School of Economic Sciences

Working Paper Series
WP 2016-14

When Induced Innovation Augments and Saves Less Expensive Inputs

Daegoon Lee, C. Richard Shumway and
Benjamin W. Cowan

August, 2016

When Induced Innovation Augments and Saves Less Expensive Inputs

Daegoon Lee
C. Richard Shumway*
Benjamin W. Cowan

Abstract
We consider a two-period cost-minimization model involving technical change in which the firm allocates resources to research in the first period and selects input quantities in the second period. The model allows us to distinguish between planned innovation and the input choice decision. We document that the induced innovation hypothesis (IIH) can fail both in Hicks' original version of inducing innovation aimed at saving the more expensive input and in the classic interpretation of actually saving the more expensive input. We find that the elasticity of substitution and the rate of trade-off between research outcomes (the research concavity parameter) are both important in determining the range over which the IIH fails. While the analytical findings support Salter's (1960) early objection toward the IIH, they also document that factor-saving behavior in response to a relative price increase is expected over a wide range of empirically-relevant substitution elasticities.

Keywords: demand, factor augmentation, induced innovation, innovation function, supply

JEL Classification: O300, D240

Daegoon Lee is a Ph.D. Graduate Research Assistant, C. Richard Shumway is a Regents Professor, and Benjamin W. Cowan is an Assistant Professor in the School of Economic Sciences, Washington State University. This research was supported by the Washington Agricultural Research Center and by the USDA National Institute of Food and Agriculture, Hatch grant WPN000275.

When Induced Innovation Augments and Saves Less Expensive Inputs

The induced innovation hypothesis (IIH) postulated by Hicks (1932) more than 80 years ago has captured sustained attention because of the theoretical appeal that prices are important not only for input choices but also for technology development to save inputs that become relatively more expensive. The hypothesis states than a change in relative input prices induces innovation to save the more expensive input (inference 1). In empirical testing, it is widely interpreted as implying technical change that actually saves the more expensive input (inference 2).

Salter (1960) challenged inference 2 and argued that the firm would reduce use of the more expensive input only if it helped increase profit (or decreased cost), which may not occur by saving the more expensive input. Based on innovation demand analysis, work by Acemoglu (2002), Funk (2002), and Armanville and Funk (2003) supported Salter and concluded that the IIH implies inference 2 if and only if the inputs are gross complements. By considering both innovation demand and innovation supply, we document that inference 2 is obtained for a wider range of input elasticities of substitution than if innovation supply is treated as exogenous; in addition, inference 1 is obtained only for a (non-monotonic) subset of substitution elasticities.

Theoretical Model

We propose a two-period model for a cost-minimizing firm that makes input choice decisions and makes (or influences) research resource allocation decisions. We start with input choice decisions in period 2 without imposing any restrictions on factor augmentation. We then turn to the research resource allocation decision in period 1 and its factor augmentation implications. By combining both components, we show how relative price changes impact both factor augmentation and subsequent input choice decisions. In doing so, we document that the

IIH's implications of augmenting and saving the more expensive input is limited to certain ranges of the firm's elasticities of substitution and the innovation function's curvature parameter.

To allow parsimonious representation of a multiple-input production function with the possibility that elasticities of substitution are not the same for all input pairs, we consider a two-level constant elasticity of substitution (CES) functional form as is frequently used in IIH tests (e.g., Kawagoe, Otsuka, and Hayami 1986, de Janvry, Sadoulet, and Fafchamps 1991, Cowan, Lee, and Shumway 2015). Consider a representative firm that produces a single output *Y* in period 2:

(1) $$Y = F\left(X_1, X_2\right) = \left[\delta X_1^{\frac{\rho-1}{\rho}} + (1-\delta) X_2^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \text{ for } \rho \in [0,\infty)$$

where ρ is the elasticity of substitution between input indices X_i, $i \in (1,2)$, and δ is the share parameter.[1] The input indices are produced respectively by pairs of inputs that also follow a CES form:

(2) $$X_i = F_i\left(x_{i1}, x_{i2}; a_{i1}, a_{i2}\right) = \left[\delta_i (a_{i1}x_{i1})^{\frac{\rho_i-1}{\rho_i}} + (1-\delta_i)(a_{i2}x_{i2})^{\frac{\rho_i-1}{\rho_i}}\right]^{\frac{\rho_i}{\rho_i-1}},\ i \in \{1,2\},$$

where x_{ij} is input $j \in (1,2)$ used in production of input index *i*, and *a* is a factor-augmenting parameter that captures technical progress.

The firm selects optimal input quantities x_{ij} that minimize the cost of producing a given output level with known input prices and technology in period 2. Since the two-level CES production function maintains homotheticity, minimizing cost provides the same optimal input ratios as maximizing profit with the same input prices and technology. Taking the first-order

[1] For greater generality, the input indices could include augmentation parameters. We suppress them because the analytical results are unaffected.

conditions and with a little reorganization documented in Appendix I, we obtain the following optimal input demand relationship:

$$(3) \quad \frac{x_{i1}^*}{x_{i2}^*} = \left(\frac{\delta_i}{1-\delta_i} \right)^{\rho_i} \left(\frac{w_{i1}}{w_{i2}} \right)^{-\rho_i} \left(\frac{a_{i1t}}{a_{i2t}} \right)^{\rho_i - 1}, \; i \in \{1,2\}.$$

where w_{ij} is the price of input x_{ij}, and the asterisk on *x* denotes the cost-minimizing input level.

Equation (3) documents that the qualitative effect of technical change (represented by the ratio of factor augmentation parameters) on the input ratio is dependent on the magnitude of the elasticity of substitution, as shown by Acemoglu (2002), Funk (2002), and Armanville and Funk (2003). Specifically, for two inputs, say labor and capital, without relative price changes, labor-augmenting technical change (that augments labor relatively more than capital) results in a labor-saving production decision if and only if the elasticity of substitution is less than one, i.e., the two factors are gross complements. However, if the inputs are gross substitutes (i.e., elasticity of substitution is greater than one), labor-augmenting technical change results in relatively greater use of labor because it is more easily substituted for capital. When the elasticity of substitution is exactly one (as in the Cobb-Douglas production function), technical change does not lead to changes in the input ratio.

We now turn to the research resource allocation decision made in period 1. In doing so, we show that the relationship between relative price and relative factor augmentation is a non-monotonic function of the elasticity of substitution. More importantly, we show that competitive cost-minimizing behavior is insufficient to always induce innovation efforts to save the more expensive input.

We consider a simple but very general homothetic innovation function, where innovation is defined as augmentation of at least one factor.[2] For a given research budget $\bar{R}$, the innovation function is given by:[3]

(4) $R_i = (c_{i1}\hat{a}_{i1})^{\theta_i} + (c_{i2}\hat{a}_{i2})^{\theta_i}$.

where R_i is expenditure on research in period 1 to augment the *i*th input index, the total research budget is assumed to be exogenously given and fully expended, i.e., $\bar{R} \equiv R_1 + R_2$, $\hat{a}_{ij}$ is the expected factor-augmentation in period 2; [4] $c_{ij} > 0$ denotes marginal research costs (or degree of research difficulty considering all research costs and probability of success) in period 1 for technology that is expected to augment x_{ij} by 1 percent in period 2; and θ_i >1 is a parameter representing the rate of trade-off between the two research outcomes $\hat{a}_{i1}$ and $\hat{a}_{i2}$, i.e., the larger the parameter, the more concave the frontier toward the origin (hereafter referred to as the "concavity parameter"). The innovation function is neutral if and only if $c_{i1} = c_{i2}$. Given a research budget, the condition $\theta_i > 1$ is sufficient to ensure that $\frac{\partial \hat{a}_{i2}}{\partial \hat{a}_{i1}} < 0$ and $\frac{\partial^2 \hat{a}_{i2}}{\partial \hat{a}_{i1}^2} < 0$ on the innovation function, as demonstrated in Appendix II.

This simple formulation of the innovation function results in an inverse relationship between degree of research difficulty and research outcome. An increase in the marginal cost of research shifts the innovation function toward the origin. Further, the innovation function is output-homothetic on the frontier, so the share of research outputs measured by the ratio of the factor-augmenting parameters does not depend on the total research budget.

---

[2] An increase in a factor augmentation parameter shifts the entire production function upward. But the shift may be greater in the region where the factor associated with the augmented parameter is used intensively, a feature similar to "localized" progress introduced by Atkinson and Stiglitz (1969) and revisited by Acemoglu (2015).
[3] This function is a modified version of the one employed by Armanville and Funk (2003, p. 1633).
[4] The factor augmentation parameter is assumed to be nonregressive, i.e., $\hat{a}_{ijt} \geq \hat{a}_{ij\tau}$ if $t > \tau$.

Considering the opportunity to invest (or influence investment) in R&D in the period (perhaps many years) before production inputs are selected and assuming a two-level CES production function as in the input decision model, the firm's cost-minimizing research resource allocation problem in period 1 can be written as follows:

(5) $$\min_{\tilde{x}_{ij},\hat{a}_{ij}} \sum_i \sum_j E(w_{ij})\tilde{x}_{ij}$$

$$s.t.\ \bar{Y} = \left[\delta X_1^{\frac{\rho-1}{\rho}} + (1-\delta) X_2^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}},$$

$$\bar{R} \equiv R_1 + R_2 ,$$

where $X_i = \left[\delta_i (a_{i1}x_{i1})^{\frac{\rho_i-1}{\rho_i}} + (1-\delta_i)(a_{i2}x_{i2})^{\frac{\rho_i-1}{\rho_i}} \right]^{\frac{\rho_i}{\rho_i-1}}$, $R_i = \left(c_{i1}\hat{a}_{i1}\right)^{\theta_i} + \left(c_{i2}\hat{a}_{i2}\right)^{\theta_i}$, a tilde on input level denotes that it is a period 2 value "*conceived*" in period 1 and thus distinguished from the value that is actually chosen by the firm in period 2 when input prices are known, E is the expectations operator $E\left[\cdot \middle| \Omega\right]$, and Ω is the firm's information set in period 1.

Combining and rearranging the first-order conditions, the optimal innovation ratios in period 1 are obtained (see Appendix I for derivation of this and the next equation):

(6) $$\frac{\hat{a}_{i1}^*}{\hat{a}_{i2}^*} = \left(\frac{\delta_1}{1-\delta_1} \right)^{\frac{\rho_i}{\psi_i}} \left(\frac{E(w_{i1})}{E(w_{i2})} \right)^{\frac{1-\rho_i}{\psi_i}} \left(\frac{c_{i1}}{c_{i2}} \right)^{\frac{-\theta_i}{\psi_i}},$$

where $\psi_i = 1 + \theta_i - \rho_i$, and the asterisk denotes an optimal value. It is apparent that the effect of an expected relative price change on the ratio of expected research outcomes depends on the sign and magnitude of $\frac{1-\rho_i}{\psi_i}$ and thus on the magnitudes of both ρ_i and θ_i (see the first row in Table

1). Since $\rho_i \geq 0$ and $\theta_i > 1$, an increase in the (expected) relative price of x_{i1} brings about factor-augmenting technical change such that the input that is expected to become more expensive is augmented more than the other whenever the two inputs are gross complements ($0 \leq \rho_i < 1$). Conversely, when the inputs are gross substitutes ($\rho_i > 1$) and the elasticity of substitution is less than 1 plus the concavity parameter ($\rho_i < \theta_i + 1$), the same increase will induce technical change that augments the input that is expected to become less expensive. However, if the inputs are gross substitutes ($\rho_i > 1$) and the elasticity of substitution is greater than 1 plus the concavity parameter ($\rho_i > 1 + \theta_i$), then the same increase will again induce technical change that augments the input that is expected to become more expensive. Thus, it is clear that firms respond not only to price incentives but also to the technological opportunity costs in making research resource allocation decisions. Further, the relationship between the elasticity of substitution and the impact of an input price change on factor augmentation is not monotonic.[5]

The firm optimizes in period 2 by solving the input choice decision problem in which the research outcomes are taken as given. Substituting the expected augmentation parameters, $\frac{\hat{a}_{i1}^*}{\hat{a}_{i2}^*}$, into the equilibrium condition (3) yields the following:

$$(7) \quad \frac{x_{i1}^*}{x_{i2}^*} = \left(\frac{\delta_i}{1-\delta_i}\right)^{\frac{\theta_i \rho_i}{\psi_i}} \left(\frac{w_{i1}}{w_{i2}}\right)^{-\rho_i} \left(\frac{E(w_{i1})}{E(w_{i2})}\right)^{\frac{-(1-\rho_i)^2}{\psi_i}} \left(\frac{c_{i1}}{c_{i2}}\right)^{\frac{\theta_i(1-\rho_i)}{\psi_i}} .$$

With this combined condition, it is now possible to distinguish optimal input choice effects of technical change caused by changes in the expected price ratio from input substitution

[5] Although not critical to the central theme of this paper, it is apparent that the qualitative impact of a change in the marginal cost of research on factor augmentation is also dependent on the magnitudes of ρ_i and θ_i.

effects caused by changes in the current price ratio. As with factor augmentation, the effect of an expected relative price change on the optimal input ratio depends on the magnitudes of both ρ_i and θ_i (see the second row in Table 1). An increase in the expected input price ratio, $\frac{E(w_{i1})}{E(w_{i2})}$, results in a production decision to save the input that becomes more expensive only if inputs are gross complements ($0 \le \rho_i < 1$) or if they are gross substitutes ($\rho_i > 1$) and the elasticity of substitution is less than 1 plus the concavity parameter ($\rho_i < \theta_i + 1$). If the inputs are gross substitutes ($\rho_i > 1$) and the elasticity of substitution is greater than 1 plus the concavity parameter ($\rho_i > 1 + \theta_i$), the increase results in a production decision to save the input that becomes less expensive.[6] In agreement with Acemoglu (2002, 2007), the latter documents that in the long run the demand curve for an input can be positively sloped.

Conclusions

We consider a two-period cost-minimization model involving technical change in which the firm allocates resources to research in the first period and selects input quantities in the second period. The model allows us to distinguish between planned innovation and the input choice decision. We document that the IIH can fail both in Hicks original version of inducing innovation aimed at saving the more expensive input and in the classic interpretation of actually saving the more expensive input. We find that the elasticity of substitution and the rate of trade-off between research outcomes (the research concavity parameter) are both important in determining the range over which the IIH fails.

[6] The qualitative impact of a change in the marginal cost of research on the input choice decision is also dependent on the magnitudes of ρ_i and θ_i and is not monotonic.

Prior literature (Acemoglu 2002, Funk 2002, Armanville and Funk 2003) based on static cost minimization documented that factor augmenting technical change results in a production decision that saves the augmented input if and only if inputs are gross complements, i.e., if the elasticity of substitution is less than one. Considering a two-period dynamic optimization that accounts both for optimal allocation of resources to research and optimal choice of production inputs, we find that the factor augmentation decision also depends on the magnitude of the elasticity of substitution. It further depends on the magnitude of the research concavity parameter. Thus, the qualitative impact of changes in relative input prices on both factor augmentation and input choice decisions do not unambiguously favor the input that becomes more expensive. While the analytical findings support Salter's (1960) early objection toward the IIH, they also document that factor-saving behavior in response to a relative price increase is expected over a wide range of substitution elasticities.

References

Acemoglu, D. 2002. "Directed Technical Change." *Review of Economic Studies* 69:781-809.

——. 2007. "Equilibrium Bias of Technology" *Econometrica* 75(5):1371-1409.

——. 2015. "Localized and Biased Technologies: Atkinson and Stiglitz's New View, Induced Innovations, and Directed Technological Change." *Economic Journal* 125(583):443-463.

Ahmad, S. 1966. "On the Theory of Induced Invention." *Economic Journal* 76:344-357.

Armanville, I., and P. Funk. 2003. "Induced Innovation: An Empirical Test." *Applied Economics* 35:1627-1647.

Atkinson, A.B., and J.E. Stiglitz. 1969. "A New View of Technical Change." *Economic Journal* 79(315):573-578.

Cowan, B.W., D. Lee, and C.R. Shumway. 2015. "The Induced Innovation Hypothesis and U.S. Public Agricultural Research." *American Journal of Agricultural Economics* 97(3):727-42.

Funk, P. 2002. "Induced Innovation Revisited." *Economica* 69:155-171

Hicks, J.R. 1932. *The Theory of Wages*. London: Macmillan.

de Janvry, A., E. Sadoulet, and M. Fafchamps. 1991. "Agrarian Structure, Technological Innovations, and the State." Chapter 18 in P.K. Bardhan, ed. *The Economic Theory of Agrarian Institutions*. Cambridge: Oxford University Press.

Kawagoe, T., K. Otsuka, and Y. Hayami. 1986. "Induced Bias of Technical Change in Agriculture: The United States and Japan, 1880-1980." *Journal of Political Economy* 94(3):523-544.

Salter, W.E.G. 1960. *Productivity and Technical Change.* Cambridge, MA: MIT Press.

Table 1. Effect of a Change in Expected Input Price Ratio, $\Delta\left(\frac{E(w_{i1})}{E(w_{i2})}\right) > 0$

Impact on	Inputs Gross Complements $0<\rho<1$	Inputs Gross Substitutes	
		$1<\rho<\theta+1$	$\rho>\theta+1>1$
Factor augmentation $\Delta\left(\frac{\hat{a}_{i1}^{*}}{\hat{a}_{i2}^{*}}\right)$	Positive	Negative	Positive
Optimal input ratio $\Delta\left(\frac{x_{i1}^{*}}{x_{i2}^{*}}\right)$	Negative	Negative	Positive

Appendix I.

We develop the two-stage optimization conditions in this appendix. A single output Y in period 2 is produced by the following two-level CES production function as in equation (1):

$$Y = F\left(X_1, X_2\right) = \left[\delta X_1^{\frac{\rho-1}{\rho}} + (1-\delta) X_2^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \text{ for } \rho \in [0,\infty),$$

where ρ is the elasticity of substitution between input indices, and δ is a share parameter. The input indices X_1 and X_2 are produced respectively by pairs of inputs that also follow a CES form:

$$X_i = F_i\left(x_{i1}, x_{i2}; a_{i1}, a_{i2}\right) = \left[\delta_i (a_{i1} x_{i1})^{\frac{\rho_i-1}{\rho_i}} + (1-\delta_i)(a_{i2} x_{i2})^{\frac{\rho_i-1}{\rho_i}}\right]^{\frac{\rho_i}{\rho_i-1}}, \ i \in \{1,2\}$$

as in equation (2), where x_{ij} is input j used in production of input index *i*, and *a* is a factor-augmenting parameter that captures technical progress.

A cost-minimization problem for period 2 can be stated as:

$$\min \sum_i \sum_j w_{ij} x_{ij} \text{ for } i, j \in \{1,2\}$$

$$s.t.\ \overline{Y}_t = F\left(X_1, X_2\right)$$

where w_{ij} is the price of input x_{ij}. This gives the Lagrangian:

$$L = \sum_i \sum_j w_{ij} x_{ij} + \lambda\left(\overline{Y}_t - F\left(X_1, X_2\right)\right)$$

where λ is the Lagrangian multiplier. First-order conditions for x_{11} and x_{12} are:

(A-1) $$\frac{\partial L}{\partial x_{11}} = w_{11} - \lambda \frac{\partial F}{\partial X_1} \frac{\partial F_1}{\partial x_{11}} = 0,$$

(A-2) $$\frac{\partial L}{\partial x_{12}} = w_{12} - \lambda \frac{\partial F}{\partial X_1} \frac{\partial F_1}{\partial x_{12}} = 0$$

where $\frac{\partial F_1}{\partial x_{11}} = \delta_1 \left(x_{11}\right)^{\frac{-1}{\rho_1}} \left(a_{11}\right)^{\frac{\rho_1 - 1}{\rho_1}}$ and $\frac{\partial F_1}{\partial x_{12}} = \left(1-\delta_1\right)\left(x_{12}\right)^{\frac{-1}{\rho_1}} \left(a_{12}\right)^{\frac{\rho_1 - 1}{\rho_1}}$. We only derive the first-order conditions for the input pair, x_{11} and x_{12}. The conditions for the other pair of inputs, x_{21} and x_{22}, can be obtained analogously. Dividing (A-1) by (A-2), we obtain:

(A-3) $$\frac{w_{11}}{w_{12}} = \frac{\delta_1}{1-\delta_1} \left(\frac{x_{11}}{x_{12}}\right)^{\frac{-1}{\rho}} \left(\frac{a_{11}}{a_{12}}\right)^{\frac{\rho_1 - 1}{\rho_1}}.$$

Solving for $\frac{x_{11}}{x_{12}}$, we obtain the condition for the optimal ratio of inputs:

(A-4) $$\frac{x_{11}^*}{x_{12}^*} = \left(\frac{\delta_1}{1-\delta_1}\right)^{\rho_1} \left(\frac{w_{11}}{w_{12}}\right)^{-\rho_1} \left(\frac{a_{11}}{a_{12}}\right)^{\rho_1 - 1}$$

which is equation (3).

Now, consider research and development opportunities in period 1. For a given research budget $\bar{R}$, the innovation function is given by equation (4):

$$R_i = \left(c_{i1}\hat{a}_{i1}\right)^{\theta_i} + \left(c_{i2}\hat{a}_{i2}\right)^{\theta_i}$$

Where R_i is expenditure on research to augment the *i*th input index, the total research budget is assumed to be exogenously given and is fully expended, i.e., $\bar{R} \equiv R_1 + R_2$, $\hat{a}_{ij}$ is the factor-augmentation parameter which is assumed to be nonregressive, i.e., $\hat{a}_{ijt} > \hat{a}_{ij\tau}$ if $t > \tau$; $c_{ij} > 0$ denotes marginal research costs in period 1 for technology that is expected to augment x_{ij} by 1 percent in period 2; and $\theta_i > 1$ is a concavity parameter.

In the two-stage optimization problem, the cost-minimization problem for the firm's research resource allocation in period 1 can be expressed, as in equation (5), by:

$$\min_{\tilde{x}_{ij},\hat{a}_{ij}} \sum_i \sum_j E(w_{ij})\tilde{x}_{ij}$$

$$s.t.\ \bar{Y} = \left[\delta X_1^{\frac{\rho-1}{\rho}} + (1-\delta) X_2^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}},\ \bar{R} \equiv R_1 + R_2$$

where $X_i = \left[\delta_i (a_{i1}x_{i1})^{\frac{\rho_i-1}{\rho_i}} + (1-\delta_i)(a_{i2}x_{i2})^{\frac{\rho_i-1}{\rho_i}} \right]^{\frac{\rho_i}{\rho_i-1}}$, $R_i = \left(c_{i1}\hat{a}_{i1}\right)^{\theta_i} + \left(c_{i2}\hat{a}_{i2}\right)^{\theta_i}$, $E\left(w_{ij}\right)$ denotes expectation for price of input x_{ij} in period 2 given information available in period 1, and $\hat{a}_{ij}$ is the expected factor augmentation parameter in period 2. A tilde is given to input levels to denote that they are period 2 values "*conceived*" in period 1 and thus are distinguished from the values that are actually chosen by the firm in period 2.

The Lagrangian is given by

$$L = \sum_i \sum_j E\left(w_{ij}\right)\tilde{x}_{ij} + \lambda_Y \left(\bar{Y}_t - F\left(X_1, X_2\right)\right) - \lambda_R \left(\bar{R} - R_1 - R_2\right)$$

where λ_Y and λ_R are Lagrangian multipliers for the constraints on production and research budget, respectively. First order conditions for $\tilde{x}_{11}$ and $\tilde{x}_{12}$ are

(A-5) $$\frac{\partial L}{\partial \tilde{x}_{11}} = E\left(w_{11}\right) - \lambda_Y \frac{\partial F}{\partial X_1} \frac{\partial G_1}{\partial \tilde{x}_{11}} = 0$$

(A-6) $$\frac{\partial L}{\partial \tilde{x}_{12}} = E\left(w_{12}\right) - \lambda_Y \frac{\partial F}{\partial X_1} \frac{\partial G_1}{\partial \tilde{x}_{12}} = 0$$

where $\frac{\partial G_1}{\partial \tilde{x}_{11}} = \delta_1 \left(\tilde{x}_{11}\right)^{\frac{-1}{\rho_1}} \left(\hat{a}_{11}\right)^{\frac{\rho_1-1}{\rho_1}}$ and $\frac{\partial G_1}{\partial \tilde{x}_{12}} = \left(1-\delta_1\right)\left(\tilde{x}_{12}\right)^{\frac{-1}{\rho_1}} \left(\hat{a}_{12}\right)^{\frac{\rho_1-1}{\rho_1}}$.

First order conditions for $\hat{a}_{11}$ and $\hat{a}_{12}$ are

(A-7) $$\frac{\partial L}{\partial \hat{a}_{11}} = -\lambda_Y \frac{\partial F}{\partial X_1}\frac{\partial G_1}{\partial \hat{a}_{11}} + \lambda_R \frac{\partial R_1}{\partial \hat{a}_{11}} = 0$$

(A-8) $$\frac{\partial L}{\partial \hat{a}_{12}} = -\lambda_Y \frac{\partial F}{\partial X_1}\frac{\partial G_1}{\partial \hat{a}_{12}} + \lambda_R \frac{\partial R_1}{\partial \hat{a}_{12}} = 0$$

where $\frac{\partial G_1}{\partial \hat{a}_{11}} = \delta_1 \left(\tilde{x}_{11}\right)^{\frac{\rho_1 - 1}{\rho_1}} \left(\hat{a}_{11}\right)^{\frac{-1}{\rho_1}}$, $\frac{\partial G_1}{\partial \hat{a}_{12}} = \delta_1 \left(\tilde{x}_{12}\right)^{\frac{\rho_1 - 1}{\rho_1}} \left(\hat{a}_{12}\right)^{\frac{-1}{\rho_1}}$, $\frac{\partial R_1}{\partial \hat{a}_{11}} = \theta_1 \left(\hat{a}_{11}\right)^{\theta_1 - 1} \left(c_{11}\right)^{\theta_1}$ and

$\frac{\partial R_1}{\partial \hat{a}_{12}} = \theta_1 \left(\hat{a}_{12}\right)^{\theta_1 - 1} \left(c_{12}\right)^{\theta_1}$. Dividing (A-5) by (A-6) yields

(A-9) $$\frac{E\left(w_{11t}\right)}{E\left(w_{12t}\right)} = \frac{\delta_1}{1-\delta_1}\left(\frac{\tilde{x}_{11}}{\tilde{x}_{12}}\right)^{\frac{-1}{\rho_1}}\left(\frac{\hat{a}_{11}}{\hat{a}_{12}}\right)^{\frac{\rho_1 - 1}{\rho_1}}$$

Dividing (A-7) by (A-8) and rearranging yields

(A-10) $$\frac{\delta_1}{1-\delta_1}\left(\frac{\tilde{x}_{11}}{\tilde{x}_{11}}\right)^{\frac{\rho_1 - 1}{\rho_1}}\left(\frac{\hat{a}_{11}}{\hat{a}_{12}}\right)^{\frac{-1}{\rho_1}} = \left(\frac{\hat{a}_{11}}{\hat{a}_{12}}\right)^{\theta_1 - 1}\left(\frac{c_{11}}{c_{12}}\right)^{\theta_1}$$

Dividing (A-9) by (A-10) and solving for $\frac{\tilde{x}_{11}}{\tilde{x}_{12}}$, we obtain

(A-11) $$\frac{\tilde{x}_{11}}{\tilde{x}_{12}} = \left(\frac{E\left(w_{11}\right)}{E\left(w_{11}\right)}\right)^{-1}\left(\frac{\hat{a}_{11}}{\hat{a}_{12}}\right)^{\theta_1}\left(\frac{c_{11}}{c_{12}}\right)^{\theta_1}$$

Substituting (A-11) into (A-9) and solving for $\frac{\hat{a}_{11}}{\hat{a}_{12}}$ we obtain

(A-12) $$\frac{\hat{a}^*_{11}}{\hat{a}^*_{12}} = \left(\frac{\delta_1}{1-\delta_1}\right)^{\frac{\rho_1}{\psi_1}}\left(\frac{E\left(w_{11}\right)}{E\left(w_{11}\right)}\right)^{\frac{1-\rho_1}{\psi_1}}\left(\frac{c_{11}}{c_{12}}\right)^{\frac{-\theta_1}{\psi_1}}$$

where $\psi_1 = 1 + \theta_1 - \rho_1$. This is equation (6). Asterisks are given to the factor-augmenting parameters to denote that they are optimal values.

By substituting (A-12) into the optimal condition for period 2 (A-4) with $\hat{a}_{1j}^{*}$ replacing a_{1j}, and rearranging, we get

(A-13) $$\frac{x_{11}^{*}}{x_{12}^{*}} = \left(\frac{\delta_1}{1-\delta_1}\right)^{\frac{\theta_1\rho_1}{\psi_1}} \left(\frac{w_{11}}{w_{12}}\right)^{-\rho_1} \left(\frac{E\left(w_{11}\right)}{E\left(w_{11}\right)}\right)^{\frac{-(1-\rho_1)^2}{\psi_1}} \left(\frac{c_{11}}{c_{12}}\right)^{\frac{\theta_1(1-\rho_1)}{\psi_1}}$$

which is equation (7). The condition for $\frac{x_{21}^{*}}{x_{22}^{*}}$ can be analogously found as (A-13):

(A-14) $$\frac{x_{21}^{*}}{x_{22}^{*}} = \left(\frac{\delta_2}{1-\delta_2}\right)^{\frac{\theta_2\rho_2}{\psi_2}} \left(\frac{w_{21}}{w_{22}}\right)^{-\rho_2} \left(\frac{E\left(w_{21}\right)}{E\left(w_{22}\right)}\right)^{\frac{-(1-\rho_2)^2}{\psi_2}} \left(\frac{c_{21}}{c_{22}}\right)^{\frac{\theta_2(1-\rho_2)}{\psi_2}} .$$

Appendix II.

In this appendix, we demonstrate that the condition $\theta_i > 1$ ensures that $\frac{\partial \hat{a}_{i2}}{\partial \hat{a}_{i1}} < 0$ and $\frac{\partial^2 \hat{a}_{i2}}{\partial \hat{a}_{i1}^2} < 0$ on the innovation function, equation (4),

$$R_i = \left(c_{i1}\hat{a}_{i1}\right)^{\theta_i} + \left(c_{i2}\hat{a}_{i2}\right)^{\theta_i},$$

for $c_{ij} > 0$, $\hat{a}_{ij} > 0$, $j = \{1, 2\}$. At a given research budget, differentiating both sides of the innovation function with respect to $\hat{a}_{i1}$ yields

$$0 = \theta_i c_{i1}^{\theta_i} a_{i1}^{\theta_i - 1} + \theta_i c_{i2}^{\theta_i} a_{i2}^{\theta_i - 1} \frac{\partial \hat{a}_{i2}}{\partial \hat{a}_{i1}}.$$

Solving for $\frac{\partial \hat{a}_{i2}}{\partial \hat{a}_{i1}}$ yields

$$\frac{\partial \hat{a}_{i2}}{\partial \hat{a}_{i1}} = -\frac{c_{i1}}{c_{i2}}\left(\frac{c_{i1}\hat{a}_{i1}}{c_{i2}\hat{a}_{i2}}\right)^{\theta_i - 1} < 0.$$

Differentiating both sides of the innovation function’s first-derivative equation again with respect to $\hat{a}_{i1}$ gives

$$0 = \theta_i\left(\theta_i - 1\right)c_{i1}^{\theta_i} a_{i1}^{\theta_i - 2} + \theta_i\left(\theta_i - 1\right)c_{i2}^{\theta_i} a_{i2}^{\theta_i - 2}\left(\frac{\partial \hat{a}_{i2}}{\partial \hat{a}_{i1}}\right)^2 + \theta_i c_{i2}^{\theta_i} a_{i2}^{\theta_i - 1}\frac{\partial^2 \hat{a}_{i2}}{\partial \hat{a}_{i1}^2}.$$

Solving for $\frac{\partial^2 \hat{a}_{i2}}{\partial \hat{a}_{i1}^2}$ yields

$$\frac{\partial^2 \hat{a}_{i2}}{\partial \hat{a}_{i1}^2} = -\frac{\theta_i\left(\theta_i - 1\right)c_{i1}^{\theta_i} a_{i1}^{\theta_i - 2} + \theta_i\left(\theta_i - 1\right)c_{i2}^{\theta_i} a_{i2}^{\theta_i - 2}\left(\frac{\partial \hat{a}_{i2}}{\partial \hat{a}_{i1}}\right)^2}{\theta_i c_{i2}^{\theta_i} a_{i2}^{\theta_i - 1}} < 0.$$