We differ from Chiappori's model by allowing household members to share labor earnings as well as non-labor income. The joint budget constraint is

(5) $$w_1\left(l_1 + h_1\right) + w_2\left(l_2 + h_2\right) + c_1 + c_2 = w_1 T + w_2 T + y \text{ with } l_1 + h_1 \leq T, l_2 + h_2 \leq T ,$$

where y is the household's non-labor income or debt. This allows us to explicitly allow one member to subsidize the other with market income, and allows both income and substitution effects from non-labor income.

II.B. An Example Identity Weight Function

A functional form that captures the characteristics we ascribe to the identity deviation weight is

$$ID_i = \left[1 - \frac{g_{ii}\left(h_i - h_i^{E_i}\right)^2 + g_{ij}\left(h_j - h_j^{E_i}\right)^2}{\left(g_{ii} + g_{ij}\right)T^2}\right], \forall i, j = 1, 2., \quad (6)$$

The subscripts identify whose expectation of whom is being measured. For example, if $i = 1$ and $j = 2$, we have the husband's identity deviation weight in the form of:

$$ID_1 = \left[1 - \frac{g_{11}\left(h_1 - h_1^{E_1}\right)^2 + g_{12}\left(h_2 - h_2^{E_1}\right)^2}{\left(g_{11} + g_{12}\right)T^2}\right], \quad (7)$$

indicating that a husband cares if he meets his identity and if his wife meets his expectation for her. On the other hand, if $i = 2$ and $j = 1$, it represents the identity deviation weight of the wife:

(8) $$ID_2 = \left[1 - \frac{g_{21}\left(h_1 - h_1^{E_2}\right)^2 + g_{22}\left(h_2 - h_2^{E_2}\right)^2}{\left(g_{21} + g_{22}\right)T^2} \right],$$

reflecting that a wife cares whether she can meet her identity and whether her husband meets her expectation of him. Note in all cases we have $ID_i \in [0,1]$ since

(9) $$0 \le h_i - h_i^{E_j} \le T,\ i = 1, 2.$$

Because we are interested in female labor market outcomes we focus on the impact of identities toward a wife's role on the distribution of household welfare. To that goal, we address a specific case in which the husband and wife have identity only on the wife. This means we set $g_{11} \equiv 0$ and $g_{21} \equiv 0$ leaving identity deviation weights of

(10) $$ID_1 = \left[1 - \frac{\left(h_2 - h_2^{E_1}\right)^2}{T^2} \right] \text{ and } ID_2 = \left[1 - \frac{\left(h_2 - h_2^{E_2}\right)^2}{T^2} \right].$$

An advantage of focusing only on the wife's identity is that the model is much more tractable. This simpler model implies no household expectations on the husband. With this simplified model if expectations are met, so $h_2 = h_2^{E_i}$, then $ID_i = \left[1 - \frac{0}{T^2} \right] = 1$ while if $h_2 - h_2^{E_i} = T$ we have $ID_i = \left[1 - \frac{T^2}{T^2} \right] = 0$. One consequence of this model is that unless $h_2^{E_2} = h_2^{E_1}$, (that is, the husband and wife share expectations about the wife's role in the household), at least one and possibly both will lose individual utility whatever the outcome of the wife's time devoted to the household.

III. HOUSEHOLD OPTIMIZATION WITH IDENTITY

In this section we look at a parametric form of utility to better understand how identity on a wife's home activity plays a role in the intra-household decision-making process and in the distribution of welfare of a household. For the individual utility functions absent any identity deviation weight we use the functional form

$$U_i = a_i \log c_i + b_i \log l_i + d_i \log h_i, \quad \forall i = 1, 2. \tag{11}$$

Our purpose is to show how any deviation of a wife's home activity h_2 from $h_2^{E_i}$ triggers identity costs and substitution behavior within the household. We first look into the case where a household shares identical identity for the wife and then extend our analysis to the case where a wife and husband have different identities. In particular, with the latter case we show how different identities on a wife's time allocation in the household brings the possibility of a substitution effect of non-labor income. We first address the case when the husband and wife share the same identity for the wife, then move to the case when they do not.

III.A. Shared Identity

When the husband and wife share an identity for the wife (indicated by h_2^E) the household's maximization problem is

$$H = U_1(c_1, l_1, h_1) ID_1(h_2, h_2^E) + \mu U_2(c_2, l_2, h_2) ID_2(h_2, h_2^E), \tag{12}$$

subject to their joint budget constraint. The general Lagrangian for the case is

$$\begin{aligned} £ = &\left[a_1 \log c_1 + b_1 \log l_1 + d_1 \log h_1 + \mu\left(a_2 \log c_2 + b_2 \log l_2 + d_2 \log h_2\right)\right]\left[1 - \frac{\left(h_2 - h_2^E\right)^2}{T^2}\right] \\ &+\lambda\left[w_1(T - l_1 - h_1) + w_2(T - l_2 - h_2) + y - c_1 - c_2\right]. \end{aligned} \tag{13}$$

Our goal is to identify how the marginal rate of substitutions between various household activities of change. To that regard we first suppose that there exists a specific condition such as the following lemma.

Lemma 1 *There exist values for w_1, w_2, y^0, and μ, such that the household's optimal choices $\{c_i^0, l_i^0, h_i^0\}$ $\forall i$ satisfy $h_2^0 = h_2^E$ and $ID_i^0 = ID^0 = 1$ $\forall i$.*

Superscript 0 indicates the optimal choices with non-labor income y^0 and specific values for w_1, w_2 and μ. *Lemma 1* supposes a specific environment in which a household's optimal outcome meets exactly at the point of their shared identity on a wife's home activity.[1] In such a case the outcomes from this model would be the same as Chiappori's joint utility maximization problem in sense that they are Pareto optimal outcomes mainly determined by pecuniary motives and the bargaining weight. Lemma 1 implies that in this specific case

(14) $$h_2 = h_2^{E_1} = h_2^{E_2}.$$

and so gives us first-order conditions

(15) $$\frac{\partial £}{\partial c_1^0} : \frac{a_1}{c_1^0} = \lambda^0$$

(16) $$\frac{\partial £}{\partial l_1^0} : \frac{b_1}{l_1^0} = \lambda^0 w_1$$

(17) $$\frac{\partial £}{\partial h_1^0} : \frac{d_1}{h_1^0} = \lambda^0 w_1$$

(18) $$\frac{\partial £}{\partial c_2^0} : \mu \frac{a_2}{c_2^0} = \lambda^0$$

(19) $$\frac{\partial £}{\partial l_2^0} : \mu \frac{b_2}{l_2^0} = \lambda^0 w_2$$

[1] Values that support *Lemma 1* are further explained in Appendix 1.

(20) $$\frac{\partial £}{\partial h_2^0} : \mu \frac{d_2}{h_2^0} = \lambda^0 w_2$$

(21) $$\frac{\partial £}{\partial \lambda^0} : w_1 \left(T - l_1 - h_1 \right) + w_2 \left(T - l_2 - h_2 \right) + y^0 - c_1 - c_2 = 0 .$$

Equations (15) to (21) imply the following relationships:

$$c_1^0 = w_1 \frac{a_1}{d_1} h_1^0 ;\ l_1^0 = \frac{b_1}{d_1} h_1^0 ;\ c_2^0 = w_2 \frac{a_2}{d_2} h_2^0 ;\ l_2^0 = \frac{b_2}{d_2} h_2^0 ;$$

(22) $$w_1 \left(T - \frac{b_1}{d_1} h_1^0 - h_1 \right) + w_2 \left(T - \frac{b_2}{d_2} h_2^0 - h_2 \right) + y^0 - w_1 \frac{a_1}{d_1} h_1^0 - w_2 \frac{a_2}{d_2} h_2^0 = 0 ;$$

$$\mu = \frac{a_1}{a_2} \frac{c_2^0}{c_1^0} = \frac{a_1}{a_2} \frac{w_2 \frac{a_2}{d_2} h_2^0}{w_1 \frac{a_1}{d_1} h_1^0} = \frac{w_2}{w_1} \frac{d_1}{d_2} \frac{h_2^0}{h_1^0} = \frac{w_2}{w_1} \frac{d_1}{d_2} \frac{h_2^E}{h_1^0} .$$

These provide a benchmark for comparison. Note that when μ=1 and agents have the same utility function our model reduces to the unitary model of Becker and the couple decides their household and market work according to their productivity.

The equations in (22) result from a very specific case. More generally, even when a couple shares identity on her role at their household production, a wife's actual choice will not necessarily coincide with their expectations. Other factors may mean the best result differs from their shared identity, thus they incur identity deviation costs. To examine this claim, we propose the following two propositions (still constrained for when a husband and wife hold the same identity for the wife).

Suppose that non-labor income decreases from y^0 to y^1 with other things constant.

Proposition 1 *When the household members share an identical identity, decreasing non-labor income from* y^0 *with* w_1, w_2, and μ *held constant, will decrease both members' consumption, leisure, and home activity. However, the proportionate change in the wife's home activity and welfare is less than the proportionate change in the wife's and husband's other choices, and the husband's welfare.*

We first show that when non-labor income falls, the family will no longer maintain the initial choice $h_2 = h_2^E$. At the initial condition before non-labor income changes $y = y^0$, the household's problem is as follows:

$$\text{(23)} \quad \max_{c_i, l_i, h_i} \pounds^0 = \left[\left(a_1 \log c_1^0 + b_1 \log l_1^0 + d_1 \log h_1^0\right) + \mu\left(a_2 \log c_2^0 + b_2 \log l_2^0 + d_2 \log h_2^0\right)\right] ID^0 + \lambda^0 \left[w_1\left(T - l_1^0 - h_1^0\right) + w_2\left(T - l_2^0 - h_2^0\right) + y^0 - c_1^0 - c_2^0\right].$$

Assuming $h_2^0 = h_2^E$ there is no identity deviation so $ID_i^0 = ID^0 = 1$ and the first order conditions give $\mu \dfrac{d_2}{h_2^E} = \lambda^0 w_2$. But suppose non-labor income changes such that $y^1 = y^0 - k$. By counter example, we show that the household cannot maintain h_2^E because of their budget constraint.

Let non-labor income reduce to y^1 but maintain $h_2 = h_2^E$. The new maximization problem is

(24) $$\max_{c_i,l_i,h_i} £^1 = \left[\left(a_1 \log c_1^1 + b_1 \log l_1^1 + d_1 \log h_1^1\right) + \mu\left(a_2 \log c_2^1 + b_2 \log l_2^1 + d_2 \log h_2^1\right)\right] ID^1$$
$$+\lambda^1\left[w_1\left(T - l_1^1 - h_1^1\right) + w_2\left(T - l_2^1 - h_2^1\right) + \left(y^0 - k\right) - c_1^1 - c_2^1\right],$$

where superscript 1 stands for the new optimal choices, after non-labor income decreases by k. Under the assumption that $h_2^1 = h_2^E$ when solving this problem, there is still no identity deviation so $ID_i^1 = ID^1 = 1$ still. First-order conditions thus give that $\lambda^1 = \frac{\mu}{w_2}\frac{d_2}{h_2^E}$ which would mean that $\lambda^1 = \lambda^0$ because $\lambda^0 = \frac{\mu}{w_2}\frac{d_2}{h_2^E}$ from the above. We also have

(25) $$c_2^1 = w_2 \frac{a_2}{d_2} h_2^E = c_2^0;\; l_2^1 = \frac{b_2}{d_2} h_2^E = l_2^0;\; h_1^1 = \frac{d_1}{a_1}\frac{c_1^1}{w_1} = h_1^0;\; c_1^1 = \frac{1}{\mu}\frac{a_1}{a_2} c_2^0 = c_1^0 \text{;and}$$
$$\mu = \frac{a_1}{a_2}\frac{c_2^1}{c_1^1} = \frac{a_1}{a_2}\frac{c_2^0}{c_1^1}.$$

The household would choose the same optimal consumption, leisure, and home activity that are the outcomes with higher non-labor income. But those choices cannot satisfy their new budget constraint, so we have a contradiction.

Since h_2^1 must change it can no longer equal h_2^E and the first-order conditions are therefore

(26) $$\frac{\partial £}{\partial c_1^1} : \frac{a_1}{c_1^1} ID^1 = \frac{a_1}{c_1^1}\left(1 - \frac{\left(h_2^1 - h_2^E\right)^2}{T^2}\right) = \lambda^1$$

(27) $$\frac{\partial £}{\partial l_1^1}: \frac{b_1}{l_1^1} ID^1 = \frac{b_1}{l_1^1}\left(1-\frac{\left(h_2^1-h_2^E\right)^2}{T^2}\right) = \lambda^1 w_1$$

(28) $$\frac{\partial £}{\partial h_1^1}: \frac{d_1}{h_1^1} ID^1 = \frac{d_1}{h_1^1}\left(1-\frac{\left(h_2^1-h_2^E\right)^2}{T^2}\right) = \lambda^1 w_1$$

(29) $$\frac{\partial £}{\partial c_2^1}: \mu\frac{a_2}{c_2^1} ID^1 = \mu\frac{a_2}{c_2^1}\left(1-\frac{\left(h_2^1-h_2^E\right)^2}{T^2}\right) = \lambda^1$$

(30) $$\frac{\partial £}{\partial l_2^1}: \mu\frac{b_2}{l_2^1} ID^1 = \mu\frac{b_2}{l_2^1}\left(1-\frac{\left(h_2^1-h_2^E\right)^2}{T^2}\right) = \lambda^1 w_2$$

(31) $$\frac{\partial £}{\partial h_2^1}: \mu\frac{d_2}{h_2^1}\left(1-\frac{\left(h_2^1-h_2^E\right)^2}{T^2}\right) + \left(V_1^1+\mu V_2^1\right)\left(-2\frac{\left(h_2^1-h_2^E\right)}{T^2}\right) = \lambda^1 w_2$$

(32) $$w_1\left(T-l_1^1-h_1^1\right)+w_2\left(T-l_2^1-h_2^1\right)+y^1-c_1^1-c_2^1=0,$$

where $V_1^1 = a_1\log c_1^1 + b_1\log l_1^1 + d_1\log h_1^1$ and $V_2^1 = a_2\log c_2^1 + b_2\log l_2^1 + d_2\log h_2^1$ are individuals' indirect utility under the lower non-labor income y^1. Now the wife's new choices incur identity costs on the household and on each individual.

To prove the first part of *Proposition 1*, we need to examine whether their all choices are decreased. Equations (26) to (28) say $w_1 = \frac{b_1}{a_1}\frac{c_1^1}{l_1^1} = \frac{d_1}{a_1}\frac{c_1^1}{h_1^1}$ and

Equations (15) to (17) say $w_1 = \frac{b_1}{a_1}\frac{c_1^0}{l_1^0} = \frac{d_1}{a_1}\frac{c_1^0}{h_1^0}$ and thus, the husband has the same marginal substitution rate with the following relations:

$$\frac{c_1^1}{l_1^1} = \frac{c_1^0}{l_1^0} \text{ and } \frac{c_1^1}{h_1^1} = \frac{c_1^0}{h_1^0}. \tag{33}$$

Since consumption is a normal good we can infer that the husband's consumption must decrease as non-labor income decreases. Accordingly, the husband's other choices such as leisure and home activity would decrease in direct proportion. The husband's marginal rate of substations between consumption and leisure and between consumption and home activity are unaffected by the (now nonzero) identity cost. Moreover, we also have $\frac{c_2^0}{c_1^0} = \frac{\mu a_2}{a_1}$ and $\frac{c_2^1}{c_1^1} = \frac{\mu a_2}{a_1}$ from (15) and (18) and (26) and (29). Since the bargaining weight μ would be treated as given at the stage of allocating their time, we can see that the decrease in the wife's consumption is also in direct proportion. This means that the marginal rate of substitution of consumption across the couple would be predetermined at the last stage regardless of the inclusion of identity. Likewise, since wage rates are unchanged, the relations $w_2 \frac{a_2}{b_2} = \frac{c_2^0}{l_2^0} = \frac{c_2^1}{l_2^1}$ from Equations (18), (19), (29), and (30) indicate that the wife's marginal rate of substitutions between consumption and leisure is unchanged, too, even when her actual choice deviates from her identity value.

However, (29) and (31) show that the wife's marginal rate of substitution of home activity and consumption does change with identity deviation. In particular, we now have

(34) $$\mu \frac{d_2}{h_2^1}\left(1-\frac{\left(h_2^1-h_2^E\right)^2}{T^2}\right)=\lambda^1 w_2+2\frac{\left(h_2^1-h_2^E\right)}{T^2}\left(V_1^1+\mu V_2^1\right).$$

To examine how the wife's home activity changes when non-labor income decreases, first note that (26) and (15) imply, respectively,

(35) $\frac{a_1}{c_1^1}=\lambda^1 / \left(1-\frac{\left(h_2^1-h_2^E\right)^2}{T^2}\right)=\frac{\lambda^1}{ID^1}$ (assuming $ID^1 \neq 0$) and $\frac{a_1}{c_1^0}=\lambda^0$.

We also know $c_1^0 > c_1^1$ so $\frac{\lambda^1}{ID^1} > \lambda^0$ and there exists $\alpha > 0$ such that $\frac{\lambda^0}{ID^0}+\alpha=\frac{\lambda^1}{ID^1}$. Using the relationship $\lambda^0+\alpha=\frac{\lambda^1}{ID^1}$ and(20) and (34) we have

(36) $$\mu d_2\left(\frac{1}{h_2^1}-\frac{1}{h_2^E}\right)+\frac{2\left(h_2^E-h_2^1\right)}{T^2}\frac{\left(V_1^1+\mu V_2^1\right)}{ID^1}=\alpha w_2>0,$$

where the plus sign on the middle term comes from switching the order of $h_2^1-h_2^E$ to $h_2^E-h_2^1$. Rational behavior implies that the indirect utilities V_1^1 and V_2^1 are non-negative. The left-hand-side (LHS) of equation (36) can be positive only if $h_2^E > h_2^1$ so, as indicated above, the wife's household commitment goes down.

Meanwhile, (36) also tells us that the marginal utility change in the wife's home activity differs from αw_2, which proves the second part of *Proposition 1*—whether the change in a wife's marginal utility of home activity is less than the other proportional changes. We know that the marginal utility of the husband's consumption changes so

$$(37) \qquad a_1\left(\frac{1}{c_1^1}-\right)\frac{1}{c_1^0}=\frac{\lambda^1}{ID^1}-\lambda^0=\alpha$$

and likewise the marginal utility of the wife's consumption changes so

$$(38) \qquad \mu a_2\left(\frac{1}{c_2^1}-\frac{1}{c_2^0}\right)=\frac{\lambda^1}{ID^1}-\lambda^0=\alpha\,.$$

In fact, all the choices of the household follow this proportionate change except the wife's home activity. For example, the change in a husband's marginal utility of home activity is

$$(39) \qquad b_1\left(\frac{1}{l_1^1}-\frac{1}{l_1^0}\right)=d_1\left(\frac{1}{h_1^1}-\frac{1}{h_1^0}\right)=\left(\frac{\lambda^1}{ID^1}-\lambda^0\right)w_1=\alpha w_1\,.$$

But, from (36), the change in a wife's home activity is less than αw_2. Specifically, since the LHS of the first equality is positive so is the RHS,

$$(40) \qquad \mu d_2\left(\frac{1}{h_2^1}-\frac{1}{h_2^E}\right)=\alpha w_2-\frac{2\left(h_2^E-h_2^1\right)}{T^2}\frac{\left(V_1^1+\mu V_2^1\right)}{ID^1}<\alpha w_2=\mu b_2\left(\frac{1}{l_2^1}-\frac{1}{l_2^0}\right).$$

We thus conclude that the decrease in non-labor income will not reduce the wife's home activity as much as it reduces her leisure. It also implies that it requires a bigger change in her consumption and her husband's consumption, leisure, and home activity. In other words, the household, on the margin, trades off consumption and leisure of both parties against her identity loss. *A pure income change has both an income and substitution effect*.

The husband's welfare after the income change is

$$
(41) \quad \begin{aligned} V_1^1 &= a_1 \log c_1^1 + b_1 \log l_1^1 + d_1 \log h_1^1 = a_1 \log \alpha^* c_1^0 + b_1 \log \alpha^* l_1^0 + d_1 \log \alpha^* h_1^0 \\ &= V_1^0 + (a_1 + b_1 + d_1) \log \alpha^*, \end{aligned}
$$

where $0 < \alpha^* = \dfrac{\lambda^0}{\lambda^1} ID^1 < 1$ and so it decreases by $(a_1 + b_1 + d_1)\log \alpha^*$. On the other hand, the wife's welfare is

$$
(42) \quad \begin{aligned} V_2^1 &= a_2 \log c_2^1 + b_2 \log l_2^1 + d_2 \log h_2^1 = a_2 \log \alpha^* c_2^0 + b_2 \log \alpha^* l_2^0 + d_2 \log \alpha^{**} h_2^0 \\ &= V_2^0 + (a_2 + b_2) \log \alpha^* + d_1 \log \alpha^{**} \end{aligned}
$$

where $0 < \alpha^* < \alpha^{**} = \dfrac{\lambda^0}{\lambda^1 - \dfrac{\left(h_2^E - h_2^1\right)}{T^2} \dfrac{\left(V_1^1 + \mu V_2^1\right)}{w_2}} ID^1 < 1$. Consequently, her indirect welfare reduction is relatively smaller, proportionately, than that of the husband.

In the next proposition we hold other factors constant and increase only non-labor income to grasp whether the impact of a change in non-labor inomce is symmetric.

Proposition 2 *When the household members share an identical identity, an increase in non-labor income to* y^2 *, all else equal, results in increases in members' consumption, leisure, and home activity. However, the proportion of the change in a wife's home activity would be smaller than the proportional change in other choices in the household, as would her welfare relative to the husband's.*

Proposition 2 also indicates that the degree of the change in a wife's home activity is smaller than the proportion of the change in the other choices by the household as *Proposition 1*.[2] Accordingly, as non-labor income increases, the husband can increase his consumption, leisure, and home activity by reducing his market work in some proportion but the wife cannot reduce her market work to the same proportion because of identity deviation. Under this circumstance, the increment in the wife's indirect utility would be relatively smaller than that of her husband:

(43) $V_1^2 = V_1^0 + \left(a_1 + b_1 + d_1\right)\log\beta$ and $V_2^2 = V_2^0 + \left(a_2 + b_2\right)\log\beta + d_2 \log\beta^*$

[2] The detail is explained in Appendix 2.

where $\beta > \beta^*$, and superscript 0 and 2 indicate the cases with non-labor incomes y^0 and y^2, respectively.

To summarize, one of the implications, expressed in *Propositions 1 and 2* is that the household has a wife trading some of her marginal utility of identity (captured in our model by home activity) against both the husband's and wife's marginal utility of other goods. Most importantly, even if wage levels are same and the bargaining weight is even $(\mu = 1)$ the wife's market work does not increase or decrease, proportionately, as much as the husbands.

III.B. Different Identities

We expand our analyses by considering household members having different identities for the wife. When members have different expectation to a wife's home activity the household's maximization problem is

(44) $$H = U_1(c_1, l_1, h_1) ID_1(h_2, h_2^{E_1}) + \mu U_2(c_2, l_2, h_2) ID_2(h_2, h_2^{E_2}).$$

Their money budget constraint remains

(45) $$w_1(T - l_1 - h_1) + w_2(T - l_2 - h_2) + y - c_1 - c_2 = 0,$$

where $h_2^{E_1}$ and $h_2^{E_2}$ are a husband's and wife's identity on a wife's home activity, respectively. The Lagrangian is:

$$(46)\quad £ = \left[a_1 \log c_1 + b_1 \log l_1 + d_1 \log h_1\right]\left[1 - \frac{\left(h_2 - h_2^{E_1}\right)^2}{T^2}\right] + \mu\left[a_2 \log c_2 + b_2 \log l_2 + d_2 \log h_2\right]\left[1 - \frac{\left(h_2 - h_2^{E_2}\right)^2}{T^2}\right]$$
$$+\lambda\left[w_1\left(T - l_1 - h_1\right) + w_2\left(T - l_2 - h_2\right) + y - c_1 - c_2\right].$$

Since a wife and husband have different identities on the wife's home activity $h_2^{E_i}$, any choice of the wife's home activity carries an identity deviation for at least one member of the household, and possibly both.

One possible outcome is for the couple to have equal amounts of identity deviation so $\left|h_2^0 - h_2^{E_1}\right| = \left|h_2^0 - h_2^{E_2}\right| = A$, which requires that h_2^0 be midway between $h_2^{E_1}$ and $h_2^{E_2}$ as shown in Figure I.[3]

[Insert Figure I Here]

We now extend our earlier discussion with Lemma 2 which considers a specific outcome, equivalent in this case to the starting point of Lemma 1

Lemma 2 *There exist values for* w_1, w_2, μ, *and* y^0, *such that a household's optimal choices are* c_1^0, c_2^0, l_1^0, l_2^0, h_1^0, *and* h_2^0 *such that* $ID_1^0 = ID_2^0$.

Subscript 1 stands for a husband and 2 represents a wife while superscript 0 is of the choices with non-labor income y^0. *Lemma 2* supposes a specific environment

[3] For discussion purposes we suppose that that the husband's identity value for his wife's home activity is larger than his wife's.

in which a household's optimal outcomes result in an equal identity deviation to each member.[4] Under *Lemma 2* the following conditions hold:

(47) $$\frac{\partial \pounds}{\partial c_1^0}:\frac{a_1}{c_1^0}ID_1^0=\frac{a_1}{c_1^0}\left[1-\frac{\left(h_2^0-h_2^{E_1}\right)^2}{T^2}\right]=\frac{a_1}{c_1^0}\left[1-\frac{\left(-A\right)^2}{T^2}\right]=\lambda^0$$

(48) $$\frac{\partial \pounds}{\partial l_1^0}:\frac{b_1}{l_1^0}ID_1^0=\frac{b_1}{l_1^0}\left[1-\frac{\left(h_2^0-h_2^{E_1}\right)^2}{T^2}\right]=\frac{b_1}{l_1^0}\left[1-\frac{\left(-A\right)^2}{T^2}\right]=\lambda^0 w_1$$

(49) $$\frac{\partial \pounds}{\partial h_1^0}:\frac{d_1}{h_1^0}ID_1^0=\frac{d_1}{h_1^0}\left[1-\frac{\left(h_2^0-h_2^{E_1}\right)^2}{T^2}\right]=\frac{d_1}{h_1^0}\left[1-\frac{\left(-A\right)^2}{T^2}\right]=\lambda^0 w_1$$

(50) $$\frac{\partial \pounds}{\partial c_2^0}:\mu\frac{a_2}{c_2^0}ID_2^0=\mu\frac{a_2}{c_2^0}\left[1-\frac{\left(h_2^0-h_2^{E_2}\right)^2}{T^2}\right]=\mu\frac{a_2}{c_2^0}\left[1-\frac{\left(A\right)^2}{T^2}\right]=\lambda^0$$

(51) $$\frac{\partial \pounds}{\partial l_2^0}:\mu\frac{b_2}{l_2^0}ID_2^0=\mu\frac{b_2}{l_2^0}\left[1-\frac{\left(h_2^0-h_2^{E_2}\right)^2}{T^2}\right]=\mu\frac{b_2}{l_2^0}\left[1-\frac{\left(A\right)^2}{T^2}\right]=\lambda^0 w_2$$

(52) $$\frac{\partial \pounds}{\partial h_2^0}:V_1^0\left(\frac{2A}{T^2}\right)-\mu V_2^0\left(\frac{2A}{T^2}\right)+\mu\frac{d_2}{h_2^0}ID_2^0=\lambda^0 w_2$$

[4] Such a possible outcome is exemplified in Appendix 3. This can be also interpreted by a distance rule, by which spatial economists consider the cost of distance in doing business.

(53) $$\frac{\partial \pounds}{\partial \lambda^0}: w_1\left(T-l_1^0-h_1^0\right)+w_2\left(T-l_1^0-h_2^0\right)+y-c_1^0-c_1^0=0,$$

where V_1^0 and V_2^0 are members' indirect utilities.

These also provide a benchmark for comparison. Equations (47) through (53) imply

(54) $$c_1^0=w_1\frac{a_1}{d_1}h_1^0;\ \ l_1^0=\frac{b_1}{d_1}h_1^0;\ c_2^0=\frac{a_2\mu ID_2^0}{\left\{V_1^0\left(\frac{2A}{T^2}\right)-\mu V_2^0\left(\frac{2A}{T^2}\right)+\mu\frac{d_2}{h_2^0}ID_2^0\right\}}w_2;$$

$$l_2^0=\frac{b_2\mu ID_2^0}{\left\{V_1^0\left(\frac{2A}{T^2}\right)-\mu V_2^0\left(\frac{2A}{T^2}\right)+\mu\frac{d_2}{h_2^0}ID_2^0\right\}};$$

$$w_1\left(T-\frac{b_1}{d_1}h_1^0-h_1\right)+w_2\left(T-\frac{b_2\mu ID_2^0}{\left\{V_1^0\left(\frac{2A}{T^2}\right)-\mu V_2^0\left(\frac{2A}{T^2}\right)+\mu\frac{d_2}{h_2^0}ID_2^0\right\}}-h_2\right)+y^0$$
$$-w_1\frac{a_1}{d_1}h_1^0-w_2\frac{a_2\mu ID_2^0}{\left\{V_1^0\left(\frac{2A}{T^2}\right)-\mu V_2^0\left(\frac{2A}{T^2}\right)+\mu\frac{d_2}{h_2^0}ID_2^0\right\}}=0;$$

and

$$\mu=\frac{a_1}{a_2}\frac{c_2^0}{c_1^0}=\frac{w_2}{w_1}\left\{\frac{d_1\mu ID_2^0}{\left\{\left(V_1^0\left(\frac{2A}{T^2}\right)-\mu V_2^0\left(\frac{2A}{T^2}\right)\right)h_2^0+d_2\mu ID_2^0\right\}}\right\}\frac{h_2^0}{h_1^0}.$$

Notice the marginal identity disutility is different across members only in terms of sign since $\frac{\partial ID_1^0}{\partial h_2} = -\frac{\partial ID_2^0}{\partial h_2}$. Hence they can be fully offset if $V_1^0 = \mu V_2^0$, in which case the maximizing conditions and the MRSs are the same as under Lemma 1.

But even in this case, where the marginal costs disappear at $h_2^0 = h_2^{E_2} + A = h_2^{E_1} - A$, both parties have lower utility because of identity deviation costs. In addition, the marginal identity deviation cost on utility cannot be fully offset, making the above relations different from those in Lemma 1. As seen in (22), even at the fair level the wife's marginal utility of home activity may additionally include total welfare cost created by marginal identity deviation. What we have is that the wife's internal MRSs are more complicated now, while husband's internal MRSs remains the same.

More importantly, when the environment changes the different identities for the wife play a role in making identity deviation of one of the parties change more. This leads to the Proposition 3:

Proposition 3 *When the household members have different identities, decreasing non-labor income from y^0 with w_1, w_2, and μ held constant, will decrease the husband's consumption, leisure, and home activity by the same proportions. On the other hand, the change in the wife's home activity depends on at how much the husband's and wife's consumption decrease.Regardless of her new level of home activity, the wife's portion of welfare in the household weighted by the bargaining* *weight and discounted by her identity deviation weight decrease more than that of the husband.*

We first show that if non-labor income falls the household cannot maintain all their initial choices while keeping the wife's home activity at the level $\left|h_2 - h_2^{E_i}\right| = A$. At the initial condition before non-labor income changes $y = y^0$, the household's problem is

$$\begin{aligned}(55)\quad \max_{c_i, l_i, h_i} £^0 &= \left[a_1 \log c_1^0 + b_1 \log l_1^0 + d_1 \log h_1^0\right] ID_1^0 + \left[\mu\left(a_2 \log c_2^0 + b_2 \log l_2^0 + d_2 \log h_2^0\right)\right] ID_2^0 \\ &+ \lambda^0 \left[w_1\left(T - l_1^0 - h_1^0\right) + w_2\left(T - l_2^0 - h_2^0\right) + y^0 - c_1^0 - c_2^0\right].\end{aligned}$$

Since we assumed $\left|h_2 - h_2^{E_i}\right| = A$ their identity deviation weights are same across members and marginal identity deviations are different only in the sign so

$$(56)\qquad ID_1^0 = ID_2^0 = ID^* \text{ and } \frac{\partial ID_1^0}{\partial h_2} = -\frac{\partial ID_2^0}{\partial h_2} = \frac{2A}{T^2}.$$

At the initial level the first order conditions give

$$(57)\qquad \frac{a_1}{c_1^0} ID^* = \lambda^0.$$

Suppose then non-labor income changes such that $y^1 = y^0 - k$ but maintain $\left|h_2 - h_2^{E_i}\right| = A$. The new maximization problem is

$$\max_{c_i, l_i, h_i} £^1 = \left[a_1 \log c_1^1 + b_1 \log l_1^1 + d_1 \log h_1^1\right] ID_1^1 + \left[\mu\left(a_2 \log c_2^1 + b_2 \log l_2^1 + d_2 \log h_2^1\right)\right] ID_2^1 + \lambda^1 \left[w_1\left(T - l_1^1 - h_1^1\right) + w_2\left(T - l_2^1 - h_2^1\right) + \left(y^0 - k\right) - c_1^1 - c_2^1\right], \tag{58}$$

where superscript 1 stands for the new optimal choices, after non-labor income decreases by k. Under the assumption that $\left|h_2 - h_2^{E_i}\right| = A$ still, when solving this problem, their identity deviation weights and marginal identity deviations are unchanged so

$$ID_1^1 = ID_2^1 = ID^* \text{ and } \frac{\partial ID_1^1}{\partial h_2} = -\frac{\partial ID_2^1}{\partial h_2} = \frac{2A}{T^2}. \tag{59}$$

The first-order conditions thus give that.

$$\frac{a_1}{c_1^1} ID^* = \lambda^1. \tag{60}$$

The household would choose their initial choices only when $\lambda^0 = \lambda^1$ holds but it is incompatible with the budget constraint reduced by k. Accordingly, the household cannot maintain all the initial outcomes.

On the other hand, the household can maintain her home activity at the initial level even after non-labor income decreases by changing consumption or leisure, or the husband's labor market activity. Keeping the wife's home activity constant, the first-order conditions under each budget condition give the following relations:

(61) $V_1^0\left(\frac{2A}{T^2}\right)-\mu V_2^0\left(\frac{2A}{T^2}\right)+\mu\frac{d_2}{h_2^0}ID^*=\lambda^0 w_2$ and $V_1^1\left(\frac{2A}{T^2}\right)-\mu V_2^1\left(\frac{2A}{T^2}\right)+\mu\frac{d_2}{h_2^1}ID^*=\lambda^1 w_2$,

where V_i^0 and V_i^1 are member i's indirect utilities from initial and new budget constraints, relatively. These relations say that

(62) $$\left(V_1^0-V_1^1\right)\left(\frac{2A}{T^2}\right)-\mu\left(V_2^0-V_2^1\right)\left(\frac{2A}{T^2}\right)+\mu d_2\left\{\frac{1}{h_2^0}-\frac{1}{h_2^1}\right\}ID^*=\left(\lambda^0-\lambda^1\right)w_2,$$

where $h_2^0=h_2^1$ is assumed so the third term on the left hand side equals 0. Since consumption is assumed to be a normal good, $\lambda^0-\lambda^1<0$ which implies that the husband's indirect utility reduces less than the wife's weighted indirect utility:

(63) $$\left(V_1^0-V_1^1\right)<\mu\left(V_2^0-V_2^1\right).$$

Therefore, her home activity at $\left|h_2-h_2^{E_i}\right|=A$ can satisfy their new budget constraint if the wife takes more burden in the distribution of household's welfare.

Next, we prove the first part of *Proposition 3*. Since h_2^1 must differ from at least one of $h_2^{E_i}, i=1,2$, the first-order conditions are

(64) $$\frac{\partial £}{\partial c_1^1}:\frac{a_1}{c_1^1}ID_1^1=\frac{a_1}{c_1^1}\left[1-\frac{\left(h_2^1-h_2^{E_1}\right)^2}{T^2}\right]=\lambda^1$$

(65) $$\frac{\partial \pounds}{\partial l_1^1} = \frac{b_1}{l_1^1} ID_1^1 = \frac{b_1}{l_1^1}\left[1 - \frac{\left(h_2^1 - h_2^{E_1}\right)^2}{T^2}\right] = \lambda^1 w_1$$

(66) $$\frac{\partial \pounds}{\partial h_1^1} : \frac{d_1}{h_1^1} ID_1^1 = \frac{d_1}{h_1^1}\left[1 - \frac{\left(h_2^1 - h_2^{E_1}\right)^2}{T^2}\right] = \lambda^1 w_1$$

(67) $$\frac{\partial \pounds}{\partial c_2^1} : \mu \frac{a_2}{c_2^1} ID_2^1 = \mu \frac{a_2}{c_2^1}\left[1 - \frac{\left(h_2^1 - h_2^{E_2}\right)^2}{T^2}\right] = \lambda^1$$

(68) $$\frac{\partial \pounds}{\partial l_2^1} : \mu \frac{b_2}{l_2^1} ID_2^1 = \mu \frac{b_2}{l_2^1}\left[1 - \frac{\left(h_2^1 - h_2^{E_2}\right)^2}{T^2}\right] = \lambda^1 w_2$$

(69) $$\frac{\partial \pounds}{\partial h_2^1} : \mu \frac{d_2}{h_2^1} ID_2^1 + \left(a_1 \log c_1^1 + b_1 \log l_1^1 + d_1 \log h_1^1\right)\frac{\partial ID_1^1}{\partial h_2^1} + \mu\left(a_2 \log c_2^1 + b_2 \log l_2^1 + d_2 \log h_2^1\right)\frac{\partial ID_2^1}{\partial h_2^1} = \lambda^1 w_2$$

(70) $$\frac{\partial \pounds}{\partial \lambda^1} : w_1\left(T - l_1^1 - h_1^1\right) + w_2\left(T - l_2^1 - h_2^1\right) + y^1 - c_1^1 - c_2^1 = 0 .$$

Equations (64) to (66) say $w_1 = \frac{b_1}{a_1}\frac{c_1^1}{l_1^1} = \frac{d_1}{a_1}\frac{c_1^1}{h_1^1}$ and (48) to (50) say $w_1 = \frac{b_1}{a_1}\frac{c_1^0}{l_1^0} = \frac{d_1}{a_1}\frac{c_1^0}{h_1^0}$ and thus, the husband has the following relations:

(71) $$\frac{c_1^1}{l_1^1} = \frac{c_1^0}{l_1^0} \text{ and } \frac{c_1^1}{h_1^1} = \frac{c_1^0}{h_1^0} .$$

Because consumption is a normal good he would reduce his consumption as non-labor income decreases. From the relations above, his leisure and home activity would also decrease in direct proportion, which implies that there exist $\gamma > 0$ and $0 < \gamma^* < 1$ such that $\frac{\lambda^0}{ID^*} + \gamma = \frac{\lambda^1}{ID_1^1}$ and $\gamma^* = \frac{c_1^1}{c_1^0} = \frac{ID_1^1}{ID_1^0}\frac{\lambda^0}{\lambda^1}$.[5] Accordingly, these relations mean that the husband's internal MRSs are not affected by the reduced non-labor income even when the members have different identities.

However, the wife's home activity does not necessarily decrease in direct proportion when non-labor income decreases. To examine the second part of *Proposition 3*, we need to consider the household's bargaining weight μ as well as the wife's internal MRSs. Because the bargaining weight is assumed to be exogenous at this stage of the household's time allocation, the weight would remain the same even after non-labor income decreases. From (47) and (50), we have

(72)
$$\mu = \frac{a_1}{a_2}\frac{c_2^0}{c_1^0}\frac{ID_1^0}{ID_2^0} = \frac{w_2}{w_1}\left[\frac{\mu d_1 ID_2^0}{\left[V_1^0\left(\frac{2A}{T^2}\right) - \mu V_2^0\left(\frac{2A}{T^2}\right)\right]h_2^0 + \mu d_2 ID_2^0}\right]\frac{h_2^0}{h_1^0},$$

[5] Unlike models without identity deviation weight, the household in our model pays a penalty λ^1 discounted by their different individual identity deviation weights ID_i^1 per unit violation of the money budget constraint. Put differently, because their identities are different, the relation to the budget constraint differ for each person. For example, each member's marginal utility of consumption does not equal to a shadow price λ^1 nor to a same discounted price $\frac{\lambda^1}{ID}$ but to individually discounted prices $\frac{\lambda^1}{ID_i^1}$, where $i = 1, 2$ —we name it "subjective shadow price".

where $\frac{2A}{T^2} = \frac{\partial ID_1^0}{\partial h_2^0} = -\frac{\partial ID_2^0}{\partial h_2^0}$ and $ID_1^0 = ID_2^0 = ID^*$. In addition, (64) and (67) say that the weight is

$$\mu = \frac{a_1}{a_2}\frac{c_2^1}{c_1^1}\frac{ID_1^1}{ID_2^1} = \frac{w_2}{w_1}\frac{\mu d_1 ID_2^1}{\left\{\left[V_1^1 \frac{\partial ID_1^1}{\partial h_2^1} + \mu V_2^1 \frac{\partial ID_2^1}{\partial h_2^1}\right]h_2^1 + \mu d_2 ID_2^1\right\}}\frac{h_2^1}{h_1^1}\frac{ID_1^1}{ID_2^1}. \tag{73}$$

Meanwhile, since consumption is a normal good, the wife's consumption would be reduced as non-labor income decreases. From (50) and (67) and (68) we have

$$\frac{b_2}{a_2 w_2} = \frac{l_2^1}{c_2^1} = \frac{l_2^0}{c_2^0}, \tag{74}$$

which implies that there exist $\delta > 0$ and $0 < \delta^* < 1$ such that $\frac{\lambda^0}{ID^*} + \delta = \frac{\lambda^1}{ID_2^1}$ and $\delta^* = \frac{c_2^1}{c_2^0} = \frac{ID_2^1}{ID_2^0}\frac{\lambda^0}{\lambda^1}$.[6] This in turn imply

$$\frac{\delta^*}{\gamma^*} = \frac{ID_2^1}{ID_1^1} \text{ and } \frac{h_2^1}{h_2^0} = \delta^* \frac{\left\{\left[V_1^1 \frac{\partial ID_1^1}{\partial h_2^1} + \mu V_2^1 \frac{\partial ID_2^1}{\partial h_2^1}\right]h_2^1 + \mu d_2 ID_2^1\right\}}{ID_2^1} \frac{ID_2^0}{\left\{\left[V_1^0\left(\frac{2A}{T^2}\right) - \mu V_2^0\left(\frac{2A}{T^2}\right)\right]h_2^0 + \mu d_2 ID_2^0\right\}}, \tag{75}$$

[6] Due to her individual time constraint $T - l_2^0 = m_2^0 + h_2^0$, where m_2^0 is her initial market work, her new and initial leisure hours should satisfy the following conditions $0 \le l_2^0 \le T$ and $0 \le \delta^* l_2^0 \le \delta^* T \le T$ and thus, δ^* cannot be greater than 1.

which shows that the change in the wife's home activity depends on their rates of change in consumption and welfare distribution change from their identity deviations in the household. We can also say that the household will adjust the wife's home activity to compensate their identity deviations by regulating their consumption. The change in the wife's home activity triggers different changes in the identity deviation weights and also affects the consumption of other goods in a household.

To identify how the wife's outcomes and the household's welfare distribution change, we suppose that her home activity changes by ε. Under the assumption that leisure and home activities are normal goods, a decrease in non-labor income reduces the demand for home activity and leisure hours, which raises hours of market work. This is the "income effect" from a decrease in non-labor income. However, a reduction in the wife's home activity, differently from her husband's, yields changes in individual identity deviation weights, which can be interpreted as the "substitution effect" of non-labor income. Given these effects of non-labor income, we cannot make an unambiguous prediction on ε when the household has different identities and thus, we examine all three possible cases: $\varepsilon = 0$, $\varepsilon > 0$, and $\varepsilon < 0$.[7]

[7] $|\varepsilon| < A$ and $A \le \frac{1}{2}T$. From Equation (75), if one of weight were zero, then the corresponding party's consumption and leisure would be also zero. Therefore, under collective rationality, the household never chooses $\varepsilon = A$, which would

Using the relationship $\frac{\lambda^0}{ID^*}+\delta=\frac{\lambda^1}{ID_2^1}$ in (52) and (69) we have

$$(76)\quad \mu d_2\left(\frac{1}{h_2^1}-\frac{1}{h_2^0}\right)-\frac{\left\{V_1^0\left(\frac{2A}{T^2}\right)-\mu V_2^0\left(\frac{2A}{T^2}\right)\right\}}{ID^*}+\frac{\left\{V_1^1\frac{\partial ID_1^1}{\partial h_2^1}+\mu V_2^1\frac{\partial ID_2^1}{\partial h_2^1}\right\}}{ID_2^1}=\delta w_2>0.$$

Because the household has different identities the second two terms of the left-hand-side of (76) would not disappear even if the household maintains the wife's home activity at the initial level $h_2^0=h_2^1$ (or equivalently, $\varepsilon=0$) in Figure I. We know that the marginal utility of the husband's consumption changes so

$$(77)\qquad a_1\left(\frac{1}{c_1^1}-\frac{1}{c_1^0}\right)=\frac{\lambda^1}{ID_1^1}-\frac{\lambda^0}{ID^*}=\gamma.$$

Since $ID_1^1=ID_2^1=ID*$ and $\frac{\lambda^0}{ID^*}+\gamma=\frac{\lambda^0}{ID^*}+\delta=\frac{\lambda^1}{ID^*}$, we can say $\gamma=\delta$. In addition, (75) says that each member's ratio of new to initial consumptions is the same so $\gamma^*=\delta^*=\frac{\lambda^0}{\lambda^1}$. These relations imply that if the household can absorb the reduction in non-labor income through the husband's and wife's consumption and leisure as well as his home activity she may not change her home activity because

mean one of identity deviation weight is zero and the other one is one if $A=\frac{T}{2}$ and $\varepsilon=-\frac{T}{2}$ so $ID_1^1=\left[1-\left(-\frac{T}{2}-\frac{T}{2}\right)^2/T^2\right]=1$ and $ID_2^1=\left[1-\left(\frac{T}{2}-\frac{T}{2}\right)^2/T^2\right]=0$.

of the cost of identity deviation. If so, her market work would increase by $m_2^1 - m_2^0 = \left(1-\delta^*\right) l_2^0$ which is less than the husband's change of $m_1^1 - m_1^0 = \left(1-\gamma^*\right)\left(l_1^0 + h_1^0\right)$.

The welfare of the husband after the non-labor income changes but her home activity remains is

(78) $$V_1^1 = V_1^0 + \left(a_1 + b_1 + d_1\right)\log \gamma^*$$

and so the husband's welfare decreases by $\left(a_1 + b_1 + d_1\right)\log \gamma^*$. On the other hand, the welfare of the wife is

(79) $$V_2^1 = V_2^0 + \left(a_2 + b_2\right)\log \delta^* + d_2 \log \delta^{**},$$

where $0 < \gamma^* = \delta^* < \delta^{**} = 1$. Consequently, the wife's indirect utility decrease proportionately less than that of the husband. Even so, however, it does not mean that her share of welfare in the household improved because these outcomes are possible only when $\left(V_1^0 - V_1^1\right) < \mu\left(V_2^0 - V_2^1\right)$. That is, although the bargaining weight μ is exogenous at their time allocation stage, μ plays a role in the distribution at the last stage in their decision-making process.

If the condition for the distribution of household's welfare does not hold at such a point, then their identity deviations can no longer remain the same because of different identities. Then, the wife's home activity changes and thus, not only their marginal identity cost but also their identity deviation weights change as well:

(80)
$$\frac{\partial ID_1^1}{\partial h_2^1} \neq \frac{\partial ID_2^1}{\partial h_2^1} \text{ and } ID_1^1 \neq ID_2^1.$$

When her home activity becomes larger than the initial level $h_2^1 > h_2^0$ (or equivalently, $\varepsilon = h_2^0 - h_2^1 = -\varepsilon' < 0$), the difference between the wife's identity and the actual choice increases as seen in Figure II.

[Insert Figure II Here]

Hence her identity deviation weight is now smaller than her initial weight, and smaller than the husband's new weight as well. We have

(81)
$$\left\{1-\frac{(A+\varepsilon')^2}{T^2}\right\} = ID_2^1 < ID^* = \left\{1-\frac{A^2}{T^2}\right\} < ID_1^1 = \left\{1-\frac{(-A+\varepsilon')^2}{T^2}\right\} < 1,$$

where $\varepsilon' = -\varepsilon > 0$. To examine whether this choice is possible for the household, we need to check (75) and (76), again. According to (75), the sign of change in her home activity would determine whose identity deviation weight is higher and their ratio of new consumption. When her home activity changes by $\varepsilon' = -\varepsilon > 0$

(82)
$$\frac{\delta^*}{\gamma^*} = \frac{ID_2^1}{ID_1^1} = \frac{\left\{1-\frac{(A+\varepsilon')^2}{T^2}\right\}}{\left\{1-\frac{(A-\varepsilon')^2}{T^2}\right\}} < 1,$$

$$1<\frac{h_2^1}{h_2^0}=\gamma^*\frac{\left\{\left[V_1^1\left\{\frac{2(A-\varepsilon')}{T^2}\right\}-\mu V_2^1\left\{\frac{2(A+\varepsilon')}{T^2}\right\}\right]h_2^1+\mu d_2 ID_2^1\right\}}{\left\{\left[V_1^0\left(\frac{2A}{T^2}\right)-\mu V_2^0\left(\frac{2A}{T^2}\right)\right]h_2^0+\mu d_2 ID_2^0\right\}}=\gamma^*\frac{\lambda^1 w_2 h_2^1}{\lambda^0 w_2 h_2^0},$$

and thus,

$$0<\delta^*=\gamma^*\frac{ID_2^1}{ID_1^1}=\gamma^*\frac{\left\{1-\frac{(A+\varepsilon')^2}{T^2}\right\}}{\left\{1-\frac{(A-\varepsilon')^2}{T^2}\right\}}<1<\gamma^*\frac{\lambda^1}{\lambda^0}.$$

indicating that her home activity can increase (or equivalently, her market work hours can decrease) because of the different identities even when non-labor income decreases. It is possible if the wife's consumption and leisure decrease at the rate of $1-\delta^*$ to compensate her increase in her home activity and the husband's consumption, leisure, and home activity less decrease at the rate of $1-\gamma^*$, where $\delta^*<\gamma^*$. That is, if she wants to increase her home activity, she would have to convince her husband to give up less of his consumption, leisure, and home activity because $1-\gamma^*$ gets smaller as γ^* increases. In addition, according to (76), we have

(83) $$\mu d_2\left\{\frac{1}{h_2^1}-\frac{1}{h_2^0}\right\}-\delta w_2=\frac{V_1^0\left(\frac{2A}{T^2}\right)-\mu V_2^0\left(\frac{2A}{T^2}\right)}{\left\{1-\frac{A^2}{T^2}\right\}}-\frac{V_1^1\left\{\frac{2(A-\varepsilon')}{T^2}\right\}-\mu V_2^1\left\{\frac{2(A+\varepsilon')}{T^2}\right\}}{\left\{1-\frac{(A+\varepsilon')^2}{T^2}\right\}}<0,$$

which implies that if the household's welfare can increase at the cost of the wife suffering additional identity deviation her home activity would increase. Since the RHS indicates that

$$(84)\qquad \frac{V_1^0\left(\dfrac{\partial ID_1^0}{\partial h_2^0}\right)}{ID_2^0}-\frac{V_1^1\left(\dfrac{\partial ID_1^1}{\partial h_2^1}\right)}{ID_2^1}<\frac{\mu V_2^0\left(\dfrac{\partial ID_2^0}{\partial h_2^0}\right)}{ID_2^0}-\frac{\mu V_2^1\left(\dfrac{\partial ID_2^1}{\partial h_2^1}\right)}{ID_2^1},$$

the wife finds her reduction in deviated-weighted utility is larger than that of the husband's. With different identities, the household might decrease her market work when non-labor income decreases if her home activity can increase the household's welfare at the cost of the wife's identity and for the benefit of husband's identity.

When her home activity becomes less than the initial level $h_2^1 < h_2^0$ (or equivalently, $\varepsilon = h_2^0 - h_2^1 > 0$), the difference between the wife's identity and the actual choice gets smaller as seen in Figure III.

[Insert Figure III Here]

So her identity deviation weight gets larger than her initial weight as well as larger than her husband's new weight;

$$(85)\qquad \left\{1-\frac{\left(-A-\varepsilon\right)^2}{T^2}\right\}=ID_1^1<ID^*=\left\{1-\frac{A^2}{T^2}\right\}<ID_2^1=\left\{1-\frac{\left(A-\varepsilon\right)^2}{T^2}\right\}<1,$$

where $\varepsilon > 0$. According to (75), it is possible to reduce her home activity when the husband reduces his consumption, leisure, and home activity at more rate of $1-\gamma^*$, so

(86) $$\frac{\delta^*}{\gamma^*} = \frac{ID_2^1}{ID_1^1} = \frac{\left\{1 - \frac{(A-\varepsilon)^2}{T^2}\right\}}{\left\{1 - \frac{(A+\varepsilon)^2}{T^2}\right\}} > 1.$$

Recall from equation (75) that

(87) $$\delta^* = \frac{h_2^1}{h_2^0} = \delta^* \frac{\left\{\left[V_1^1 \frac{\partial ID_1^1}{\partial h_2^1} + \mu V_2^1 \frac{\partial ID_2^1}{\partial h_2^1}\right] h_2^1 + \mu d_2 ID_2^1\right\}}{ID_2^1} \frac{ID_2^0}{\left\{\left[V_1^0 \left(\frac{2A}{T^2}\right) - \mu V_2^0 \left(\frac{2A}{T^2}\right)\right] h_2^0 + \mu d_2 ID_2^0\right\}},$$

which implies that

(88) $$\frac{\left[V_1^1 \frac{\partial ID_1^1}{\partial h_2^1} + \mu V_2^1 \frac{\partial ID_2^1}{\partial h_2^1}\right]}{ID_2^1} \delta^* = \frac{\left[V_1^0 \left(\frac{2A}{T^2}\right) - \mu V_2^0 \left(\frac{2A}{T^2}\right)\right]}{ID_2^0}.$$

Using the relationship $\frac{\lambda^0}{ID^*} + \delta = \frac{\lambda^1}{ID_2^1}$ in (76), we have, again, that

(89) $$\mu d_2 \left(\frac{1}{h_2^1} - \frac{1}{h_2^0}\right) - \delta w_2 = \frac{\left\{V_1^0 \left(\frac{2A}{T^2}\right) - \mu V_2^0 \left(\frac{2A}{T^2}\right)\right\}}{ID^*} - \frac{\left\{V_1^1 \frac{\partial ID_1^1}{\partial h_2^1} + \mu V_2^1 \frac{\partial ID_2^1}{\partial h_2^1}\right\}}{ID_2^1} < 0$$

and thus, the reduction in her weighted indirect utility is also larger than the husband's when being discounted by her identity deviation weight and weighted by the bargaining weight.

One implication of *Proposition 3* is that when a household has different identities, the substitution effect may dominate the income effect, so that the wife decreases her market work hours even when non-labor income decreases. The increase, maintenance, or decrease in the wife's home activity (or equivalently, her market work) depends on the relative reduction rates of the husband's and wife's consumption because the change in the wife's home activity accompanies additional identity cost to one member, and that member needs to be compensated. However, in any case the reduction in her indirect utility is always larger than that of the husband regardless of her new home activity level h_2^1. This discussion is summarized in Table I.

[Insert Table I Here]

IV. CONCLUSION

Our study shows how identity issues—narrowly defined as the expectation toward a wife's home activity— affects a household's time allocation irrespective of income, wage differentials, or bargaining power by incorporating the concept of identity of Akerlof and Kranton (2000) into Chiappori's collective model of household decision making (1988, 1992). Under the assumptions of a joint budget constraint and that household utility consists of individual utilities weighted by identity deviation and bargaining power, our model shows how identities adds an additional dimension to the trade-offs individuals make in the household decision.

Our fundamental result is that when household members have an identity for the wife, a pure exogenous income change can have both income and substitution, both of which can affect the distribution of welfare. Our main findings are as follows: 1) If a household shares identical identity on a wife's home activity, then the income effect of non–labor income always dominates the substitution effect, but the rate of change in the wife's home activity would be less than that of the husband's; and 2) if the husband and wife have different views of the wife's identity, the wife will trade off her identity deviation of home activity with her consumption and leisure. In this case it is possible for the substitution effect to dominate the income effect, which means the impact on the wife's home activity (and thus on her market work) is an empirical issue. In all cases, when identities differ, most of the welfare burden from different identities falls on the wife.

The second case, when husbands and wives have different expectations about a wife's role in the household (and hence in the labor market), can have import for social policy. For example, in-kind transfers may generally be viewed as (exogenous) changes in income. We have shown that exogenous income changes can have a substitution effect that diminishes wives' labor market participation. Hence, in-kind transfers like subsidized childcare, normally viewed as supporting wives taking on market work may perversely reduce their labor market participation.

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APPENDIX 1: LEMMA 1

To find values that support *Lemma 1* we and solve the following problem:

$$\max_{c_i, l_i, h_i} \pounds^0 = \left[\left(a_1 \log c_1^0 + b_1 \log l_1^0 + d_1 \log h_1^0 \right) ID_1^0 + \mu \left(a_2 \log c_2^0 + b_2 \log l_2^0 + d_2 \log h_2^E \right) ID_2^0 \right]$$
$$+ \lambda^0 \left[w_1 \left(T - l_1^0 - h_1^0 \right) + w_2 \left(T - l_2^0 - h_2^E \right) + y^0 - c_1^0 - c_2^0 \right].$$

where the superscript 0 indicates for the choices with non-labor income y^0. We have $ID^0 = ID_1^0 = ID_2^0$ and impose the condition that $h_2^0 = h_2^{E_1} = h_2^{E_2} = h_2^E$. Taking the first-order conditions and imposing the value for h_2^0 gives

$$\frac{\partial \pounds}{\partial c_1^0} : \frac{a_1}{c_1^0} ID^0 = \lambda^0$$

$$\frac{\partial \pounds}{\partial c_2^0} : \frac{\mu a_2}{c_2^0} ID^0 = \lambda^0$$

$$\frac{\partial \pounds}{\partial l_1^0} : \frac{b_1}{l_1^0} ID^0 = \lambda^0 w_1$$

$$\frac{\partial \pounds}{\partial l_2^0} : \frac{\mu b_2}{l_2^0} ID^0 = \lambda^0 w_2$$

$$\frac{\partial \pounds}{\partial h_1^0} : \frac{d_1}{h_1^0} ID^0 = \lambda^0 w_1$$

$$\frac{\partial \pounds}{\partial h_2^0} = \mu \frac{d_2}{h_2^E} ID^0 = \lambda^0 w_2$$

$$\frac{\partial \pounds}{\partial \lambda^0} = w_1 \left(T - l_1^0 - h_1^0 \right) + w_2 \left(T - l_2^0 - h_2^E \right) + y^0 - c_1^0 - c_2^0 = 0$$

Solving these gives

$$l_1^0 = \frac{b_1}{a_1} \frac{c_1}{w_1}; \; h_1^0 = \frac{d_1}{a_1} \frac{c_1^0}{w_1}; \; c_2^0 = \mu \frac{a_2}{a_1} c_1^0; \;\; l_2^0 = \mu \frac{b_2}{a_1} \frac{c_1^0}{w_2};$$

$$h_2^E = \mu \frac{d_2}{a_1} \frac{c_1^0}{w_2}$$

$$w_1 \left(T - \frac{b_1}{a_1} \frac{c_1}{w_1} - \frac{d_1}{a_1} \frac{c_1^0}{w_1} \right) + w_2 \left(T - \mu \frac{b_2}{a_1} \frac{c_1^0}{w_2} - \mu \frac{d_2}{a_1} \frac{c_1^0}{w_2} \right) + y^0 = c_1^0 + \mu \frac{a_2}{a_1} c_1^0$$

$$y^0 = c_1^0 + \mu \frac{a_2}{a_1} c_1^0 - w_1 \left(T - \frac{b_1}{a_1} \frac{c_1}{w_1} - \frac{d_1}{a_1} \frac{c_1^0}{w_1} \right) - w_2 \left(T - \mu \frac{b_2}{a_1} \frac{c_1^0}{w_2} - \mu \frac{d_2}{a_1} \frac{c_1^0}{w_2} \right)$$

$$y^0 = \frac{a_1 + \mu a_2 + b_1 + d_1 + \mu b_2 + \mu d_2}{a_1} - w_1 T - w_2 T$$

$$y^0 = \left(\frac{a_1 + b_1 + d_1 + \mu \left(a_2 + b_2 + d_2 \right)}{a_1} \right) c_1^0 - \left(w_1 + w_2 \right) T$$

$$= \left(\frac{a_1 + b_1 + d_1 + \mu \left(a_2 + b_2 + d_2 \right)}{a_1} \right) \frac{a_1}{d_2} \frac{w_2}{\mu} h_2^E - \left(w_1 + w_2 \right) T$$

This last equation defines a locus of points between y^0 and c_1^0 or h_2^E. Any point along this locus for which $0 \leq h_2^E \leq T$ provides a value for y^0 that supports *Lemma 1*. Note that there is no requirement that non-labor income is positive. If $a_1 + b_1 + d_1 + \mu \left(a_2 + b_2 + d_2 \right) < 2d_2 \left(w_1 + w_2 \right) \frac{\mu}{w_2}$ when $h_2^E = \frac{T}{2}$, for example, then non-labor income is negative $y^0 < 0$.

APPENDIX 2: PROPOSITION 2

The Lagrangian is

$$\max_{c_i, l_i, h_i} £^2 = \left[\left(a_1 \log c_1^2 + b_1 \log l_1^2 + d_1 \log h_1^2 \right) + \mu \left(a_2 \log c_2^2 + b_2 \log l_2^2 + d_2 \log h_2^2 \right) \right] ID^2$$
$$+ \lambda^2 \left[w_1 \left(T - l_1^2 - h_1^2 \right) + w_2 \left(T - l_2^2 - h_2^2 \right) + y^2 - c_1^2 - c_2^2 \right]$$

where the superscript 2 indicates for the choices with increased non-labor income y^2. We have $ID^2 = ID_1^2 = ID_2^2$ and impose the condition that $h_2^2 = h_2^0 + \varepsilon = h_2^E + \varepsilon,$ where $h_2^0 = h_2^{E_1} = h_2^{E_2} = h_2^E$ and $\varepsilon > 0$. The first-order conditions are

$$\frac{\partial £}{\partial c_1^2} : \frac{a_1}{c_1^2} ID^2 = \lambda^2$$

$$\frac{\partial £}{\partial l_1^2} : \frac{b_1}{l_1^2} ID^2 = \lambda^2 w_1$$

$$\frac{\partial £}{\partial h_1^2} : \frac{d_1}{h_1^2} ID^2 = \lambda^2 w_1$$

$$\frac{\partial £}{\partial c_2^2} : \mu \frac{a_2}{c_2^2} ID^2 = \lambda^2$$

$$\frac{\partial £}{\partial l_2^2} : \mu \frac{b_2}{l_2^2} ID^2 = \lambda^2 w_2$$

$$\frac{\partial £}{\partial h_2^2} : \mu \frac{d_2}{h_2^2} ID^2 + \left(V_1^2 + \mu V_2^2\right) \frac{\partial ID^2}{\partial h_2^2} = \lambda^2 w_2$$

$$\frac{\partial £}{\partial \lambda^0} : w_1 \left(T - l_1^2 - h_1^2\right) + w_2 \left(T - l_2^2 - h_2^2\right) + y^2 - c_1^2 - c_2^2 = 0$$

where $V_1^2 = a_1 \log c_1^2 + b_1 \log l_1^2 + d_1 \log h_1^2$ and $V_2^2 = a_2 \log c_2^2 + b_2 \log l_2^2 + d_2 \log h_2^2$.

Solving these gives

$$l_1^2 = \frac{b_1}{a_1} \frac{c_1^2}{w_1};\ h_1^2 = \frac{d_1}{a_1} \frac{c_1^2}{w_1};\ c_2^2 = \mu \frac{a_2}{a_1} c_1^2;\ l_2^2 = \mu \frac{b_2}{a_1} \frac{c_1^2}{w_2};$$

$$h_2^E + \varepsilon = \frac{d_2 \mu \left(T^2 - \varepsilon^2\right) c_1^2}{a_1 w_2 \left(T^2 - \varepsilon^2\right) + 2\varepsilon \left(V_1^2 + \mu V_2^2\right) c_1^2}$$

$$\Rightarrow c_1^2 = \frac{a_1 w_2 \left(h_2^E + \varepsilon\right)\left(T^2 - \varepsilon^2\right)}{\mu d_2 \left(T^2 - \varepsilon^2\right) - 2\varepsilon \left(h_2^E + \varepsilon\right)\left(V_1^2 + \mu V_2^2\right)};$$

$$w_1\left(T-\frac{b_1}{a_1}\frac{c_1^2}{w_1}-\frac{d_1}{a_1}\frac{c_1^2}{w_1}\right)+w_2\left(T-\mu\frac{b_2}{a_1}\frac{c_1^2}{w_2}-\left(\frac{d_2\mu\left(T^2-\varepsilon^2\right)c_1^2}{a_1w_2\left(T^2-\varepsilon^2\right)+2\varepsilon\left(V_1^2+\mu V_2^2\right)c_1^2}\right)\right)$$

$$+y^2-c_1^2-\mu\frac{a_2}{a_1}c_1^2=0$$

$$y^2=\left\{\frac{a_1+b_1+d_1+\mu a_2+\mu b_2}{a_1}\right\}c_1^2+\left(\frac{d_2\mu w_2\left(T^2-\varepsilon^2\right)c_1^2}{a_1w_2\left(T^2-\varepsilon^2\right)+2\varepsilon\left(V_1^2+\mu V_2^2\right)c_1^2}\right)$$

$$-\left(w_1+w_2\right)T$$

$$=\left\{\frac{a_1+b_1+d_1+\mu a_2+\mu b_2}{a_1}\right\}\left\{\frac{a_1w_2\left(h_2^E+\varepsilon\right)\left(T^2-\varepsilon^2\right)}{\mu d_2\left(T^2-\varepsilon^2\right)-2\varepsilon\left(h_2^E+\varepsilon\right)\left(V_1^2+\mu V_2^2\right)}\right\}$$

$$+\left(h_2^E+\varepsilon\right)w_2-\left(w_1+w_2\right)T$$

This last equation defines a locus of points between y^2 and $\left\{h_2^E+\varepsilon\right\}$ given the values of the other parameters. Any point along this locus for which $0\le h_2^E\le T-\varepsilon$ provides a value for y^2 that supports *Proposition 2*.

APPENDIX 3: LEMMA 2

The Lagrangian is

$$\max_{c_i,l_i,h_1}£^0=\left[\left(a_1\log c_1^0+b_1\log l_1^0+d_1\log h_1^0\right)ID_1^0+\mu\left(a_2\log c_2^0+b_2\log l_2^0+d_2\log h_2\right)ID_2^0\right]$$

$$+\lambda^0\left[w_1\left(T-l_1^0-h_1^0\right)+w_2\left(T-l_2^0-h_2\right)+y^0-c_1^0-c_2^0\right],$$

where the superscript 0 indicates for the choices with non-labor income y^0. The first-order conditions assuming are

$$\frac{\partial £}{\partial c_1^0}:\frac{a_1}{c_1^0}ID_1^0=\lambda^0$$

$$\frac{\partial £}{\partial l_1^0}:\frac{b_1}{l_1^0}ID_1^0=\lambda^0 w_1$$

$$\frac{\partial \pounds}{\partial h_1^0}: \frac{d_1}{h_1^0} ID_1^0 = \lambda^0 w_1$$

$$\frac{\partial \pounds}{\partial c_2^0}: \mu \frac{a_2}{c_2^0} ID_2^0 = \lambda^0$$

$$\frac{\partial \pounds}{\partial l_2^0}: \mu \frac{b_2}{l_2^0} ID_2^0 = \lambda^0 w_2$$

$$\frac{\partial \pounds}{\partial h_2^0}: \mu \frac{d_2}{h_2^0} ID_2^0 = \lambda^0 w_2$$

$$\frac{\partial \pounds}{\partial \lambda^0}: w_1 \left(T - l_1 - h_1\right) + w_2 \left(T - l_2 - \frac{h_2^{E_1} - h_2^{E_2}}{2}\right) + y^0 - c_1 - c_2 = 0 .$$

We are looking for an optimal distribution of non-labor income when the distance between their identities is $2A$ and the solution is midway between their desires, that is, $h_2^0 = \frac{h_2^{E_1} - h_2^{E_2}}{2}$ where without loss of generality we assume $h_2^{E_1} > h_2^{E_2}$. Given that $h_2^0 = \frac{h_2^{E_1} - h_2^{E_2}}{2}$ and $|A| = \frac{\left|h_2^{E_1} - h_2^{E_2}\right|}{2}$, we have $\left\{1 - \frac{\left(h_2^{E_1} - h_2\right)^2}{T^2}\right\} = ID_1^0 = ID^* = ID_2^0 = \left\{1 - \frac{\left(h_2^{E_2} - h_2\right)^2}{T^2}\right\}$. Then the first-order conditions can be written

$$\frac{\partial \pounds}{\partial c_1^0}: \frac{a_1}{c_1^0} ID^* = \lambda^0$$

$$\frac{\partial \pounds}{\partial l_1^0}: \frac{b_1}{l_1^0} ID^* = \lambda^0 w_1$$

$$\frac{\partial \pounds}{\partial h_1^0}: \frac{d_1}{h_1^0} ID^* = \lambda^0 w_1$$

$$\frac{\partial £}{\partial c_2^0} : \mu \frac{a_2}{c_2^0} ID^* = \lambda^0$$

$$\frac{\partial £}{\partial l_2^0} : \mu \frac{b_2}{l_2^0} ID^* = \lambda^0 w_2$$

$$\frac{\partial £}{\partial h_2^0} : \mu \frac{d_2}{\left\{\frac{h_2^{E_1} - h_2^{E_2}}{2}\right\}} ID^* = \lambda^0 w_2$$

$$\frac{\partial £}{\partial \lambda^0} : w_1\left(T - l_1 - h_1\right) + w_2\left(T - l_2 - \frac{h_2^{E_1} - h_2^{E_2}}{2}\right) + y^0 - c_1 - c_2 = 0 .$$

Solving these gives

$$l_1^0 = \frac{b_1}{a_1}\frac{c_1^0}{w_1};\ h_1^0 = \frac{d_1}{a_1}\frac{c_1^0}{w_1};\ c_2^0 = \mu\frac{a_2}{a_1}c_1^0;\ l_2^0 = \mu\frac{b_2}{a_1}\frac{c_1^0}{w_2};$$

$$\frac{h_2^{E_1} - h_2^{E_2}}{2} = \mu\frac{d_2}{a_1}\frac{c_1^0}{w_2} \Rightarrow c_1^0 = \left\{\frac{h_2^{E_1} - h_2^{E_2}}{2}\right\}\frac{a_1}{d_2}\frac{w_2}{\mu};$$

$$w_1\left(T - \frac{b_1}{a_1}\frac{c_1^0}{w_1} - \frac{d_1}{a_1}\frac{c_1^0}{w_1}\right) + w_2\left(T - \mu\frac{b_2}{a_1}\frac{c_1^0}{w_2} - \mu\frac{d_2}{a_1}\frac{c_1^0}{w_2}\right) + y^0 - c_1^0 - \mu\frac{a_2}{a_1}c_1^0 = 0$$

$$y^0 = \left\{\frac{\left(a_1 + b_1 + d_1 + \mu a_2 + \mu b_2 + \mu d_2\right)}{a_1}\right\}c_1^0 - \left(w_1 + w_2\right)T$$

$$= \frac{\left(a_1 + b_1 + d_1 + \mu a_2 + \mu b_2 + \mu d_2\right)}{d_2}\left\{\frac{h_2^{E_1} - h_2^{E_2}}{2}\right\}\frac{w_2}{\mu} - \left(w_1 + w_2\right)T.$$

This last equation defines a locus of points between y^0 and $\left\{\frac{h_2^{E_1} - h_2^{E_2}}{2}\right\}$ given the values of the other parameters. Given $0 \le h_2^0 = \left\{\frac{h_2^{E_1} - h_2^{E_2}}{2}\right\} \le T$, any points along this locus for

$$\log\left\{\frac{h_2^{E_1}-h_2^{E_2}}{2}\right\}=\frac{\left\{a_1\log c_1^0+b_1\log l_1^0+d_1\log h_1^0-a_2\log c_2^0-b_2\log l_2^0\right\}}{d_2}$$

—which implies that the wife's choice can offset their additional marginal effect by $V_1^0\frac{\partial ID_1^0}{\partial h_2^0}=V_2^0\frac{\partial ID_2^0}{\partial h_2^0}$—provides a value for y^0 that supports *Lemma 2*.

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TABLE I: SUMMARY OF THE HOUSEHOLD'S NEW POSSIBLE OUTCOMES AND WELFARE DISTRIBUTION

	$h_2^0 = h_2^1$	$h_2^0 < h_2^1$	$h_2^0 > h_2^1$
The rates of the decrease in consumption	$0 < \delta^* = \gamma^* < 1$	$0 < \delta^* < \gamma^* < 1$	$0 < \gamma^* < \delta^* < 1$
Identity deviation weights	$ID^* = ID_1^1 = ID_2^1$	$ID_2^1 < ID^* < ID_1^1$	$ID_1^1 < ID^* < ID_2^1$
New consumption level of a husband	$c_1^1 = \gamma^* c_1^0$	$c_1^1 = \gamma^* c_1^0$	$c_1^1 = \gamma^* c_1^0$
New consumption level of a wife	$c_2^1 = \delta^* c_2^0$	$c_2^1 = \delta^* c_2^0$	$c_2^1 = \delta^* c_2^0$
New domestic work level of a wife	$h_2^1 = h_2^0$	$h_2^1 = \frac{1}{\delta^*} h_2^0$	$h_2^1 = \delta^* h_2^0$
The new welfare of a husband	$V_1^1 = V_1^0 + (a_1 + b_1 + d_1)\log\gamma^*$	$V_1^1 = V_1^0 + (a_1 + b_1 + d_1)\log\gamma^*$	$V_1^1 = V_1^0 + (a_1 + b_1 + d_1)\log\gamma^*$
The new welfare of a wife	$V_2^1 = V_2^0 + (a_2 + b_2)\log\delta^*$	$V_2^1 = V_2^0 + (a_2 + b_2 - d_2)\log\delta^*$	$V_2^1 = V_2^0 + (a_2 + b_2 + d_2)\log\delta^*$
The change in a wife's market work	$m_2^1 - m_2^0 = (1-\delta^*)l_2^0$	$m_2^1 - m_2^0 = (1-\delta^*)l_2^0 + \left(1 - \frac{1}{\delta^*}\right)h_2^0$	$m_2^1 - m_2^0 = (1-\delta^*)(l_2^0 + h_2^0)$

The welfare distribution	$\frac{V_1^0\left(\frac{\partial ID_1^0}{\partial h_2^0}\right)}{ID_2^0}-\frac{V_1^1\left(\frac{\partial ID_1^1}{\partial h_2^1}\right)}{ID_2^1}<\frac{\mu V_2^0\left(\frac{\partial ID_2^0}{\partial h_2^0}\right)}{ID_2^0}-\frac{\mu V_2^1\left(\frac{\partial ID_2^1}{\partial h_2^1}\right)}{ID_2^1}$

FIGURE I: DIFFERENT IDENTITIES AND THE WIFE'S CHOICE AT THE FAIR LEVEL

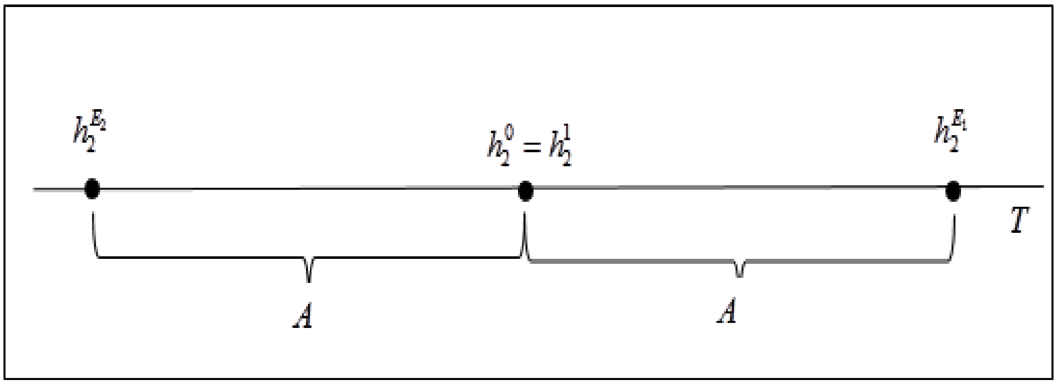

FIGURE II: DIFFERENT IDENTITIES AND THE INCREASE IN A WIFE'S HOME ACTIVITY

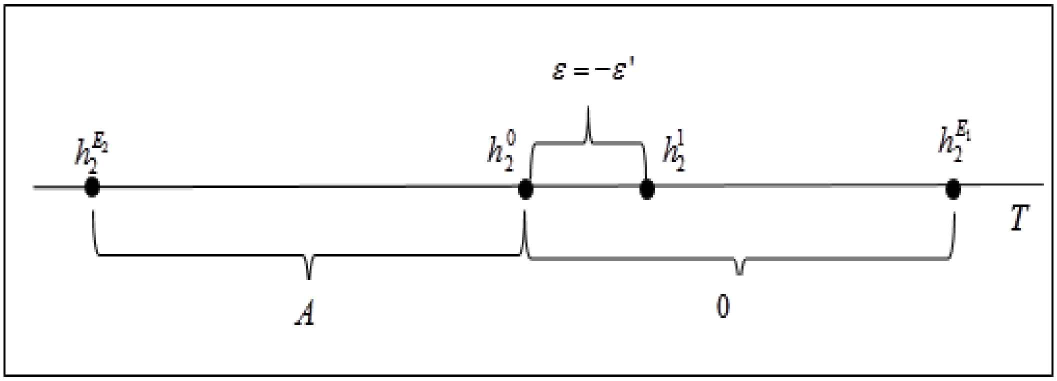

FIGURE III: DIFFERENT IDENTITIES AND THE DECREASE IN A WIFE'S HOME ACTIVITY