Rationalizing Time Inconsistent Behavior: The Case of Late Payments

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Abstract

Consumers often sign contracts in which they consume a good over a period of time, paying for it with a fee due at a later period. Although contracts normally specify that customers are expected to pay the bill on time, and most impose a penalty if the fee is paid late, many customers pay late, despite anecdotal evidence that customers intend to pay on time. In this paper, we find that such behavior, which we characterize as a preference reversal, can be explained by present bias, but only under restrictive parameter conditions. However, allowing for “memory” shocks (consumers forget a bill is due) helps rationalize such behavior for less restrictive parameter values. We further show how a seller can increase profits by setting fees and penalties that lead consumers to fall prey to preference reversals over time.

Keywords: Contracts, Penalties, Present Bias, Shocks, Memory Loss.
JEL Classification: D01, D03, D21, D84, D86.

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1 Introduction

Consumers often sign contracts in which they consume a good over a period of time, paying for it with a fee due at a later period. Examples include such goods as cable TV, internet, and cell phone. Although contracts normally specify that customers are expected to pay the bill on time, and most impose a penalty if the fee is paid late, many customers pay late, despite anecdotal evidence that customers intend to pay on time, and can afford to. In the U.S., for instance, 28.4 million households pay at least one bill late per month (Wall Street Journal 3/15/2007), and penalties for late payment increased from $7 billion in 2000 to $22 billion in 2004 (Wall Street Journal April 2004). In the same line, according to the Citi Simplicity Survey (2013) 59% of Americans have paid a bill late in their lifetime (including credit card, utility, cable, etc.), and 88% of those have done so in the past 12 months. Interestingly, the most common reason that consumers use to justify their late payments is forgetfulness (61%), and being busy with work and family obligations (39%). Lack of available funds (42%), while significant, is not the main reason a bill is paid late.

From these reports, it appears people who could pay on time, and sign a contract expecting so do so, often end up paying their bills late. Hence, while some of this consumer behavior can be attributed to financial reasons, it may be that often individuals are simply so busy with family and work that they forget to pay their bills. In this paper, we present two different explanations for such behavior. We also identify the conditions under which a seller can use such consumer behavior by strategically designing fees and penalties to exploit late payments and increase profits. Our paper seeks to answer two questions:

1. Why do consumers sign contracts anticipating paying them on time, but when payment is due, choose to not pay, thus exhibiting dynamically inconsistent behavior and incurring additional penalties? and,

2. Under what conditions can sellers increase profit by strategically designing fees and penalties that induce consumers to pay their bills late?

While considering the first question, an immediate answer is that consumers exhibit present bias. One explanation for present bias is hyperbolic discounting (Laibson, 1997, O’Donoghue and Rabin, 1999). When consumers care more about the present than the future, it can lead them to pay bills late. However, this does not provide a complete answer to the question. In particular, as the due date for a bill approaches, there is a small time difference between the current and the future (when consumers are penalized). For this small time period, we show that present bias can only explain late bill payments under restrictive parameter conditions.1 Hence, as a further explanation we hypothesize that some consumers pay their bill late due to nonfinancial “memory” shocks, such as forgetting or family obligations, which, according to the above Citi Survey, account for 61% and 39%, respectively of why payments are late.

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1 Another possibility is procrastination. While procrastination can result from hyperbolic discounting (Murooka and Schwarz, 2016), Herweg and Muller (2011) establish that procrastination can be viewed as rational behavior when the cost of paying today outweighs the costs of paying the penalty.
There is evidence that consumers, over time, often forget contract details, including due dates and late penalties. Decreasing attention abilities have been well documented in the psychological literature since Ebbinghaus (1885) and Craik and Lockhart (1972) both in short and long periods of time (for a survey of this literature see Schacter, 1999). More recently, Ericson (2011) experimentally shows that individuals suffer from overconfidence in their own memory, that is, they overestimate their ability to remember future events. As a result of this, they are unwilling to incur costs (such as setting a reminder) required to remember due dates. Consumers’ overconfidence in their own ability to remember future tasks can, hence, affect their paying bills on time and avoiding late payment penalties in a contract.

Lack of knowledge or understanding of late payment penalties can also be characterized as memory shocks. There is good evidence that consumers do not read contracts carefully. According to Smithers (2011), for instance, 93% of British citizens recognize that they did not carefully read terms and conditions before signing up online for a product or a service. Tugend (2013) finds that software contracts are, on average, 74,000 words long (similar in length to the first Harry Potter book), leading most customers to click on the “I Agree” button to accept the contract without reading it. Moreover, playing off of Herweg and Muller’s (2011) “rational” procrastination when the costs of paying on time exceed the costs of paying late fees, and recognizing that costs are not always monetary, as shown in O’Donoghue and Rabin (2001), procrastination can be one of the behavioral factors causing the shock we consider.

To answer the second question, we consider how consumer behavior can be exploited by firms. In a market where the probability of a memory shock is high, the seller can increase her profit by setting stringent penalties for late payment. However, if the probability of a shock is low, the seller may keep penalties so low that present bias alone drives all individuals to pay late. In essence, the seller’s choice is between a low penalty that extracts a small surplus from all individuals, and a high penalty that extracts a late payment penalty from only a few customers. Our findings suggest that, under large parameter conditions, sellers optimally charge a high penalty, thus fully exploiting those individuals who suffer a shock. This analysis is consistent with Chetty (2015) who shows how firms can extract welfare from present biased consumers, but goes beyond because our analysis does not rely on present biasedness, instead focusing on shocks.

Firms’ optimizing penalties can be substantial, consistent with what is observed in the market.

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2 In the experiment, subjects had to choose between receiving a large payment, conditional on them remembering to claim their payment with a six-month delay, or a smaller payment that would automatically be sent to them after six months. Ericson (2011) finds that, while three quarters of the subjects choose the large payment and said they would remember to claim their payment after six months, only half of them claimed it, thus reflecting overconfidence in their own memory.

3 There is good empirical evidence that reminders help people meet their commitments when the reminders came at no set-up cost to the consumers. For instance, Karlan et al. (2016) examine data from three banks in Peru, Bolivia and the Philippines, and show that, among people who have recently opened a savings account, reminders increase the probability of meeting their commitments. Similarly, Calzolari and Nardotto (2012) conducted a field experiment on a sample of individuals joining a gymnasium, and found that a weekly email reminder increases attendance by up to 25%.

4 In the U.S., for example, Time Warner Cable (TV services) and Verizon (phone) charge a penalty of up to 1.5% of the monthly fee in case of late payment, i.e., $K = 0.015$. Penalties are larger in other countries; Reliance India
Hence, we extend our analysis by evaluating how legal limits on the penalty can improve the welfare of consumers who suffer a shock, without inducing others to stop honoring the contract. In this effort we support Armstrong and Vickers (2012), who discuss the importance of consumer protection on contingent charges, which play a similar role as penalties in our model, although Armstrong, Vickers and Zhou (2009) point out that excessive consumer protection can have undesirable welfare effects.

Section 2 describes the model, and section 3 presents the consumer’s problem for each of our behavioral settings. In section 4 we examine the seller’s problem for each type of consumer he may face. Section 5 discusses our main conclusions and policy implications.

2 Model

Consider an individual who discounts future payoffs by a discount factor $\delta \in (0,1]$, and exhibits a present bias parameter $\beta \in [0,1]$. Present bias is absent if $\beta = 1$; as in Liabson (1997). A monopolist offers a contract at time $t = 0$. The consumer can choose to either sign it or not. If he signs the contract, he gets one unit of the good in every period $t$ thereafter. This yields him a utility of $u_t$ in each period. He pays for this service at a later time $t = n$. This setting embodies as special cases contracts in which payment is due immediately after signing (i.e. $n = 1$) as well as those allowing the customer to enjoy the good for multiple periods by having to pay at a "due date" period $n$ (e.g., $n = 30$ for monthly). Examples for such goods are cable, internet and cell phone services. The bill is repeated a total on $zi$ where $i \geq 1$.

For generality, we assume that, if the individual is late in his payment at $t = xn$ where $x \in \{1,2,...,z\}$, he pays in the subsequent period $xn + k$, where $k \geq 1$. If he pays one period late, the fee $F$ increases to $K^1F$, where penalty $K^1$ satisfies $K^1 > 1$. Similarly, if he is $k$ periods late, he pays a total fee of $K^kF$, thus indicating that penalties increase in time. We also allow the service to be suspended if the bill is not paid $S$ periods after being due, i.e., if $k > S$. In order to guarantee that the individual pays by $S$, we assume that the penalty he would suffer at $S$, denoted by $C$, is sufficiently large to ensure no one pays beyond $S - 1$. Hence, if the the buyer has not already paid, he pays the bill in period $S - 1$ even if he suffers a shock in that period. Any late payment is either due to present bias or due to shocks like forgetfulness or being too busy; rather than financial shocks that prevent the customer to pay his bill. This model assumes every customer has the financial ability from paying the amount in any period. The probability of such a shock occuring is given by $0 \leq q \leq 1$. We assume that shocks are i.i.d. in every period, and their effects last only for one period. In our paper, we first analyze the case without shocks, i.e. $q = 0$, and then extend our results to a setting with shocks, $q > 0$. Applying backward induction, we start by

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5 In the U.S., for instance, Time Warner Cable (TV services) and Verizon (phone) charge a penalty of up to 1.5% of the monthly fee in case of late payment, i.e., $K^1 = 1.015$. These penalties are larger in other countries. Reliance India Mobile, for example, charges a late penalty equivalent to 2.5% of the monthly fee in case of late payment.

6 The analytical solution for $S$ is shown in the proof of Lemma 1.
solving the consumer’s problem (whether he signs the contract and, if so, when does he pay) and then move on to the seller (designing the optimal contract).

3 Consumer’s problem

Lemma 1 (No Shocks.) When the buyer suffers no shocks, $q = 0$, he expects, at any period $t$, to pay on time the bill that is due at the end of that billing cycle (rather than paying $k$ periods later) if $K^k \geq K^k$. However, once the bill is due, he prefers to pay $k$ periods late if $K^k < K^k$, where $\bar{K}^k = \frac{1}{\delta^k}$ and $\tilde{K}^k = \frac{1}{\beta} K^k$.

In order to understand the results in Lemma 1, consider the stream of payoffs that the individual faces. If he pays the bill due at the end of the billing cycle (at period $xn$), he obtains a discounted stream of utilities from consuming the good, and pays a discounted fee. If, instead, the consumer does not pay the bill due at the end of the billing cycle, his stream of utilities and fees coincide with those above, except for the fact that: (1) he faces no fees during the $k - 1$ periods that it takes him to pay his unpaid bill; and (2) when he eventually pays at period $n + k$, he suffers a fee augmented by penalty $K^k$. If the penalty $K^k$ is sufficiently high, larger than $\bar{K}^k$, the consumer plans to pay his bill due at $xn$. Importantly, note that when obtaining cutoff $\bar{K}^k$, the individual compares, at any period $t$, the stream of payoffs happening in the distant future, that is, whether to pay at moment when the bill is due (at period $xn$) or $k$ periods later (at $xn + k$). As a consequence, present bias does not affect the consumer’s decision at period $t$ on whether to pay the bill due at $xn$. In addition, since the buyer has the financial ability to pay at any time, his income or wealth is not part of the cutoffs sustaining this equilibrium.

Once the bill is due at period $xn$, the discounted stream of payoffs from paying on time is similar to the one above, with the only difference that the fee is not discounted (since it is paid at the current period). If instead, he did not pay the bill due at $xn$, he would benefit from not paying that bill during $k$ periods, but eventually pay it at a higher penalty $K^k$. Hence, at period $tn$, the consumer does not pay the bill in that period if penalty $K^k$ is relatively low (below $\bar{K}^k$). Unlike $\bar{K}^k$, our analysis to obtain $\tilde{K}^k$ is dependent on present bias. Intuitively, when choosing whether to pay the bill at period $xn$ or $k$ periods later, the consumer compares streams of payoffs that only differ in the fact that paying on time entails a current (undiscounted) fee, while paying the bill late $k$ periods late is implies a discounted future penalty, thus allowing present bias to affect his decision.

The above lemma identifies a region of penalties in any late period $k$, $K^k$, for which time inconsistent behavior arises. Intuitively, if such penalty is intermediate the individual prefers to pay on time the bills that are due at the end of that billing cycle (at period $xn$), but does not pay them once they are due. This behavior arises because of present bias, $\beta < 1$, but would not arise if present bias was absent. In particular, if $\beta = 1$ both cutoffs on $K^k$ collapse to $\frac{1}{\delta^k}$, thus implying that the range on $K^k$ for which time inconsistent behavior can emerge is nil. In contrast, lower values of $\beta$, entail a larger range of penalties $K^k$’s for which time inconsistent behavior exists.
We next explore how the above results are affected if we allow for shocks. For tractability, we hereafter assume that the penalty of paying a bill \( k \) periods late is given by 
\[
K_k = a^{k-1}K 
\]
where \( K \) represents the penalty for paying a bill one period late, and \( a \geq 1 \) denotes the growth pattern in penalties as the individual pays the bill later, e.g., \( a^2, a^3 \), and similarly for later periods.\(^7\)

**Proposition 1 (Shocks).** When the buyer suffers shocks, \( q > 0 \), he expects, at any period \( t \), to pay on time the bill that is due at the end of that billing cycle (rather than paying \( k \) periods later) if \( K_k \geq KS_k \). However, once the bill is due, he prefers to pay \( k \) periods late if \( K_k < KS_k \), where
\[
K_k = \frac{(1 - q)}{(1 - q^k)} \left[ \left( (1 - q) \sum_{x=0}^{S-2} \delta^x a^{x-k} q^{x-k} \right) + \delta^{S-1} a^{S-2} q^{S-1-k} \right] - (1 - q) \sum_{x=1}^{k-1} \delta^x a^{x-1} q^x
\]
and \( KS_k = \frac{1}{\beta} K_k \).

First, note that when the consumer does not suffer shocks, \( q = 0 \), the cutoffs in Proposition 1 coincide with those identified in Lemma 1. Intuitively, when the individual suffers no shocks, \( q = 0 \), time inconsistent behavior can be sustained under relatively restrictive penalties, but the range of penalties expands as shocks are more likely.

Second, our results show that time inconsistent behavior can only arise if the individual exhibits present bias, \( \beta < 1 \); otherwise the range of penalties \( K \) supporting time inconsistent behavior \( K^{TI,k} \equiv \overline{K}_k - K_k = \frac{1-\beta}{\beta} K_k \) collapses to zero when \( \beta = 1 \). However, for most combinations of parameter values (\( \delta, \beta, a, S, \) and \( k \)), the difference \( K^{TI,k} = \overline{K}_k - K_k \) increases in the probability of a shock, \( q \); as depicted in figures 1a and 1b.\(^8\) In particular, figure 1a considers \( \delta = 0.95, \beta = 0.92, a = 1.05, S = 30, \) and \( k = 5 \); whereas figure 1b alters one parameter at at time. Figure 1b suggests that \( K^{TI,k} \) decreases, and thus time inconsistent behavior can be sustained under a larger range of penalties, when \( S \) decreases from \( S = 30 \) in figure 1a to \( S = 10 \) in figure 1b. Intuitively, if the service is discontinued quickly after the buyer does not pay a bill, the consumer is more likely to pay his bills on time. \( K^{TI,k} \) also decreases as factor \( a \) increases from \( a = 1.05 \) to \( a = 1.15 \), where \( K_k = a^{k-1}K \), since the buyer is willing to pay bills on time when penalties grow faster. In contrast, \( K^{TI,k} \) grows in \( k \), the number of periods that it takes the consumer to pay his unpaid bills; as the buyer’s discount factor \( \delta \) decreases; and as he exhibits a stronger present bias.\(^9\)

\(^7\)In other words, we introduce the penalty as a surcharge rate on the initial fee

\(^8\)Differentiating \( K^{TI,k} \) with respect to parameters \( \delta, \beta, a, S, \) and \( k \) produces non-compact expressions difficult to sign under general conditions. Other numerical simulations produce similar results as those in figures 1a and 1b, and can provided by the authors upon request.

\(^9\)Respectively, \( k \) increases from 5 periods in figure 1a to 10 in figure 1b; \( \delta \) decreases from 0.95 in figure 1a to 0.8 in figure 1b; and \( \beta \) decreases from 0.92 in figure 1a to 0.8 in figure 1b.
4 Seller’s Problem

4.1 Finding the optimal penalty

The seller has three decisions to make in this problem; finding the optimal $K$, $F$ and $a$. He does this using backward induction. We first find the optimal value of the penalty $K$, then the fee $F$ and finally the rate of growth of the fee $a$.

4.2 Finding the optimal penalty

When setting the optimal stream of penalties $(K^1, K^2, ..., K^{S-1})$, the seller can anticipate the above results on the consumer’s problem. In this setting, the seller can either: (1) charge low penalties that induce every buyer to pay late, both those who suffer shocks and those who do not, inducing all of them to pay in the last period $S - 1$; or (2) set high enough penalties such that only those who suffer a shock pay late (those suffering a shock pay late by definition, except in period $S - 1$).

In the case where shocks are absent, $q = 0$, the seller can only choose option (1) option. We will now carefully examine the optimal penalties in each scenario.

4.2.1 Optimal penalty to induce everyone to pay late

If the seller wants to induce late payment in everyone, she must keep the penalty low enough to induce late payments until period $S - 1$. This will give her the maximum possible profit from exploiting only the time inconsistent behavior due to present bias. Here, buyers pay late as long as penalties satisfy $K^k \leq \bar{K}^k$ for every late period $k$. Since $K^k = a^{k-1}K$ by definition, the previous condition becomes $a^{k-1}K \leq \bar{K}^k$ for every $k$. Hence, the seller sets an initial penalty $K = K_{Late}$.
that induces all individuals to pay late, where $K_{\text{Late}}$ satisfies

$$K^k \geq a^{k-1}K_{\text{Late}} \quad \text{and} \quad K^l \geq a^{l-1}K_{\text{Late}} \text{ for at least one period } l.$$  

Figure 2 depicts cutoff $K^k$ as a function of period $k$, evaluated at the same parameter values as Figure 1, and suggests one initial penalty satisfying the above two conditions.

![Figure 2. Cutoff $K^k$ as a function of $k$.](image)

Therefore, the seller’s profit from inducing late payment at period $S - 1$ is $FK^{S-1}$, which can be rewritten as $Fa^{S-2}K$, since $K^{S-2} \equiv a^{S-1}K$. In addition, $Fa^{S-2}K$ satisfies

$$Fa^{S-3}K \leq FK^{S-2}$$

by definition of $K$ being the lowest of all $K^k$; and

$$Fa^{S-2}K \leq FaK^{S-2}$$

since $K^{S-2} \equiv a^{S-3}K$ and $a > 1$. In addition, the terms in the right-hand side of the above two inequalities satisfy $Fa^{S-3}K^{S-2} \leq FaK^{S-2}$ since $K^{S-2} \leq K^{S-2}$; otherwise late payment would not occur. Hence, the highest profit from inducing late payment at $S - 1$ is $\pi_{\text{Late}} \equiv Fa^{S-2}K^{S-2}$.

### 4.2.2 Optimal penalty so only those suffering a shock pay late

The seller’s profit if only those suffering a shock pay late is

$$\pi_{\text{Shock}}(K) = F(1-q) + Fq(1-q)K^1 + \ldots + Fq^{S-2}(1-q)K^{S-2} + Fq^{S-1}K^{S-1}$$
Using $K^k = a^{k-1}K$, the above profit becomes

$$\pi_{Shock}(K) = F(1-q) + FK \left[ (1-q) \left( \sum_{x=1}^{S-2} q^x a^{x-1} \right) + q^{S-1} a^{S-2} \right]$$

Thus, it is profitable to only induce those who suffer a shock to pay late if $\pi_{Shock}(K) \geq \pi_{Late}$, or

$$K \geq \hat{K} \equiv \frac{a^{S-2}K_{Late} - (1-q)}{(1-q) \left( \sum_{x=1}^{S-2} q^x a^{x-1} \right) + q^{S-1} a^{S-2}}$$

Here, if the optimal $K$ that maximizes $\pi_{Shock}(K)$ exceeds the legal limit $K = K_{Legal}$, the seller sets it at $K = K_{Legal}$. As we illustrate in our parametric examples at the end of the section, this is the most common case. This choice of penalty $K$, however, only occurs if the legal limit satisfies $K_{Legal} \geq \hat{K}$. Otherwise, the seller prefers to set the penalty at $K = K_{Late}$, which induces all individuals to pay their bills late, and wait until $S - 1$ to pay their debts. As a consequence, when legislators set a low limit on late penalties, $K_{Legal} < \hat{K}$, it is more likely that the profitable option for the seller is to induce all individuals to pay late. However, when such legal limit is relatively high, the profitable move by the seller is to have only individuals suffering a shock pay their bills late. The following proposition summarizes our above results.

**Proposition 2.** When the legal limit on penalties, $K_{Legal}$, satisfies $K_{Legal} < \hat{K}$, the seller sets the initial penalty at $K = K_{Late}$, which induces all individuals to pay their bills late and wait until period $S - 1$ to pay their debts. In contrast, when $K_{Legal} \geq \hat{K}$, the seller sets an initial penalty of $K = K_{Legal}$, which leads only those individuals suffering a shock to pay their bills late.

Cutoff $\hat{K}$ decreases in $q$ for most parameter combinations. Figure 3 evaluates this cutoff at the same parameter values as Figure 1. Intuitively, as shocks become relatively rare, i.e., $q \to 0$, the seller’s expected profit from individuals who suffered a shock is low, leading the seller to set relatively low penalties that induce all individuals to pay late.\(^{10}\) In contrast, when shocks become more frequent (higher $q$), the expected profit from individuals who suffered a shock increases, leading the seller to set relatively high penalties that induce only this group of customers to pay late.\(^ {11}\) A similar argument holds when service is suspended relatively quickly after the buyer does not pay a bill ($S$ decreases from $S = 30$ in the benchmark case to $S = 10$), whereby cutoff $\hat{K}$ also decreases, thus expanding the region of parameter values for which $K_{Legal} \geq \hat{K}$ and the seller focuses on individuals suffering a shock. The opposite argument applies when present bias increases;\(^ {10}\)

\(^{10}\)In particular, cutoff $\hat{K}$ is extremely high when $q$ is small, thus expanding the range of parameter values for which $K_{Legal} < \hat{K}$ holds, which induces the seller to set a initial penalty at $K = K_{Legal}^{S-2}$; as described in Proposition 2.

\(^{11}\)Specifically, as cutoff $\hat{K}$ decreases, the range of parameter values for which $K_{Legal} \geq \hat{K}$ holds expands, which leads the seller to set a penalty $K = K_{Legal}$; as described in Proposition 2.
and when the rate at which penalties grow across periods, $a$, increases.$^{12}$

![Cutoff K as a function of q.](image)

4.3 Finding the optimal fee

Given the optimal penalties found in Proposition 2, we now analyze the seller’s optimal choice of fee $F$. The fee is defined by the consumer’s participation constraint because the seller tries to charge the maximum $F$ to the consumer while still ensuring his participation. The consumer’s participation constraint is

$$u_0 + \beta \sum_{x=1}^{n} \delta^x u_x - \beta \delta^n F \left[ (1-q)F + \delta q(1-q)FK_1 + \delta^2 q^2(1-q)FK_2 + \ldots + \delta^{S-1} q^{S-1}FK^{S-1} \right] \geq 0$$

which must hold with equality; otherwise the seller could still raise $F$. After solving for $F$, we find

$$F(K^k) = \frac{\bar{u} \left( 1 + \beta \sum_{x=1}^{n} \delta^x \right)}{\beta \delta^n \left[ (1-q) + \delta q(1-q)K_1 + \delta^2 q^2(1-q)K_2 + \ldots + \delta^{S-1} q^{S-1}K^{S-1} \right]}$$

Using the functional form $K^k = a^{k-1}K$, the above expression can be rewritten as

$$F(K, a) = \frac{\bar{u} \left( 1 + \beta \sum_{x=1}^{n} \delta^x \right)}{\beta \delta^n \left[ (1-q) + \delta q(1-q)K + \delta^2 q^2(1-q)aK + \ldots + \delta^{S-1} q^{S-1}a^{S-1}K^{S-1} \right]}$$

which is a function of penalty $K$ and $a$. The seller then evaluates $F(K)$ at the optimal penalty $K$ found in Proposition 2, either $K = K_{Legal}$ or $K = K_{Late}$. The next proposition summarizes this result.

$^{12}$In particular, the present bias parameter $\beta$ decreases from 0.92 in the benchmark case to 0.8; and parameter $a$ increases from 1.05 in the benchmark case to 1.15.
Proposition 3. The seller sets fee \( F = F(K_{\text{Late}}, a) \) if \( K_{\text{Legal}} < \hat{K} \) and \( K_{\text{Late}} > 1 \), but \( F = F(K_{\text{Legal}}, a) \) otherwise.

In the case in which shocks are absent, \( q = 0 \), fee \( F(K) \) reduces to

\[
F(K, a) = \frac{\bar{u}(1 + \beta \sum_{x=1}^{n} \delta^x)}{\beta \delta^a}
\]

which is unaffected by penalty \( K \) since in this context the consumer is not late on his bills, and also independent on \( a \).

4.4 Optimal value of \( a \)

We find the seller’s optimal choice of \( a \), where \( a \in [0, \bar{a}] \), by evaluating his profit function in the optimal expressions of \( K \) (from Proposition 2) and \( F \) (from Proposition 3). For each of the cases examined in Propositions 2-3, we obtain profits \( \pi_{\text{Late}}(a) \) and \( \pi_{\text{Shock}}(a) \). Differentiating \( \pi_{\text{Late}}(a) \) with respect to \( a \), we obtain

\[
\pi'_{\text{Late}}(a) = \pi_{\text{Late}}^S K_{\text{Late}}(a) + a^{S-2} \left[ F'(a) K_{\text{Late}}(a) + F(a) K'_{\text{Late}}(a) \right] = 0
\]

and similarly differentiating \( \pi_{\text{Legal}}(a) \) with respect to \( a \), we find

\[
\pi'_{\text{Legal}}(a) = (1 - q) F'(a)
\]

\[
+ (1 - q) \left[ F(a) K_{\text{Legal}}(a) \left( \sum_{x=1}^{S-2} (x - 1) q^x a^{x-2} \right) + \sum_{x=1}^{S-2} q^x a^{x-1} (F'(a) K_{\text{Legal}}(a) + F(a) K'_{\text{Legal}}(a)) \right]
\]

\[
+ (S - 2) F(a) K_{\text{Legal}}(a) q^{S-2} a^{S-3} + q^{S-1} a^{S-2} (F'(a) K_{\text{Legal}}(a) + F(a) K'_{\text{Legal}}(a)) = 0
\]

Since the profit function becomes a polynomial of a high order in \( a \), obtaining an analytical solution is not tractable. We next provide a numerical example, in which we show how to approach this problem practically.

4.5 Numerical example

The following list describes the algorithm to calculate the optimal \( K \) and \( a \):

1. Assume that \( \bar{K}^k = a^{k-1} K_{\text{Late}} \). (That is, cutoff \( \bar{K}^k \) coincides with \( a^{k-1} K_{\text{Late}} \) at exactly \( k \), but lies below otherwise.)

2. Assume that \( \hat{K} > K_{\text{Legal}} \), then by Proposition 2 the seller sets a penalty \( K = a^{k-1} K_{\text{Late}} \) by definition. The seller then obtains the fee \( F(a^{k-1} K_{\text{Late}}, a) \) as described in section 4.1.

3. Insert fee \( F(a^{k-1} K_{\text{Late}}, a) \) into the seller’s profit to solve

\[
\max_{a \geq 0} \pi \left( F(a^{k-1} K_{\text{Late}}, a), a \right)
\]
and obtain the optimal $a_{\text{Late}}^*$.

4. **Optimal $a$.** Use $a_{\text{Late}}^*$ to find $\hat{K}(a_{\text{Late}}^*) \equiv \hat{K}(K_{\text{Late}}, a_{\text{Late}}^*)$, and $K_{\text{Late}} \equiv K_{\text{Late}}(a_{\text{Late}}^*)$.

   (a) If $\hat{K}(a_{\text{Late}}^*) > K_{\text{Legal}}$ and $K_{\text{Late}} > 1$, then the $a_{\text{Late}}^*$ found in step (3) is profit-maximizing.

   (b) Otherwise, the seller sets $K = K_{\text{Legal}}$, and obtains the fee $F(K_{\text{Legal}}, a)$ as described in section 4.2. Repeat step (3) evaluating the profit function at $K_{\text{Legal}}$ in order to find the optimal $a_{\text{Legal}}^*$.

5. **Optimal penalty.**

   (a) If $\hat{K}(a_{\text{Late}}^*) > K_{\text{Legal}}$ and $K_{\text{Late}} > 1$, then the optimal penalty is $K_{\text{Late}}(a_{\text{Late}}^*)$.

   (b) Otherwise, the optimal penalty is $K_{\text{Legal}}$.

6. **Optimal fee.**

   (a) If $\hat{K}(a_{\text{Late}}^*) > K_{\text{Legal}}$ and $K_{\text{Late}} > 1$, then the optimal fee is $F_{\text{Late}} \equiv F(K_{\text{Late}}(a_{\text{Late}}^*), a_{\text{Late}}^*)$.

   (b) Otherwise, the optimal fee is $F_{\text{Legal}} \equiv F(K_{\text{Legal}}, a_{\text{Legal}}^*)$.

7. Evaluate equilibrium profits in this context (that is, when the assumption in step (1) holds).

8. Repeat steps (1)-(7) for every period $k = 1, ..., S - 3$. Then find $\pi_{\text{Late}} \equiv \max_k \pi(a^{k-1}, K_{\text{Late}})$ and $\pi_{\text{Legal}} \equiv \pi(a_{\text{Legal}}, K_{\text{Legal}})$.

   (a) If $\pi_{\text{Late}} \geq \pi_{\text{Legal}}$, the seller chooses $K_{\text{Late}}$ and $a^{k-1}$ that solves $\pi_{\text{Late}}$, inducing all consumers to pay late.

   (b) Otherwise, the seller sets $K_{\text{Legal}}$ and $a_{\text{Legal}}$, inducing only consumers who suffered a shock to pay late.

**Example 1.** Only consumers suffering a shock pay late. Consider that $q = 0.3$, $\beta = 0.97$, $\delta = 0.95$, $S = 15$, $n = 10$, $\bar{u} = 1$, $\sigma = 1.2$ and $K_{\text{Legal}} = 1.4$. We take the following steps to optimize profits. First we solve the above algorithm for $k = 5$. Therefore, we evaluate $\frac{\hat{K}^5}{K_{\text{Late}}} = a^4K_{\text{Late}}$, and assume that $\hat{K} > K_{\text{Legal}}$. As explained in step (2), the seller sets a penalty $K = a^4K_{\text{Late}}$, which yields an optimal $a$ of $a_{\text{Late}}^* = 1.1688$. In step (4), we use $a_{\text{Late}}^*$ to find $\hat{K}(a_{\text{Late}}^*) = 1.742 > 1.4 = K_{\text{Legal}}$, and $K_{\text{Late}} \equiv K_{\text{Late}}(a_{\text{Late}}^*) = 0.1628$. Since, $K_{\text{Late}} < 1$, the seller chooses $a_{\text{Legal}}$ in step (4), and penalty $K_{\text{Legal}}$ in step (5). Therefore, the optimal fee in step (6) becomes $F(K_{\text{Legal}}, a_{\text{Legal}}^*) = 12.8552$ and equilibrium profit, in step (7), is $\pi = 14.904$. A similar argument applies to all periods $k$, where the same $(a, K, F)$ are optimal, and the same equilibrium profits $\pi = 14.904$ arise. In this setting, the seller only induces late payment from individuals suffering a shock, by setting the highest legal penalty, $K_{\text{Legal}}$, and obtaining the corresponding $a$ and $F$ values.
Table I summarizes our results in the top row, and repeats the algorithm changing one parameter at a time. First, a reduction in the probability of a shock (from \( q = 0.3 \) to \( q = 0.2 \)) still induces the seller to induce late payment only from individuals suffering a shock, but he sets a higher fee and penalty when this type of consumer is late in his payments. A similar result arises when the consumer’s discount factor decreases, that is, the seller can charge a higher fee and penalty in the event that the consumer is late, still inducing participation from the less patient consumer. When increasing \( n \) to 20, we find that the optimal result is for the seller to choose \( K_{\text{Late}} \) if \( 1 < K_{\text{Late}} \leq \tilde{K} \). The noticeable change here is in the value of fee \( F \), as the fee is heavily influenced by \( n \). Since profits are in turn influenced by \( F \), we obtain high profits in this example relative to other cases. A similar result holds by decreasing \( \beta \) to 0.9 (optimal to choose \( K_{\text{Late}} \) if \( 1 < K_{\text{Late}} \leq \tilde{K} \) but, \( K_{\text{Late}} < 1 \)). Once again, it is optimal to choose the legal values and induce only those suffering shocks to pay late. Finally, we increase the value of \( S \), which provides us with results as changing other parameter values.\(^{13}\)

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline
Example 1 & 5 & 1.168 & 1.742 & 0.16 < 1 & Legal & 12.85 & 14.90 & 14.90 \\
Example 1, with \( q = 0.2 \) & 6 & 1.122 & 2.512 & 0.29 < 1 & Legal & 13.43 & 14.71 & 14.71 \\
Example 1, with \( S = 17 \) & 6 & 1.103 & 1.831 & 0.29 < 1 & Legal & 12.85 & 14.90 & 14.90 \\
Example 1, with \( \delta = 0.93 \) & 5 & 1.199 & 2.629 & 0.14 < 1 & Legal & 14.66 & 17.00 & 17.00 \\
Example 1, with \( n = 20 \) & 5 & 1.168 & 1.741 & 0.16 < 1 & Legal & 32.79 & 32.79 & 32.79 \\
Example 1, with \( \beta = 0.9 \) & 5 & 1.169 & 2.045 & 0.17 < 1 & Legal & 12.97 & 15.04 & 15.04 \\
Example 2 & 8 & 1.12 & 63.86 & 1.52 < 1 & Late & 38.00 & 529.18 & 529.18 \\
\hline
\end{tabular}
\caption{Seller’s optimal choices.}
\end{table}

**Example 2.** All consumers pay late. Consider now \( q = 0.2 \), \( \beta = 0.99 \), \( \delta = 0.8 \), \( S = 21 \), \( n = 8 \), \( \tilde{u} = 1 \), \( \bar{\pi} = 1.2 \), and \( K_{\text{Legal}} = 1.55 \); and let us start by considering \( k = 8 \). Following similar steps as in the previous example, we obtain \( a_{\text{Late}}^* = 1.1235 \). In step (4), we find \( \hat{K} (a_{\text{Late}}^*) = 54.74 > 1.4 = K_{\text{Legal}} \), and \( K_{\text{Late}} \equiv K_{\text{Late}} (a_{\text{Late}}^*) = 1.523 \), where \( K_{\text{Legal}} > K_{\text{Late}} > 1 \). Therefore, in step (5), the seller chooses \( K_{\text{Late}} \), which entails a fee \( F_{\text{Late}} = 38.0079 \) and profits \( \pi = 529.18 \). All other values of \( k \) yield case (b) in steps (4)-(6), thus yielding profit \( \pi (a_{\text{Legal}}, K_{\text{Legal}}) = 63.86 \).\(^{14}\) Since in this context profits from inducing every consumer to pay late are higher, the seller sets \( a_{\text{Late}}^* = 1.1235 \), a penalty of \( \hat{K} (a_{\text{Late}}^*) = 54.74 \), and a fee of \( F_{\text{Late}} = 38.0079 \). Overall, we see that to find settings where \( K_{\text{Late}} \) is optimal, we would need very specific parameter values; which is consistent with the observation that most buyers are not induced to pay late regardless of their suffering a shock.

\(^{13}\)However, a larger \( S \) in this context increases the number of total number of times we must run steps 1-7 in the algorithm, thus making it more likely that results change within reasonable parameter values. However, raising \( S \) significantly places computational constraints while finding \( a_{\text{Late}}^* \)

\(^{14}\)In this case, the seller sets \( a_{\text{Legal}} = \bar{\pi} = 1.2 \), a penalty of \( K_{\text{Legal}} = 1.4 \), and a fee of \( F_{\text{Legal}} = 411.802 \).
5 Discussion

In this paper we offer two different explanations for why consumers with financial means pay contractual fees late, even when suffering substantial penalties. When there are no shocks, consumers only pay late if there is substantial present bias. However, if consumers can suffer (nonfinancial) “memory” shocks, paying late can be the norm even when present-biasedness is small if sellers impose a low penalty. However, they do so only for a very specific set of parameter combinations, when enticing all consumers to pay late increases profit by high volume—in effect the elasticity of paying late with respect to a low penalty must be substantial. More commonly, sellers do better by setting a high penalty, extracting substantial rents from fewer consumers.

Given sellers’ ability and incentive to use high penalties to extract rents from consumers, a common policy response is to limit late penalties. We show that the range of an effective cap is limited. Set too high, and it does not limit sellers’ exploitation of consumers. Set too low, and sellers cannot punish customers who do not honor their contracts, hurting sellers’ incentive to participate in the market.

Our analysis assumes a fixed fee for the service, limiting its direct applicability to cases where the price of the good is regulated by the market or by policymakers. Otherwise, besides choosing penalties, price-setting firms would choose the fee as well, and it is likely there would be tradeoff between the price of the good and the penalties. A high price may attract fewer customers, limiting the pool of consumers from which rents could be extracted with high late fees. At the same time, a high price with a percentage-based penalty could amount to substantial rents collected from those who do pay late. Overall, considering late fees in the profit maximizing problem facing a firm with substantial market power adds an additional tradeoff that would allow the firm to increase its profits further.

6 Appendix

6.1 Proof of Lemma 1

In every period $t$, the stream of payoffs that the consumer obtains from paying the bill due at the end of that period is

$$u_t + \beta \sum_{x=t}^{\infty} \delta^x u_x - \beta F (\delta^{n-t} + \delta^{2n-t} + \ldots + \delta^{zn-t}) = u_t + \beta \sum_{x=t}^{\infty} \delta^x u_x - \beta F \sum_{x=1}^{z} \delta^{zn-t}. \quad (6.1)$$

If, instead, the consumer does not pay at period $t$, his stream of discounted payoffs becomes

$$u_t + \beta \sum_{x=t}^{\infty} \delta^t u_t - \beta F \left(0 + 0 + \ldots + 0 + \delta^{n+k-t} K^k + \delta^{2n-t} + \ldots + \delta^{zn-t}\right)$$
where we allow the individual to pay his unpaid bill \( k \) periods after being late, and where penalty \( K^k > K \). The above analysis assumes that the payment period \( k \) happens before the service is suspended, \( S \) periods from the last payment (at \( t-1 \)). If, however, \( S < k \), the service is suspended, the firm takes action (e.g., bringing the case to court) and the consumer faces a penalty \( C \), yielding a stream of payoffs

\[
    u_0 + \beta \sum_{x=1}^{S_n} \delta^x u_x - \beta (0 + 0 + \ldots + \delta^{S_n} C) = u_0 + \beta \sum_{x=t}^{S_n} \delta^x u_x - \beta \delta^{S_n} C.
\]

In order to guarantee that the individual pays at least before the suspension period \( S \), we assume that penalty \( C \) satisfies \( C > \frac{u_t + \sum_{x=1}^{S_n} \delta^x u_x}{\beta \delta^S} \), which holds by definition, as described in the Model section.

Therefore, the individual prefers to pay at period \( t \) than not pay (and pay after \( k \) periods of being late), if and only if (1) is larger than (2), that is,

\[
    u_t + \beta \sum_{x=t}^{2n} \delta^x u_x - \beta F \sum_{x=1}^{z} \delta^{x-n-t} \geq u_t + \beta \sum_{x=t}^{2n} \delta^x u_x - \beta F \delta^{n+k-t} K^k - \beta F \sum_{x=2}^{z} \delta^{x-n-t}
\]

which reduces to \( K^k \geq K^k \equiv \frac{1}{\beta^k} \).

Once the consumer reaches any paying period \( x_n \), for any \( x = \{1, 2, \ldots, z\} \), he pays his bill due at \( x_n \) rather than paying it \( k \) periods later, at period \( x_n + k \), if and only if

\[
    u_{x_n} - F + \beta \sum_{x=1}^{2n} \delta^x u_x - \beta F \sum_{x=2}^{z} \delta^x \leq u_{x_n} + \beta \sum_{x=t}^{2n} \delta^x u_x - \beta F \delta^k K^k - \beta F \sum_{x=2}^{z} \delta^x
\]

which simplifies to \( K^k < K^k \equiv \frac{1}{\beta^k} \), where the number of delay periods, \( k \), satisfies \( k < S_n \) by definition. Then, the consumer expects to pay on time at the beginning of every paying cycle if \( K^k \geq \frac{1}{\beta^k} \), but chooses to pay at period \( t_n + k \) rather than at the due date \( t_n \) if \( K^k \leq \frac{1}{\beta^k} \). □

### 6.2 Proof of Proposition 1

In a setting with shocks, the stream of payoffs that, at every period \( t \), the consumer obtains from paying the bill due at the end of that billing cycle is

\[
    u_t + \beta \sum_{x=t}^{2n} \delta^x u_x
\]

\[
    -\beta \delta^{n-t} \left[ (1-q)F + \delta q (1-q)FK^1 + \delta^2 q^2 (1-q)FK^2 + \ldots + \delta^{S-1} q^{S-1} FK^{S-1} \right]
\]

\[
    -\beta \delta^{2n-t} \left[ (1-q)F + \delta q (1-q)FK^1 + \delta^2 q^2 (1-q)FK^2 + \ldots + \delta^{S-1} q^{S-1} FK^{S-1} \right] - \ldots
\]
\[-\beta \delta^{z_{n-1}} [(1 - q)F + \delta q(1 - q)FK^1 + \delta^2 q^2(1 - q)FK^2 + \ldots + \delta^{S-1} q^{S-1} FK^{S-1}]\]

\[= u_t + \beta \sum_{x=t}^{2n} \delta^x u_x\]

\[-\beta \sum_{x=1}^{z} \delta^{z_{n-1}} [(1 - q)F + \delta q(1 - q)FK^1 + \delta^2 q^2(1 - q)FK^2 + \ldots + \delta^{S-1} q^{S-1} FK^{S-1}]\quad (6.3)\]

In words, the individual pays fee \(F\) at period \(n\) if he does not suffer a shock at that period, which occurs with probability \(1 - q\). If, instead, he suffers a shock, with probability \(q\), then he pays the fee (augmented by the penalty) one period later, \(FK^1\), if he does not suffer a shock on that subsequent period, which happens with probability \((1 - q)\). A similar argument applies if he suffers two subsequent shocks, with probability \(q^2\), and thus pays \(FK^2\) three two periods after the bill was due, which occurs with probability \(q^2(1 - q)\). Finally, if the individual suffers shocks until the period in which the service is suspended, \(S - 1\), we assume that the bill is paid.

If, instead, at period \(t\), the consumer plans to not pay the bill that is due at period \(tn\) despite not suffering a shock (but pay it \(k\) periods after it was due), his stream of discounted payoffs becomes

\[u_t + \beta \sum_{x=t}^{2n} \delta^x u_x\]

\[-\beta \delta^{z_{n-1}}[(1 - q)(0 + 0 + \ldots + \delta^k FK^k) + \delta q(1 - q)FK^1 + \delta^2 q^2(1 - q)FK^2 + \ldots + \delta^{S-1} q^{S-1} FK^{S-1}] - \beta \delta^{2z_{n-1}}[(1 - q)F + \delta q(1 - q)FK^1 + \delta^2 q^2(1 - q)FK^2 + \ldots + \delta^{S-1} q^{S-1} FK^{S-1}] - \ldots\]

\[-\beta \delta^{zn-1}[(1 - q)F + \delta q(1 - q)FK^1 + \delta^2 q^2(1 - q)FK^2 + \ldots + \delta^{S-1} q^{S-1} FK^{S-1}]\quad (6.4)\]

Comparing (3) and (4), we conclude that, at any period \(t\), the consumer plans to pay the next bill, due at \(tn\), if and only if

\[K \geq K \equiv \frac{(1 - q)}{(1 - q^k) \left[ \delta^x a^{-1} - \delta^{S-1} a^{-2} q^{S-1-k} \right] - (1 - q) \sum_{x=1}^{k-1} \delta^x a^{-1} q^x} \]

Note that when \(q = 0\) this cutoff reduces to \(K \geq K^k \equiv \frac{1}{\delta^x}\), which coincides with the cutoff found in the case with no shocks.

Once the bill is due at period \(tn\), his discounted stream of payoffs from paying the bill on time becomes

\[u_{tn} - F + \beta \sum_{x=tn}^{zn} \delta^x u_x\]

\[-\beta \sum_{x=t}^{z-1} \delta^{z_{n-1}} [(1 - q)F + \delta q(1 - q)FK^1 + \delta^2 q^2(1 - q)FK^2 + \ldots + \delta^{S-1} q^{S-1} FK^{S-1}]\quad (6.5)\]

Note that the above expression assumes that the individual does not suffer a shock at period \(tn\). Otherwise, he would not pay the bill due at \(tn\). If, instead, he chooses to pay \(k\) periods late, his
discounted stream of payoffs is

\[ u_{tn} + \beta \sum_{x=t}^{\infty} \delta^x u_x - \beta \delta \left[ (0 + 0 + \ldots + (1 - q)\delta^{k-1}FK^k) + \delta q(1 - q)FK^{k+1} + \ldots + \delta^{S-k}q^{S-k}FK^{S-1} \right] \]

\[-\beta \sum_{x=t}^{\infty} \delta^x \left[ (1 - q)F + \delta q(1 - q)FK + \delta^2 q^2(1 - q)FK^2 + \ldots + \delta^{S-1}q^{S-1}FK^{S-1} \right] \]

(6.6)

Comparing (5) and (6), we conclude that, at any period \( t_n \) when bills are due, the consumer does not pay if and only if

\[ K < \frac{1}{\beta} \frac{(1 - q)}{(1 - q^k) \left[ (1 - q) \sum_{x=k}^{\infty} \delta^x a^{x-1} q^{x-k} \right] + \delta^{k-1} a^{k-2} q^{k-1} - (1 - q) \sum_{x=1}^{k-1} \delta^x a^{x-1} q^x} \]

Note that when \( q = 0 \) this cutoff reduces to \( K < \frac{1}{\beta \delta^k} \), which coincides with that in the case with no shocks.

References


