Appendix:

DESIGNING A PRIVATIZED MARKET:

THE CASE OF SAN FRANCISO BAY AREA AIRPORTS

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This appendix describes the technical details of BLP GMM estimation of the demand and marginal cost parameters, as well as the existence, uniqueness, and the computation of equilibrium in the airport privatization game.

BLP GMM Estimation

Let $\Gamma_D\left(Z_j^D\right)$ and $\Gamma_C\left(Z_j^C\right)$ denote the vector-valued functions of the instruments; the moment condition in equation (22) of the text implies $E\left(\xi_j \cdot \Gamma_D\left(Z_j^D\right)\right)=0$ and $E\left(\eta_j \cdot \Gamma_C\left(Z_j^C\right)\right)=0$ such that a GMM estimator can be constructed by the following empirical analog of the moment conditions

$$\chi_D(\Theta)=M^{-1}\sum_{m=1}^{M}\sum_{j=1}^{N_m}\xi_j(\Theta)\cdot\Gamma_D\left(Z_j^D\right) \quad \text{and} \quad \chi_C(\Theta)=M^{-1}\sum_{m=1}^{M}\sum_{j=1}^{N_m}\eta_j(\Theta)\cdot\Gamma_C\left(Z_j^C\right) \tag{A1}$$

where Θ denotes the vector of unknown parameters (demand parameters and the remaining marginal cost parameters). For some weighting matrix Σ, the BLP GMM estimator of the unknown parameters, Θ_{GMM}^{BLP}, is the solution to the minimization problem

$$\Theta_{GMM}^{BLP} = \arg\min_{\Theta} \begin{bmatrix}\chi_D(\Theta)\\ \chi_C(\Theta)\end{bmatrix}' \cdot \Sigma \cdot \begin{bmatrix}\chi_D(\Theta)\\ \chi_C(\Theta)\end{bmatrix}. \tag{A2}$$

To evaluate the GMM objective function, it is necessary to invert the share equation in equation (10) of the text to express the unobservables in demand, ξ, as a function of the observed data and unknown parameters. Following Berry and Jia (2010), we can solve $\boldsymbol{\xi}_\mathbf{m} \equiv \left(\xi_1, \xi_2, \ldots, \xi_{J_m}\right)$ by iterating the contraction mapping

$$\boldsymbol{\xi}_{\mathbf{m}}^{\mathbf{t+1}} = \boldsymbol{\xi}_{\mathbf{m}}^{\mathbf{t}} + \lambda \cdot \left[\ln\left(\mathbf{S}_{\mathbf{m}}^{*}\right) - \ln\left(\mathbf{S}_{\mathbf{m}}\left(\boldsymbol{\xi}_{\mathbf{m}}^{\mathbf{t}}, \Theta, data\right)\right)\right] \tag{A3}$$

until $\boldsymbol{\xi}_{\mathbf{m}}^{\mathbf{t+1}}$ and $\boldsymbol{\xi}_{\mathbf{m}}^{\mathbf{t}}$ are sufficiently close. In equation (A3), $\mathbf{S}_{\mathbf{m}}^{*} \equiv \left(S_1^{*}, \ldots, S_{J_m}^{*}\right)'$ is the vector of observed product market shares in market m and $\mathbf{S}_{\mathbf{m}}\left(\boldsymbol{\xi}_{\mathbf{m}}^{\mathbf{t}}, \Theta, data\right)$ is the vector of predicted markets shares that is calculated using equation (10) of the text.

We employ the nested-fixed point (NFP) algorithm in Berry, Levinsohn, and Pakes (1995) to implement BLP GMM estimation of the model.

Algorithm 1: The NFP algorithm of BLP GMM estimation

Initialization. Make an initial guess Θ^0

Inner loop. Given any Θ^0,

Step 1: Solve the demand shocks given Θ^0 denoted by $\boldsymbol{\xi}_{\mathbf{m}}\left(\Theta^0\right) \equiv \left(\xi_1\left(\Theta^0\right), \ldots, \xi_{J_m}\left(\Theta^0\right)\right)$. The solution can be obtained by starting from a guess $\boldsymbol{\xi}_{\mathbf{m}}^{\mathbf{0}}$ and iterating the contraction mapping in equation (A3) until $\max\left\{\left|\xi_1^{t+1} - \xi_1^{t}\right|, \ldots, \left|\xi_1^{t+1} - \xi_1^{t}\right|\right\} \le \varepsilon_{Inner}$.

Step 2: Compute the mark-ups (the third term of the left hand side of equation (21) in the text) for each carrier in each market given Θ^0 and $\boldsymbol{\xi}_{\mathbf{m}}\left(\Theta^0\right)$.

Step 3: Compute the cost shocks $\boldsymbol{\eta}_{\mathbf{m}}\left(\Theta^0\right) \equiv \left(\eta_1\left(\Theta^0\right), \ldots, \eta_{J_m}\left(\Theta^0\right)\right)$ from equation (21) in the text.

Step 4: Interact demand and cost shocks with instruments to obtain the moment functions in equation (A1).

Outer loop. Search for the Θ that solves the minimization problem in equation (A2). The minimization problem is solved by MATLAB's unconstrained nonlinear optimization package, which implements the BFGS Quasi-Newton method with a cubic line search procedure. The convergence tolerance of the outer loop is denoted by ε_{Outer}.

Recent papers by Knittel and Metaxoglou (2008, 2011) and Dube, Fox, and Su (2011) have raised several numerical issues associated with the NFP algorithm, which we address here.

Numerical errors in the inner loop. In the NFP algorithm, the optimization procedure in the outer loop maximizes an objective function that is constructed by the demand and cost shocks that are solved from the inner loop. The solutions from the inner loop are subject to numerical errors because demand shocks are solved by iterating the contraction mapping and because numerical errors associated with the solution are determined by the chosen tolerance ε_{Inner}. Dube, Fox, and Su (2011) point out that a loose inner loop tolerance may cause the optimization procedure in the outer loop not to converge or to converge to a point that is not a valid local minimum under a loose outer loop tolerance. Following their suggestion, we choose stringent tolerances for both the inner and outer loops. Specifically, we set $\varepsilon_{Inner} = 10^{-12}$ and $\varepsilon_{Outer} = 10^{-5}$ in the estimation. We also tried $\varepsilon_{Inner} = 10^{-14}$ and $\varepsilon_{Outer} = 10^{-6}$ and the estimation results were virtually unaffected. However, the optimization procedure converged to a different point under the tolerances $\varepsilon_{Inner} = 10^{-12}$ and $\varepsilon_{Outer} = 10^{-3}$, indicating that the objective function of the estimation has an irregular surface and that a loose tolerance in the outer loop may lead to inaccurate results**.**

Multiple local minimums and saddle points. The minimization problem in BLP GMM estimation is highly nonlinear and has many parameters. Knittel and Metaxoglou (2008) found that in practice BLP GMM estimation usually faces multiple local minimums and saddle points. Estimation results can be sensitive to the initial guess of the parameters and to the employed optimization algorithm. Those numerical concerns are partly addressed by the stringent tolerances that are chosen. We address the concerns further by randomly varying the initial guess of the parameters and by employing a more robust optimization procedure—the Nelder-Mead simplex (direct search) method. Under the stringent tolerances that are chosen, our estimation results are robust to the different starting values of the parameters and the optimization algorithms.

Existence and uniqueness of a pure-strategy Nash equilibrium to the Bertrand game. In the BLP GMM estimation, the objective function of the minimization problem includes moment conditions from both the demand and cost side. The moment conditions from the cost side are derived from the first-order conditions of Bertrand competition. Such an exercise requires the existence of a pure-strategy equilibrium to Bertrand competition in each market. As we indicate later, we are not aware of a result that shows the existence of a pure-strategy equilibrium to the airline price competition that we are considering. If a pure-strategy equilibrium to the Bertrand game does not exist, then incorporating moment conditions that are derived from the first-order conditions would lead to estimates that are not consistent with the observed market outcomes.

We addressed this concern in two ways. First, similar to Berry and Jia (2010), we dropped the cost-side moment conditions from the estimation to see how the demand estimates would be affected and we did not find any significant changes in those parameter estimates. Second, we used the estimation results to compute the pure-strategy Nash equilibrium of Bertrand competition in each of the 120 city-pair markets given the observed flight frequencies. The computation procedure follows step 3 of Algorithm 1 in this appendix. The computation of a firm's best response to the prices of the products offered by other firms requires us to solve a system of non-linear equations defined by the first-order conditions in equation (5) of the text. We used MATLAB's *fsolve* function, which implements a dogleg trust-region version of Newton's method (Nocedal and Wright (1999)), and used a stringent termination tolerance (1e-6) and tried multiple starting values. The computation procedure always converged to the same equilibrium for every market in those experiments. More importantly, the convergent equilibrium outcomes in the 120 city-pair markets replicated the observed prices and demands very closely. Thus the results suggest that the model estimates from the BLP GMM estimation are consistent with the existence of a unique equilibrium. Although a unique equilibrium is not essential for model estimation, it is essential for our policy simulations.

Two-stage estimation and the GMM standard errors. The supply side moment conditions in the BLP GMM estimation are derived from equation (21) of the text, which is computed by using the estimated parameters in equation (14) of the text. Thus, the statistical uncertainty of the parameters in the first stage carries through to the second stage GMM estimation and requires the GMM standard errors to be corrected (the parameter estimates are still consistent).

One acceptable approach is the bootstrap technique. The reported standard errors in tables 4 and 6 are estimated by

$$Var\left(\hat{\Theta}\right)=\left(\Psi'\Sigma\Psi\right)^{-1}\left(\Psi'\Sigma V\Sigma\Psi\right)\left(\Psi'\Sigma\Psi\right)^{-1} \tag{A4}$$

where Ψ is the Jacobian matrix of the moment function evaluated at the GMM estimates; the weighting matrix takes the form of

$$\Sigma = diag\left(\left(\left(Z^{D}\right)'\left(Z^{D}\right)\right)^{-1},\left(\left(Z^{C}\right)'\left(Z^{C}\right)\right)^{-1}\right)$$

and $V=\sum_{m=1}^{M}\sum_{j=1}^{N_m}\Gamma_C\left(Z_j^C\right)'\Gamma_D\left(Z_j^D\right)'\left(\xi_j\left(\hat{\Theta}\right),\eta_j\left(\hat{\Theta}\right)\right)'\left(\xi_j\left(\hat{\Theta}\right),\eta_j\left(\hat{\Theta}\right)\right)\Gamma_D\left(Z_j^D\right)\Gamma_C\left(Z_j^C\right)$. To implement the bootstrap: take a draw from the estimated asymptotic distribution of the parameter estimates in equation (14) of the text; compute the residuals $\left(\varepsilon_{if}^{Z}\right)$ given the draw; evaluate the marginal aircraft operating costs by using equation (15) given the draw and the resulting residuals; substitute the computed marginal aircraft operating costs into equation (21) to obtain the supply-side equation in the BLP GMM estimation; and run the BLP GMM estimation in equation (A2). The preceding steps can be repeated and the variance-covariance matrix of the parameters in (A4) can computed for each repetition. The averaged variance-covariance matrix could then be used to construct the standard errors of the BLP GMM estimates to take account of the statistical uncertainty in estimating the marginal aircraft operating costs.

Unfortunately, although the bootstrap technique is a valid approach, it is not feasible here because BLP GMM estimation is so time consuming. However, we can comment on why our standard errors would not be affected much by this correction. First, the parameter estimates in the marginal aircraft operating cost equation are highly statistically significant. Second, as noted, dropping the cost-side moments in the BLP GMM estimation had little effect on the demand estimates. Third, most of the cost parameter estimates in table 6 are highly statistically significant. Finally, as a partial test, we conditioned on the mark-up estimates from the BLP GMM estimation, used OLS to re-estimate the supply-side parameters in equation (21), used the preceding bootstrap procedure to account for the statistical uncertainty associated with the marginal aircraft operating costs, and found that the standard errors of the cost parameter estimates changed very little.

Computing the SPE to the Three-Stage Game of Airport Privatization under Different Scenarios

Computing the SPE to the three-stage game of airport privatization under different scenarios requires an algorithm to compute the SPE to the airline service-price subgame given airport charges. Definition 1 of the text summarizes the necessary conditions of the SPE under Assumption 4. We develop an algorithm to locate points that satisfy the necessary conditions. The computation is done by iterating the fixed-point equation that is defined in equation (6) of the text. Details of the algorithm are as follows.

Algorithm 2: Fixed-point iteration to compute the SPE to airline capacity-price competition

Step 1: *Initialization.* Initialize capacities on the spoke-routes that are connected to the three SF airports.

Step 2: *Updating flight frequencies and airport delays*. Taking the average aircraft size as given, we determine the frequencies of each product and the airport delays. The flight frequencies for products using connecting flights (d_i) are determined by the flight frequencies on both segments based on regressing d_i on the number of daily airline departures on each flight segment:

$$d_i = \begin{cases} 0.3482 \times x_1 + 0.6153 \times x_2 & if \quad x_1 \geq x_2 \\ \\ 0.8199 \times x_1 + 0.0231 \times x_2 & if \quad x_1 < x_2 \end{cases} \tag{A5}$$

where x_1 and x_2 are the flight frequencies on the first and second segment respectively. Daily flights at the three SF airports are divided equally into 15 minute intervals; equation (17) in the text is then used to calculate average airport delays during operating hours, which are calibrated to obtain the observed delays at equilibrium.[1]

Step 3: *Computing the equilibrium of price competition given flight frequencies*. Given the flight frequencies and airport delays, we solve for the competitive equilibrium prices in each of the 120 markets. We compute market equilibrium prices iteratively by allowing each carrier in the market to optimize the prices of the air travel products it offers taking the other carriers' prices of their products as fixed. An iteration of this computational procedure then involves F optimization problems, where F is the number of carriers in the market, which are solved by using the first-order condition in equation (5) of the text. Marginal costs in the first-order condition are calculated by equation (3)[2] and the mark-ups are calculated by using the third term of the left hand side of equation (21)[3] in the text. Let $\mathbf{p}_\mathbf{m}^{\mathbf{t+1}}$ and $\mathbf{p}_\mathbf{m}^{\mathbf{t}}$

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[1] The average departure and arrival delays (in minutes) in the third quarter of 2007 are 17 and 5 at SFO, 10 and 2 at OAK, and 8 and 3 at SJC.

[2] We include the estimated random component $\hat{\eta}_j$, which captures the shift in marginal costs caused by unobserved product attributes.

[3] We include the estimated random component $\hat{\xi}_j$, which captures travelers' preferences for unobserved product attributes.

denote the price vectors of market m from two successive iterations, equilibrium is reached when $\left\| \mathbf{p}_{\mathbf{m}}^{\mathbf{t+1}} - \mathbf{p}_{\mathbf{m}}^{\mathbf{t}} \right\|_{\infty} \leq 0.01$.

Step 4: *Convergence check*. Given equilibrium prices from step 3, compute demand quantities for the 20830 products using equations (10), (11), and (12) in the text and the estimated parameters. Given the realized demand, each carrier updates the capacity on its spoke routes that are connected to the three SF airports to achieve a load factor of 70%. Let $\mathbf{T}^{\mathbf{t}+1}$ and $\mathbf{T}^{\mathbf{t}}$ denote the vectors of spoke capacities from two successive iterations, we check for convergence as follows: if $\left\| \mathbf{T}^{\mathbf{t+1}} - \mathbf{T}^{\mathbf{t}} \right\|_{\infty} \leq 1$, stop; otherwise set $\mathbf{T}^{\mathbf{t}} = \mathbf{T}^{\mathbf{t}+1}$ and return to step 2.

The algorithm can locate an equilibrium only if Bertrand equilibrium exists for each market given spoke capacities and if a fixed point of the self-map $\mathbf{H}(\cdot)$ exists given the load factor of 70%. Because the capacity constraint in equation (1) of the text is not binding at the equilibrium, we confine our discussion of the equilibrium of the price subgame to a given market with multi-product firms facing discrete-choice demands. Caplin and Nalebuff (1991) and Mizuno (2003) have studied the existence and uniqueness of pure strategy equilibrium in a price game among single-product firms facing discrete-choice demands (including the nested-logit model used in this paper). In the case of multi-product firms, Allon, Federgruen, and Pierson (2010) have shown that when total product costs are linear, price competition among multi-product firms with multinomial logit demand functions and random coefficients is supermodular and has a unique Nash equilibrium when prices are bounded by $\tilde{\mathbf{p}}$, where each element in the upper bound is the sum of the product's marginal cost and the absolute value of the inverse of the marginal utility of price. Using a fixed-point approach, Morrow and Skerlos (2010) prove the existence of Bertrand-Nash equilibrium for multi-product firms that face multinomial logit demand with an underlying utility function that can be nonlinear.

Starting from different initial guesses, we found that step 3 of algorithm 1 can always locate a unique equilibrium for each market given spoke capacities. This result suggests that within the price space that we are exploring, a unique equilibrium exists for airline price competition in each city-pair market. For the outer loop that iterates the fixed-point equation $\mathbf{H}(\cdot)$ given a 70% load factor, we checked the robustness of the results by varying the initial guesses of spoke capacities and found no significant changes to the results. This is plausible because $\mathbf{H}(\cdot)$ is expected to be monotonically increasing. Larger spoke capacities indicate more frequent flights and higher service quality; in equilibrium, greater demand for air travel products exists when spoke capacities are larger. And when $\mathbf{H}(\cdot)$ is monotonically increasing, there is at most one fixed point for $\mathbf{H}(\cdot)$. The algorithm can also locate a unique solution if we increase the target load factor to 80% and 90%. However, we were unable to find a solution if we reduced the target load factor to 60%.[4]

With Algorithm 1, we can compute the SPE to the three-stage game of airport privatization under different scenarios. When only SFO is privatized, we use the grid search to determine the SPE to the three-stage game. Specifically, we first consider a wide range of airport charges at SFO [2, 200]. We then take 50 equally spaced points from the interval. We do a loop by setting the airport charge at SFO at each of the 50 points and in an iteration of the loop, airport charges at OAK and SJC are set as the current weight-based landing charges. Algorithm 1 is used to compute the equilibrium outcomes of the airlines' capacity-price subgame given airport charges at each of the iterations. The data set from the loop is used to fit curves by spline interpolation, which represent the objective values (airport profits, social welfare, and airline profits under the equilibrium outcomes of the capacity-price subgame) as

[4] This finding is plausible because as shown in equation (6) of the text, the inverse of the load factor is the multiplier that maps a set of product demands to a set of spoke capacities. A smaller load factor implies a larger multiplier, such that a small disturbance can cause the dynamic system $\mathbf{H}(\cdot)$ to move away from the fixed-point when the load factor is small. Hence, the fixed-point is likely to be unstable when the load factor is small and the numerical algorithm cannot locate the fixed-point.

functions of the SFO charge. For the scenario of the SFO profit-maximizing charge subject to a non-negative change in consumer surplus (the last column of Table 8), the grid search is done over a much smaller interval [2, 20] because the solution to the problem is expected to result in a small charge.

When the three airports are privatized and purchased by the same firm, the private owner charges landing and takeoff fees at the three airports to maximize certain objectives. Again, we use the grid search to solve for the optimal charges at the three airports to maximize a certain objective. The grid search is on a three-dimensional space and we take 6 equally spaced points from the interval [2, 150] from each dimension. In total, we have 6^3 different airport charges and we use Algorithm 1 to compute the equilibrium outcomes of the capacity-price subgame of the airlines given each of those airport charges. We then use our data to approximate a surface by 3-dimension spline interpolation, which represents the objective value as a function of the charges at the three airports.

When the three airports are sold to different owners, the three airports engage in Bertrand competition. We develop an algorithm to compute the SPE to the three-stage oligopoly airport pricing game and the SPE is defined in Definition 2 of the text. An allocation of travelers across products under $\Psi(\varsigma)$ (an equilibrium of the service-price subgame of airlines given a set of airport charges ς) is denoted by the set $\{q_j(\Psi(\varsigma))\}_j$. We can define the demand (number of passengers) for airport n as

$$Q_n\left(\varsigma_n^A, \varsigma_{-n}^A\right) = \sum_{s \in \Phi_n} \left\{ \sum_f \left(\sum_{j \in J_{sf}} q_j\left(\Psi\left(\varsigma_n^A, \varsigma_{-n}^A\right)\right) \right) \right\} \quad . \tag{A6}$$

Properties of the demand function, such as own-price and cross-price elasticities, cannot be characterized analytically because a change in airport charges causes a change in $\Psi(\varsigma)$, which cannot be characterized analytically. Travelers respond to the change in equilibrium by adjusting their choices of air travel products to maximize utility.

As shown in Topkis (1979), the sufficient conditions for such a game to be a supermodular game are:

C1: $Q_n\left(\varsigma_n^A, \varsigma_{-n}^A\right)$ is an isotone function of $\varsigma_{n'}^A$ for each $n' \neq n$; and

C2: $-Q_n\left(\varsigma_n^A, \varsigma_{-n}^A\right)$ has antitone differences in $\left(\varsigma_n^A, \varsigma_{n'}^A\right)$ for each $n' \neq n$.

The first condition says that increasing the landing and takeoff charge at airport n' will increase the demand for airport n. The second condition says that the demand for airport n is more sensitive to its own landing and takeoff charge when the charge at airport n' is lower (and thus airport n' is more competitive with airport n).

Although we cannot verify analytically whether the demand function in (A6) satisfies those two conditions, it is reasonable to assume that it does. When an airport increases its landing and takeoff charges, the marginal aircraft operating costs of products that originate or terminate at the airport will increase, reducing the attractiveness of those products compared with substitutes using the other two airports (at the equilibrium of airline capacity-price competition given the airport charges) because those products will be relatively more expensive and less convenient (less flight frequency). The discrete choice demand model implies that travelers will respond to those changes by shifting to substitute products that use the other two airports; shifts to substitute products that use another airport are more likely when the charge at that airport decreases and charges at alternative airports are fixed.

Another assumption that we make to simplify the computation is that airport charges take discrete values when they are expressed in dollars. Under this assumption the incremental change of the landing and takeoff charges at an airport is \$1.

The preceding assumptions ensure a nonempty set of the SPE (defined in Definition 2) exists that has the least and largest element. By definition, the three airports' charges at the least equilibrium point are the lower bound of airport charges at all equilibrium points of the set. At the least equilibrium point, travelers' surplus is the largest among all equilibrium points. The algorithm that computes the least point of the set of SPE is defined as follows.

Algorithm 2: Computing the SPE to the three-stage oligopoly airport price competition

Step 1: *Initialization*. Set $\boldsymbol{\varsigma}^0 \equiv \left({\varsigma_{SFO}^{A}}^{0}, {\varsigma_{SJC}^{A}}^{0}, {\varsigma_{OAK}^{A}}^{0} \right) = (0,0,0)$;

Step 2: *Updating*. Given airport charges from the last iteration $\boldsymbol{\varsigma}^t \equiv \left({\varsigma_{SFO}^{A}}^{t}, {\varsigma_{SJC}^{A}}^{t}, {\varsigma_{OAK}^{A}}^{t} \right)$

i) Solve the equilibrium of the capacity-price competition of airlines when airport charges are $\boldsymbol{\varsigma}^{try} \equiv \left({\varsigma_{SFO}^{A}}^{t} + 1, {\varsigma_{SJC}^{A}}^{t}, {\varsigma_{OAK}^{A}}^{t} \right)$ using Algorithm 1 and evaluate airport profits at the equilibrium. If $\pi_{SFO}^{A}\left(\boldsymbol{\varsigma}^t\right) < \pi_{SFO}^{A}\left(\boldsymbol{\varsigma}^{try}\right)$, set ${\varsigma_{SFO}^{A}}^{t+1} = {\varsigma_{SFO}^{A}}^{t} + 1$; otherwise, set ${\varsigma_{SFO}^{A}}^{t+1} = {\varsigma_{SFO}^{A}}^{t}$. Finally, set $\boldsymbol{\varsigma}^t = \left({\varsigma_{SFO}^{A}}^{t+1}, {\varsigma_{SJC}^{A}}^{t}, {\varsigma_{OAK}^{A}}^{t} \right)$.

ii) Solve the equilibrium of the capacity-price competition of airlines when airport charges are $\boldsymbol{\varsigma}^{try} \equiv \left({\varsigma_{SFO}^{A}}^{t+1}, {\varsigma_{SJC}^{A}}^{t} + 1, {\varsigma_{OAK}^{A}}^{t} \right)$ using Algorithm 1 in the appendix and evaluate airport profits at the equilibrium. If $\pi_{SJC}^{A}\left(\boldsymbol{\varsigma}^t\right) < \pi_{SJC}^{A}\left(\boldsymbol{\varsigma}^{try}\right)$, set ${\varsigma_{SJC}^{A}}^{t+1} = {\varsigma_{SJC}^{A}}^{t} + 1$; otherwise, set ${\varsigma_{SJC}^{A}}^{t+1} = {\varsigma_{SJC}^{A}}^{t}$. Finally, set $\boldsymbol{\varsigma}^t = \left({\varsigma_{SFO}^{A}}^{t+1}, {\varsigma_{SJC}^{A}}^{t+1}, {\varsigma_{OAK}^{A}}^{t} \right)$.

iii) Solve the equilibrium of the capacity-price competition of airlines when airport charges are $\boldsymbol{\varsigma}^{try} \equiv \left({\varsigma_{SFO}^{A}}^{t+1}, {\varsigma_{SJC}^{A}}^{t+1}, {\varsigma_{OAK}^{A}}^{t} + 1 \right)$ using Algorithm 1 in the appendix and evaluate airport profits at the equilibrium. If $\pi_{OAK}^{A}\left(\boldsymbol{\varsigma}^t\right) < \pi_{OAK}^{A}\left(\boldsymbol{\varsigma}^{try}\right)$, set ${\varsigma_{OAK}^{A}}^{t+1} = {\varsigma_{OAK}^{A}}^{t} + 1$; otherwise, set ${\varsigma_{OAK}^{A}}^{t+1} = {\varsigma_{OAK}^{A}}^{t}$.

iv) Set $\boldsymbol{\varsigma}^{t+1} = \left({\varsigma_{SFO}^{A}}^{t+1}, {\varsigma_{SJC}^{A}}^{t+1}, {\varsigma_{OAK}^{A}}^{t+1} \right)$.

Step 3: *Convergence check*. If $\boldsymbol{\varsigma}^{t+1} = \boldsymbol{\varsigma}^t$, stop; otherwise, return to step 2.

The algorithm generates three monotone sequences $\varsigma_n^{A^0} \le \varsigma_n^{A^1} \le \cdots \le \varsigma_n^{A^t} \le \varsigma_n^{A^{t+1}} \le \cdots$, $\forall n \in \{SFO, SJC, OAK\}$. When the oligopoly airport pricing game is supermodular, the sequences converge to $\boldsymbol{\varsigma}^* \equiv \left({\varsigma_{SFO}^{A}}^{*}, {\varsigma_{SJC}^{A}}^{*}, {\varsigma_{OAK}^{A}}^{*} \right)$, which is the lowest point of the equilibrium set.

We also consider negotiation scenarios between airports and airlines, where airlines that operate at an airport form a bargaining unit. One extreme case of those scenarios is that the landing and takeoff charges at each airport seek to maximize an airport's profits, but they are subject to the constraint of a nonnegative change (compared with the base case under the current weight-based landing fees) in the aggregate profits of the airlines that operate at the airport. Thus the constraint set faced by an airport is $\Phi_n \equiv \left\{\varsigma_n^A : \pi_n^F\left(\Psi\left(\varsigma_n^A, \varsigma_{-n}^A\right)\right) - \pi_n^F\left(\Psi\left(\hat{\varsigma}_n^A, \hat{\varsigma}_{-n}^A\right)\right) \geq 0\right\}, \forall n \in \{SFO, SJC, OAK\}$, where $\pi_n^F\left(\Psi\left(\varsigma_n^A, \varsigma_{-n}^A\right)\right)$ and $\pi_n^F\left(\Psi\left(\hat{\varsigma}_n^A, \hat{\varsigma}_{-n}^A\right)\right)$ are the aggregate profits of the airlines operating at airport n evaluated at the equilibria $\Psi\left(\varsigma_n^A, \varsigma_{-n}^A\right)$ and $\Psi\left(\hat{\varsigma}_n^A, \hat{\varsigma}_{-n}^A\right)$, which in turn are the equilibria of the capacity-price competition of airlines given airport charges $\varsigma \equiv \left(\varsigma_n^A, \varsigma_{-n}^A\right)$ and given current weight-based landing fees $\hat{\varsigma} \equiv \left(\hat{\varsigma}_n^A, \hat{\varsigma}_{-n}^A\right)$. We claim that the constraint set satisfies the following condition:

C3: Let $\varsigma^0 \equiv \left(\varsigma_n^A, \varsigma_{n'}^{A^0}, \varsigma_{-n,-n'}^A\right)$ and $\varsigma^1 \equiv \left(\varsigma_n^A, \varsigma_{n'}^{A^1}, \varsigma_{-n,-n'}^A\right)$, where $\varsigma_{-n,-n'}^A$ is the set of airport charges excluding airport n and n'. If $\varsigma_{n'}^{A^1} > \varsigma_{n'}^{A^0}$, we have

$$\Phi_n^0 \equiv \left\{\varsigma_n^A : \pi_n^F\left(\Psi\left(\varsigma^0\right)\right) - \pi_n^F\left(\Psi\left(\hat{\varsigma}\right)\right) \geq 0\right\} \subset \Phi_n^1 \equiv \left\{\varsigma_n^A : \pi_n^F\left(\Psi\left(\varsigma^1\right)\right) - \pi_n^F\left(\Psi\left(\hat{\varsigma}\right)\right) \geq 0\right\}$$

The condition says that the constraint set faced by an airport expands when another airport increases its landing and takeoff charges. Under the assumption that the demand function in (A6) satisfies condition C1, the constraint set faced by each airport satisfies C3 such that Algorithm 3 discussed below can be easily modified by incorporating the check for the constraint in step 2. The modified algorithm finds the lowest point of the equilibrium set.

Another extreme case of the negotiation scenarios is that the landing and takeoff charges at each airport seek to maximize the aggregate profits of airlines operating at the airport, but they are subject to the constraint that the airport's profits are nonnegative. Conditions C1-C3 are satisfied in this bargaining game and therefore the analysis of the equilibrium to this game is the same as in the previous analysis.

Table A1. Sensitivity analysis to changes in aircraft size after privatization

	No change in aircraft size after privatization **Bertrand competition: charges for commercial flights maximize airports' profits subject to a non-negative change in airline profits at each airport and given a price-cap on the GA charge**	*Increase aircraft size 50% after privatization* **Bertrand competition: charges for commercial flights maximize airports' profits subject to a non-negative change in airline profits at each airport and given a price-cap on the GA charge**
Airport charge for commercial airlines($/seat) [a]		
SFO	50	75
SJC	49	61
OAK	49	48
Airport charge for general aviation ($/flight) [b]		

SFO	280	280
SJC	280	280
OAK	280	280
Airport Delay (min.)		
SFO		
Departure delay	10	7
Arrival delay	3	3
SJC		
Departure delay	3	1
Arrival delay	1	1
OAK		
Departure delay	3	2
Arrival delay	2	2
Change in Airport Profits (million $/quarter)		
SFO	136.16	176.35
SJC	80.56	97.14
OAK	183.97	206.05
Total	400.69	479.54
Change in Airlines' Profits by airport (million $/quarter)		
By airports		
SFO	0.61	3.47
SJC	0.00	1.53
OAK	10.16	43.87
Total	10.77	48.87
Consumer surplus change (million $/quarter)[c]		
Business travelers	-2.14	-2.29
Leisure travelers	-1.16	-1.23
General aviation travelers	-5.84	-5.54
Total	-9.14	-9.06
Change in social welfare (million $/quarter)[c]	402.32	519.35

[a] The airport charge in the base case is the 2007 weight-based landing fee that is charged when a commercial carrier lands at an airport. The carrier is not charged when it takes off from an airport. Travelers pay passenger facility charges that are included in the fare. In the privatization scenarios, the weight-based landing charge and the passenger facility charges are replaced with the following charges. When an aircraft takes off from a San Francisco Bay Area airport, it is assessed the charge indicated in the column heading (e.g., airport profit maximizing charge) as well as the weight-based landing charge at the non-San Francisco Bay Area airport. When an aircraft takes off from a non-San Francisco Bay Area airport it is not assessed a charge by that airport but it is assessed the charge indicated in the column heading (e.g., airport profit maximizing charge) for its landing at a San Francisco Bay Area airport.

[b] The airport charge at SFO for general aviation in the base case is the current minimal charge when a general aviation aircraft lands at the airport. A general aviation aircraft is not charged when it takes off from SFO. In the privatization scenarios, the current charge is replaced by a charge that is applied to both take-off and landing.

[c] Measured as the change from the base case.