The Prudent Principal*

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Abstract

This paper re-examines executive incentive compensation, using a principal-agent model in which the principal is downside risk averse, or prudent (as a number of empirical facts and scholarly works suggest it should be), instead of risk neutral (as it has been commonly assumed so far in the literature). We find that optimal incentive pay should then be ‘approximately concave’ in performance, the approximation being closer the more prudent the principal is relative to the agent. This means that an executive should face higher-powered incentives while in the bad states, but be given somewhat weaker incentives when things are going well. Such a statement runs counter to current evidence that incentive compensation packages are often convex in performance. We show that this disparity can be justified under certain limited liability and taxation regimes. Implications for public policy and financial regulation are briefly discussed.

Keywords: Executive compensation, downside risk aversion, prudence, approximately concave functions

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1 Introduction

The 2008 financial crisis has put again the spotlight on executive pay. One highlighted feature is the increasing *convexification* over the past decades - through the more and more widespread use of call options, notably - of the pay-performance relationship in incentive packages: in other words, managerial rewards have generally become very responsive to upside gains but relatively immune to poor results. This asymmetry is now being questioned by several scholars (see, e.g., Yermack 1995; Jensen and Murphy 2004; Murphy and Jensen 2011; Boyer 2011). In its January 2011 report, the National Commission in charge of investigating the causes of the financial and economic downturn maintained that:

> Compensation systems - designed in an environment of cheap money, intense competition, and light regulation - too often rewarded the quick deal, the short-term gain - without considerations of long-term consequences. Often those systems encouraged the big bet - where the payoff *on the upside could be huge and the downside limited*. This was the case up and down the line - from the corporate boardroom to the mortgage broker on the street. (emphasis added)

One might impute this state of affairs to managerial power (Finkelstein 1992; Bebchuck and Fried 2003) or other systematic behavioral and governance failures (Gervais et al. 2011; Jensen et Murphy 2004, p. 50-81; Ruiz-Verdú 2008), although these views are increasingly being questioned (Holmstrom 2005, Kaplan 2012). In this paper, we reconsider the main framework to deal with executive compensation - the principal-agent model. The ’principal’ in this context stands for shareholders or the corporate board and has been commonly viewed as being risk-neutral. This assumption can be challenged on at least two grounds. First, since Roy (1952) and Markowitz (1959), a number of economists have contended that investors react asymmetrically to gains and losses. Corroborating this, Harvey and Siddique (2000), Ang et al. (2006), and others report that stock returns do reflect a premium for bearing downside risk. Second, corporate law and jurisprudence endow corporate board members with *fiduciary duties of loyalty and care* towards their corporation (Clark 1985; Gutierrez 2003; Adams et al. 2010; Lan and Herakleous 2010; Corporate Law Committee 2011). The latter charge notably confers board directors and officers a key role

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1 As far as CEO (not traders) compensation is concerned, this assertion has now been qualified (see, e.g., Murphy 2012). Whether and when convex incentive schemes are appropriate remains, however, a fundamental issue.

2 In addition to the managerial power and principal-agent approaches, other theories of incentive pay include contests and tournaments (Shen et al. 2010), career concerns (Hermalin 2005), organizational structure (Santaló and Kock 2009), pay-fortune (Bertrand and Mullainathan 2001), firms-managers matching (Gabaix and Landier 2008), social influence (O’Reilly and Main 2010), the contracting environment (Cadman et al. 2010), and government policy (Murphy 2011). Recent accounts of these theories can be found in the indicated references. Principal-agent theory remains, however, the dominant approach to analyze executive compensation from either a positive or a normative perspective. For general surveys of CEO compensation, see Baker et al. (1988), Murphy (1999), Jensen and Murphy (2004), Edmans and Gabaix (2009), and Adams et al. (2010).
in preventing and managing crisis situations (Mace 1971; Williamson 2007; Adams et al 2010). Directors and officers should accordingly “(...) exercise that degree of care, skill, and diligence which an ordinary, prudent man would exercise in the management of his own affairs.” (Clark 1985, p. 73; emphasis added). This requirement should drive corporate boards (which are chiefly responsible in setting top executives’ compensation) to weigh differently the risks correlated with downside losses versus those linked to upside gains. Such behavior is of course inconsistent with risk neutrality, so one might reasonably question some of the prescriptions from current and past principal-agent analyses of incentive pay.

In what follows, we therefore examine executive compensation using a principal-agent model in which the principal is ‘prudent,’ in the sense introduced in economics by Kimball (1990). This indeed portrays the principal as a downside risk averse entity.\(^3\) A prudent decision maker dislikes mean and variance-preserving transformations that skew the distribution of outcomes to the left (Menezes et al 1980; Crainich and Eeckhoudt 2008). Equivalently, she prefers additional volatility to be associated with good rather than bad outcomes (Eeckhoudt and Schlesinger 2006; Denuit et al 2010). Formally, someone is prudent when her marginal utility function is strictly convex (it is of course constant in the risk neutral case). As a characteristic of the agent’s (not the principal’s) preferences - the agent standing here for an executive or a top manager, prudence has already been dealt with and found relevant in the literature, especially in contingent monitoring (Fagart and Sinclair-Desgagné 2007) and background risk (Ligon and Thistle 2008) situations, and to explain the composition of incentive pay (Chaigneau 2012).\(^4\) To our knowledge, this is the first time prudence is taken to also be an attribute of the principal.

In a benchmark model, we obtain that incentive compensation should be approximately concave (as defined by Páles 2003) in performance, the approximation being closer the more prudent the principal is relative to the agent. This complements Hemmer et al (2000)’s and Chaigneau (2012)’s respective propositions that relate convexity of the agent’s remuneration to the agent’s prudence under a risk neutral principal, and Hau (2011)’s converse statement that a risk averse principal may ‘concavify’ the reward function of a risk neutral agent. The principle underneath our general statement seems straightforward: whoever is relatively more prudent should bear less downside risk. A convex incentive scheme, being very sensitive to performance in upbeat situations and rather flat in the range where results are mediocre, shelters a prudent agent against downside volatility which must then be born by the principal. A concave scheme, by contrast, rewards performance improvements much more strongly under adverse circumstances and makes the agent bear significant downside risk; a prudent principal thereby decreases her own exposure

\(^3\) An alternative way to capture loss aversion would be to assume that utility declines sharply (albeit at a decreasing rate) below some reference point, as in prospect theory (Kahneman and Tversky 1979). Dittman et al (2010), for instance, use this representation of an agent’s preferences to analyze executive compensation; the principal is risk neutral in their model.

\(^4\) Empirical evidence that executives are prudent can be found in McAnally et al (2011), Garvey and Milbourn (2006), and the references therein.
to downside risk by firmly pushing her agent to get away from dangerous territory.

Whether the principal is more or less prudent relative to the agent should therefore be an important practical matter in setting optimal compensation contracts. Yet, the principal’s prudence does not seem to matter much so far in most industries. According to Garvey and Milbourn (2006, p. 198), “(...) the average executive loses 25-45% less pay from bad luck than is gained from good luck.” In other words, convex executive contracts are quite common, owing notably to the widespread use of stock options (see, e.g., Hall and Murphy 2003) and performance shares (Equilar 2012 a and b). We argue below that this can be justified under some common government policies such as limited liability and progressive taxation. In cases where the agent/executive cannot be inflicted negative revenues or the principal/corporation can be refunded when net profits are negative (which can be seen as a rough proxy for the 2008 TARP - Trouble Asset Relief Program - rescue of financial institutions), approximately concave contracts are no longer optimal even if the principal is very prudent. A similar conclusion holds when the principal’s profits are taxed (which corresponds to a British government’s proposal concerning banks’ profits). When executive income is subject to progressive taxation (which roughly reproduces suggestions actively debated in the U.S. and implemented in France), the upshot is even more radical: a prudent principal might squarely offer convex rewards in order to circumvent the effect of taxation and properly encourage the agent to pursue the better states of nature.

The rest of the paper unfolds as follows. Section 2 presents the benchmark model - a static principal-agent model where the agent is effort and risk averse while the principal is both risk averse and prudent; we assume throughout that the first-order approach, as justified in Rogerson (1985), is valid. Our central proposition - that the optimal contract should then be approximately concave, thereby seeking a balance between the agent’s and the principal’s respective prudence - is established in Section 3. Sections 4 and 5 next show that limited liability and taxation can respectively produce deviations from this prescription. Section 6 contains some concluding remarks and policy recommendations. All proofs are in the Appendix.

5 A notable exception is the utility sector, where the pay-performance relation is actually concave (Murphy 1999). In this context, regulation and public pressure can be suspected to add to fiduciary duties to ultimately make corporate boards quite prudent.

6 Tax laws have been pointed out by many (e.g., Smith and Watts 1982; Hall and Murphy 2003; Murphy 2011) to be one explanation of the sudden wave of executive option grants which convexify incentive compensation schemes. In a more recent study, however, Kadan and Swinkels (2008) find mitigated evidence of this. Concerning liability regimes, Dittman and Maug (2007)’s theoretical and empirical work suggests that bankruptcy risks tend to reduce the convexity of incentive schemes. These issues are further discussed in Sections 4 and 5 respectively.
2 The benchmark model

Consider an agent - standing for a CEO or a top executive - whose preferences can be represented by a Von Neumann-Morgenstern utility function \( u(\cdot) \) defined over monetary payments. We assume this function is three-times differentiable, increasing and strictly concave, formally \( u'(\cdot) > 0 \) and \( u''(\cdot) < 0 \), so the agent is risk averse.

This agent can work for a principal - in this case, a corporate board acting for a given company - whose preferences are represented by the Von Neumann-Morgenstern utility function \( v(\cdot) \) defined over net final wealth. We suppose this function is increasing and strictly concave, i.e. \( v'(\cdot) > 0 \) and \( v''(\cdot) < 0 \), so the principal is risk averse. Moreover, let the marginal utility \( v'(\cdot) \) be convex, i.e. \( v'''(\cdot) > 0 \), which means that the principal is downside risk averse or (equivalently) prudent.

The principal’s profit depends stochastically on the agent’s effort level \( a \). The latter cannot be observed, however, and the agent incurs a cost of effort \( c(a) \) that is increasing and convex \( (c'(a) > 0 \) and \( c''(a) > 0) \). The principal only gets a verifiable signal \( s \), drawn from a compact subset \( S \) of \( \mathbb{R} \), which is positively correlated with the agent’s effort \( a \) through the conditional probability distribution \( F(s; a) \) with density \( f(s; a) \) strictly positive on \( S \). Based on observing \( s \), she can infer a realized profit \( \pi(s) \), which we suppose increasing and concave or linear in \( s \) \( (\pi'(s) > 0 \) and \( \pi''(s) \leq 0) \), and pays the agent a compensation \( w(s) \).

To fix intuition, one may think of \( s \) as a sales forecast; if the firm has some market power, then it is reasonable to expect profit to be concave in output, hence in sales.\(^7\)

The principal’s problem is to find a smooth (i.e. twice differentiable) reward schedule or incentive scheme \( w(s) \) that maximizes profit, under the constraints that the agent will maximize his own expected utility (the incentive compatibility constraint) and must receive an expected payoff that is not inferior to some external one \( U_0 \) (the participation constraint). This can be written formally as follows:

\[
\max_{w(s), a \in S} \int_S v(\pi(s) - w(s))dF(s; a) \tag{1}
\]

subject to

\[
a \in \arg \max_{e} \int_S u(w(s))dF(s; e) - c(e)
\]

\[
\int_S u(w(s))dF(s; a) - c(a) \geq U_0
\]

As it is commonly done in the literature for tractability reasons, we replace the incentive compatibility constraint by the first-order necessary condition on the agent’s utility-maximizing effort \( a \). This transforms

\(^7\) We thank Justin Leroux for this example.
the principal’s initial problem into the following one:

$$\max_{w(s), a \in S} \int v(\pi(s) - w(s))dF(s; a)$$

subject to

$$\int_{s \in S} u(w(s))dF_u(s; a) - c'(a) \geq 0, \quad (\gamma)$$

$$\int_{s \in S} u(w(s))dF(s; a) - c(a) \geq U_0, \quad (\mu)$$

where $\gamma$ and $\mu$ are the constraints’ respective Lagrange multipliers. This so-called ‘first-order approach’ delivers a valid solution under the following sufficient conditions (see Rogerson 1985).$^8$

**Assumption 1** [Concave Monotone Likelihood Ratio Property]: The ratio $\frac{f_a(s, a)}{f(s, a)}$ is non decreasing and concave in $s$ for each value of $a$.

**Assumption 2** [Convexity of the Distribution Function Condition]: At every $a$ and $s$, we have that $F_{aa}(s, a) \geq 0$.

Before ending this section, let us write $R_u = -\frac{u''}{u'}$ and $R_v = -\frac{v''}{v'}$ the Arrow-Pratt measures of risk aversion corresponding to the agent’s and the principal’s utility functions $u$ and $v$ respectively, and $P_u = -\frac{u''}{u'}, \ P_v = -\frac{v''}{v'}$ the analogous measures of prudence proposed by Kimball (1990). Observe, finally, that the product $P_u R_v = \frac{u''(\cdot)}{v'(\cdot)} = d_v$, a coefficient introduced by Modica and Scarsini (2005) to measure someone’s degree of local downside risk aversion (or local prudence). A higher coefficient $d_v$ means the principal would be ready to pay more to insure against a risk with greater negative skewness. As shown by Crainich and Eeckhoudt (2008), $d_v$ increases if the utility function $v$ becomes more concave while the marginal utility $v'$ becomes more convex.$^9$

This completes the description of our benchmark model. Let us now proceed to characterize the optimal incentive scheme in this context.

### 3 Approximately concave incentive schemes

This section will now establish that a principal who is sufficiently prudent compared with the agent (in a sense to be made precise very soon) should set an incentive compensation package that is approximately

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$^8$Assumption 1 is actually due to Jewitt (1988); Rogerson (1985)’s article does not suppose that the likelihood ratio is concave. For economic interpretations and examples of distributions that satisfy the above two assumptions, see LiCalzi and Spaeter (2003).

$^9$A somewhat different measure is the ‘index of downside risk aversion’ $S_v = d_v - \frac{3}{2} R_v^2$ due to Keenan and Snow (2005). This index does not have the properties $d_v$ has, but it recalls the Arrow-Pratt measure of risk aversion in the sense that its value increases under monotonic downside risk averse transformations of the utility function $v$. 

concave in outcome. The implications of such a contract are discussed below. To begin with, note that the Kuhn-Tucker necessary and sufficient conditions require that a solution to program (2) meet the equation:

\[
\frac{v'(\pi(s) - w(s))}{u'(w(s))} = \mu + \gamma \frac{f_a(s; a)}{f(s; a)}, \quad \forall s
\]  

(3)

The multipliers \( \mu \) and \( \gamma \) are non-negative by construction, and the right-hand-side of (3) is increasing and concave in the signal \( s \), by Assumption 1. This allows to state the following.

**Lemma 1** The optimal reward schedule \( w^*(s) \) is increasing in the performance signal \( s \).

The proof consists in taking the first derivative of the left-hand-side of expression (3), knowing it must be positive by Assumption 1.

Similarly taking the second derivative, which must in turn be negative by Assumption 1, will yield the central result of this section. Beforehand, we need to define two key items. The first one concerns the principal’s and the agent’s relative prudence.

**Definition 1** If \( k \cdot d_u < d_v \) for some integer \( k > 0 \), the principal is said to be more prudent than the agent by a factor \( k \).

We next borrow from the literature on approximately concave functions (Hyers and Ulam 1952; Páles 2003).

**Definition 2** If \( I \) is a subinterval of the real line \( \mathbb{R} \) and \( \delta, \rho \) are nonnegative numbers, a function \( g : I \rightarrow \mathbb{R} \) is called \( (\delta, \rho) \)-concave on \( I \) if

\[
tg(x) + (1-t)g(y) \leq g(tx + (1-t)y) + \delta t(1-t) |x-y| + \rho \quad \text{for all} \ x, y \in I \quad \text{and} \ t \in [0,1].
\]

Clearly, the function \( g \) is concave when \( \delta = \rho = 0 \). The literature uses the term \( \rho \)-concave when \( \delta = 0 < \rho \). The following characterization, which combines Páles (2003)’s theorems 3 and 4, will help visualize better the case where \( \delta > 0 = \rho \), which is the relevant one in this paper.

**Lemma 2** Let \( I \) be a subinterval of the real line \( \mathbb{R} \) and \( \delta \) a nonnegative number. A function \( g : I \rightarrow \mathbb{R} \) is \( (\delta,0) \)-concave at \( x \in I \) if and only if there exists a non-increasing function \( q : I \rightarrow \mathbb{R} \) such that

\[
g(y) \leq g(x) + q(x)(y-x) + \frac{\delta}{2} |y-x| \quad \text{for all} \ y \in I.
\]

The function \( q \) in the lemma bears a close resemblance to a subgradient, and the literature indeed says that \( g \) is \( (\delta,0) \)-subdifferentiable when such a function exists. If the lemma holds on each subinterval of the entire domain of the function \( g \), then \( g \)'s graph might look like the one shown in Figure 1.

**Insert Figure 1 about here.**

Our main result is now at hand.
Theorem 1 Suppose that the principal is more prudent than the agent by a factor $k$. Then the optimal wage schedule $w^*(s)$ is $(\delta(k), 0)$-concave at any $s \in S$, where the number $\delta(k)$ decreases with $k$ and tends to 0 as $k$ grows to infinity.

The proof shows, actually, that convergence to concavity is not asymptotic: when $\left(\frac{\pi'(s)-w'(s)}{w'(s)}\right)^2 \geq \frac{1}{k}$, i.e. the CEO’s earnings do not grow too fast with respect to the firm’s net profit, then we must have $\delta(k) = 0$ so $w^*(s)$ concave at $s$. Meanwhile, moreover, the $(\delta(k), 0)$-subgradient of the wage schedule $w^*(s)$ will be the derivative $\pi'(s)$ of the profit function, so the CEO’s incentives will remain well-aligned on the firm’s interest.

The theorem’s conclusion holds vacuously - hence the optimal incentive scheme is concave - when the agent is not prudent (for $u'' \leq 0$, hence $d_u \leq 0$, in this case). If the agent is prudent (i.e. $u'' > 0$), the theorem says that he may still have to bear more downside risk when the principal exhibits enough local prudence. In this case, incentive compensation will be approximately concave, so generally more responsive to performance under unfavorable than under positive circumstances. By offering such a contract, the prudent principal motivates the agent to keep away from, not only the bad, but indeed the very bad outcomes. 

While Theorem 1 recommends to set approximately concave contracts under certain conditions, non-concave or even convex compensation modes seem rather prevalent in practice. Hall and Murphy (2003, p. 49), for instance, report that: “In 1992, firms in the Standard & Poor’s 500 granted their employees options worth a total of $11 billion at the time of grant; by 2000, option grants in S&P 500 firms increased to $119 billion.” This phenomenon per se does not invalidate our result, since we adopt here a normative standpoint. The current model may simply not capture key elements of the corporate landscape that would make non-concave incentive schemes optimal. In the following sections, we successively examine two sets of reasons which, when added to the benchmark model, might indeed justify why pay-performance concavity should have become the exception rather than the rule.

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*We don’t mean to say here that pay-performance concavity renders the agent less eager to take risks. As Ross (2004, p. 209-211) pointed out, the overall effect of an incentive scheme $w(s)$ compared to an alternative $z(s)$ on the agent’s behavior towards risk depends on whether the utility function $u(w(s))$ displays more or less risk aversion than the utility function $u(z(s))$. Suppose, for instance, that the latter scheme takes the form of a call option (a convex contract) $z(s) = \max \{s - r, 0\}$ with $r$ the exercise price, while the former is the put option (a concave contract) $w(s) = \min \{b - r + s, b\}$ with $b$ a fixed fee and $r$ the exercise price. An agent whose risk aversion decreases with wealth (prudence is a necessary condition for this) will then be less locally risk averse at the exercise price $r$ under contract $w(\cdot)$ than under contract $z(\cdot)$.*
4 Limited liability and non-concavity

As a first departure from our benchmark model, let’s allow either the agent or the principal to bear limited losses. In the first subsection, the agent will always earn nonnegative revenue. In the second subsection, the principal will be rescued whenever net profits are falling below zero.

4.1 The judgment-proof agent

Suppose the agent’s revenue is bounded from below, so he cannot bear very high penalties when performance is bad. Management remuneration is frequently subject to this type of constraint.\textsuperscript{11} An agent with limited wealth, for instance, can file for bankruptcy if he cannot afford paying some penalty. Golden parachutes and other devices (like retirement benefits) have also been introduced to compensate top managers in case employment is terminated. Executives who own large amounts of their company’s stock can now often hedge their holdings to contain losses if the value of the stock plunges (Bisin et al. 2008; Acharya and Bisin 2009; Gao 2010). Recently, some CEOs have also been offered insurance to offset the costs of investigations or liability for certain criminal mischiefs such as foreign corruption and bribery (Boyer and Tennyson 2011).\textsuperscript{12} And in certain contexts, institutions that prevent an agent from breaching his contract under bad circumstances might simply not exist.

Without loss of generality, let us then normalize the agent’s minimum revenue to zero. The principal’s optimization problem now becomes:

$$\max_{w(s), a s \in S} \int v(\pi(s) - w(s))dF(s; a)$$  \hspace{1cm} (4)

subject to

$$\int_{s \in S} u(w(s))dF_a(s; a) - c^a(a) \geq 0, \hspace{1cm} (\gamma)$$

$$\int_{s \in S} u(w(s))dF(s; a) - c(a) \geq U_0, \hspace{1cm} (\mu)$$

$$w(s) \geq 0, \forall s \hspace{1cm} (\lambda(s))$$

where $\lambda(s)$ is the Lagrange multiplier associated with the nonnegative wage constraint in state $s$.

Let $a^{**}$ denote the agent’s new choice of effort (to be soon compared with $a^*$, the agent’s optimal effort in the benchmark model). The Kuhn-Tucker conditions for a solution to this problem (4) lead this

\textsuperscript{11}Hence, since Holmstrom (1979) and especially Sappington (1983)’s seminal works, analyzing the impact of an agent’s limited liability remains a rather well-covered topic in the principal-agent literature. For a recent account of this literature, see Poblete and Spulber (2011). In most articles, however, both the principal and the agent are assumed to be risk neutral.

\textsuperscript{12}Marsh & McLennan created such a policy, in order to allow people and businesses to cover the cost of investigations under the U.S. Foreign Corrupt Practices Act and the U.K.’s Bribery Act.
time to the equation:

$$v'(\pi(s) - w(s)) \frac{w'(w(s))}{w'(w(s))} = \mu + \gamma \frac{f_a(s; a)}{f(s; a)w'(w(s))} + \frac{\lambda(s)}{f(s; a)w'(w(s))}, \quad \forall s$$

with $\lambda(s)w(s) = 0$ at all $s$. This, and some extra computation, entail the following conclusions.

**Proposition 1** Suppose that the agent is protected by limited liability, and that the principal is more prudent than the agent by a factor $k$. Then:

a) The optimal wage schedule is such that (i) $w^*(s) = 0$ for any signal $s$ lower than some threshold $s_0$, and (ii) $w^*(s)$ is $(\delta(k), 0)$-concave in $s > s_0$, where the number $\delta(k)$ decreases with $k$ and tends to 0 as $k$ grows to infinity.

b) For any incentive wage schedule, $a^{**} < a^*$ so the agent’s effort is lower than in the benchmark case.

Hence, allowing the agent to have limited liability should not change the pay-performance $(\delta(k), 0)$-concavity established in Theorem 1 in the range where performance signals induce strictly positive rewards. Some convexity is nevertheless introduced through the floor payment $w(s) = 0$ when $s \leq s_0$. The upshot is a decrease in the agent’s effort relative to the benchmark situation where no limited liability exists.

### 4.2 The sheltered principal

Consider now a situation where it is the principal’s losses which are limited. Many countries actually possess, implicitly or explicitly, rescue programs aimed at supporting their so-called ‘strategic’ or ‘too big to fail’ enterprises when they are on the verge of collapse. In 2008, for example, at the heart of the financial crisis, the United States government - under the Troubled Assets Relief Program (TARP) - purchased hundreds of billions of dollars in assets and equity from distressed financial institutions. In 2004, the engineering and manufacturing company Alstom, which had experienced a string of business disasters, received 2.5 billion euros in rescue money from the French government (as part of a plan previously approved by the European Commission). Such state interventions usually raise concerns that they will fuel moral hazard from the sheltered firms. As we shall see, they might at first have an effect on the incentives of senior executives and CEOs.

Suppose there is a state of nature $s_1$ in which $\pi(s_1) = 0$; profits being increasing in $s$ by assumption, we have that $\pi(s) < 0$ when $s < s_1$ and $\pi(s) \geq 0$ for $s \geq s_1$. We postulate that the principal will be rescued after she has compensated the agent.\footnote{As an illustration, recall that, during the 2008 financial crisis, some banks disclosed their financial losses after having set aside provisions to pay their traders.} Let’s assume (rather safely) that $|w(s)| < |\pi(s)|$ for any $s \in S$, so the principal never pays the agent an amount larger than her gains or lower than her losses; net profits $\pi(s) - w(s)$ remain therefore positive for $s > s_1$, equal to zero at $s = s_1$ and negative when $s < s_1$.\footnote{As an illustration, recall that, during the 2008 financial crisis, some banks disclosed their financial losses after having set aside provisions to pay their traders.}
Consider now the two subsets:
\[ S_1 = \{ s \in S; \pi(s) - w(s) \geq 0 \} \]
\[ \overline{S_1} = \{ s \in S; \pi(s) - w(s) < 0 \} , \]

A sheltered principal must then solve the following problem:
\[
\max_{w(s), a} \int_{s \in S_1} v(\pi(s) - w(s))dF(s; a) + v(0).F(s_1; a)
\]
subject to
\[
\int_{s \in S} u(w(s))dF_a(s; a) - c'(a) \geq 0, \quad (\gamma)
\]
\[
\int_{s \in S} u(w(s))dF(s; a) - c(a) \geq U_0, \quad (\mu)
\]

The Kuhn-Tucker conditions applied to this problem give rise to two distinct expressions. That is:
\[
\frac{v'(\pi(s) - w(s))}{u'(w(s))} = \mu + \gamma \frac{f_a(s; a)}{f(s; a)}, \quad \forall s \in S_1
\]
and
\[
u'(w(s)) \cdot f(s; a) \left( \mu + \gamma \frac{f_a(s; a)}{f(s; a)} \right) \geq 0 \text{ for } \hat{s} < s \leq s_1 \text{ and } \leq 0 \text{ for } s \leq \hat{s}
\]
Condition (7) is the same as in the benchmark case while condition (8) entails a constant floor wage. The optimal incentive scheme is thus similar, on the upside, to the one prescribed in Proposition 1.

**Proposition 2** Suppose that net profits are prevented to fall below zero, and that the principal is more prudent than the agent by a factor k. The optimal incentive scheme will then be such that (i) \( w^{**}(s) = w^{**}(s_1) = 0 \) for all adverse signals \( s \in S_1 \), and (ii) \( w^{**}(s) \) is \( (\delta(k), 0) \)-concave on \( \overline{S_1} = S \setminus S_1 \), where the number \( \delta(k) \) decreases with \( k \) and tends to 0 as \( k \) grows to infinity.

Limiting the principal’s losses induces therefore some convexity in the pay-performance relationship of the agent’s wage schedule, as the agent will partly benefit (when profits \( \pi(s) \) become negative) from the principal’s protection.

All in all, it seems, however, that a CEO’s remuneration should remain approximately concave on the upside when either he or his company is subject to limited liability. This does not fully match common practices in some industries (investment banking, notably). We shall now examine whether taxation can support the evidence further.

### 5 Taxation and convexity

The second departure from our benchmark model is to introduce personal and corporate taxes. Tax laws are often pointed out as a key rationale for the popularity of stock options and other components of
executive compensation (see, e.g., Smith and Watts 1982; Murphy 1999; Hall and Murphy 2003; Dittman and Maug 2007; Murphy 2011) and for discrepancies in executive pay across countries (Thomas 2008). Taxation - progressive taxation notably - is also periodically mentioned as a means to moderate what many people regard as excessive rewards to some corporate members (see Rose and Wolfram 2002 for an empirical study). In 2009, for example, U.S. President Barack Obama and U.K. Prime Minister Gordon Brown respectively gazed at taxing traders or banks to precisely meet this concern. In what follows, we investigate the ramifications such proposals could have for the agent’s incentive scheme.

5.1 Income taxation

Assume that the agent has to pay a tax when her income is positive, the tax rate \( \tau(s) \) being positive and nondecreasing in the agent’s revenue. Such progressive taxation exists in several countries. In the United States, for example, the effective tax rate for revenues around $266,000 is 20.1%, while it is 20.9% for incomes around $610,000.\(^{14}\) In the Netherlands, the first 200,000 euros of taxable income are subject to a tax rate of 20%, and the rate on further income is 25.5%. Meanwhile, France’s new president François Hollande has made it a binding electoral promise to impose a special tax rate of 75% on annual incomes larger than one million euros.

Since the agent’s payoff grows with the signal \( s \), let us then write the tax rate as a function \( \tau(s) \) of \( s \) with \( \tau'(s) \geq 0 \) at all \( s \) where \( w(s) > 0 \). To keep matters simple, we suppose that \( \tau''(s) = 0 \). Using the notation \( S^+ = \{ s \in S; w(s) > 0 \} \) and \( S^- = \{ s \in S; w(s) \leq 0 \} \), the optimal incentive scheme must now solve the following problem:

\[
\max_{w(s), a \in S} \int v(\tau(s) - w(s))dF(s; a) \tag{9}
\]

subject to

\[
\int_{s \in S^+} u((1 - \tau(s))w(s))dF_a(s; a) + \int_{s \in S^-} u(w(s))dF_a(s; a) - c'(a) \geq 0, \quad (\gamma)
\]

\[
\int_{s \in S^+} u((1 - \tau(s))w(s))dF(s; a) + \int_{s \in S^-} u(w(s))dF(s; a) - c(a) \geq U_0, \quad (\mu)
\]

The first-order conditions are then given by:

\[
\frac{v'(\tau(s) - w(s))}{(1 - \tau(s))w'(1 - \tau(s))w(s))} = \mu + \gamma \frac{f_a(s; a)}{f(s; a)}, \quad \forall s \in S^+ \tag{10}
\]

and

\[
\frac{v'(\tau(s) - w(s))}{w'(w(s))} = \mu + \gamma \frac{f_a(s; a)}{f(s; a)}, \quad \forall s \in S^- \tag{11}
\]

\(^{14}\)The US Congressional Budget Office calculates effective tax rates by dividing taxes paid with comprehensive household income. The latter “equals pretax cash income plus income from other sources. Pretax cash income is the sum of wages, salaries, self-employment income, rents, taxable and nontaxable interest, dividends, realized capital gains, cash transfer payments, and retirement benefits (...). Other sources of income include all in-kind benefits (Medicare, Medicaid, employer-paid health insurance premiums, food stamps, school lunches and breakfasts, housing assistance, and energy assistance).”
Taking the first and second derivatives of the left-hand side of the latter expressions leads to a perhaps surprising (albeit intuitive) conclusion.

**Proposition 3** Assume that the principal is more prudent than the agent by a factor $k$, that a non-decreasing linear tax rate $\tau(s)$ applies to the agent’s positive income, and that the agent’s net income $(1 - \tau(s))w(s)$ is nondecreasing in $s$. Then:

(i) When the agent is risk neutral and the tax rate is constant, the optimal wage schedule is concave at any state $s$ in $S$.

(ii) When the agent is risk-averse, the principal being more prudent than the agent by a factor $k$ no longer suffices to make the optimal wage schedule $(\delta(k), 0)$-concave on $S$.

This proposition compares a situation with (i) a constant tax rate to another (ii) where it is progressive. Each fiscal policy has of course a specific impact on the agent’s behavior and affects therefore the optimal compensation scheme set by the principal. In the former case ($\tau'(s) = 0$), the principal’s prudence prevails and the pay-performance relationship remains $(\delta(k), 0)$-concave (and concave when the agent is risk neutral). For any remuneration package, however, an increasing tax function ($\tau'(s) > 0$) weakens more and more the agent’s incentives as his efforts yield better results. This might induce even a principal that is a lot more prudent than the agent to find approximately concave incentive pay inappropriate and offer instead a reward function that becomes steeper as $s$ goes up. Progressive taxation might thus bring about convex reward schemes, despite the fact that the principal is prudent, and despite the often-explicit intent of such fiscal policy to curb executive revenues.\footnote{The same distortive power of taxation has already been emphasized by Eeckhoudt et al. (2009) in a different setting (with perfect information). Their result is that a progressive (increasing and concave) taxation scheme can actually make an otherwise risk neutral manager behave as if he were risk averse and prudent.} We shall comment further on this finding in the conclusion.

### 5.2 Profit taxation

Suppose now that the principal’s positive net profit is subject to a constant tax rate $\theta$. An optimal incentive scheme must then solve:

\[
\max_{w(s), a} \int_{s \in \mathbb{S}_1} v((1 - \theta)(\pi(s) - w(s)))dF(s; a) + \int_{s \in \mathbb{S}_2} v(\pi(s) - w(s))dF(s; a)
\]

subject to

\[
\int_{s \in S} u(w(s))dF_a(s; a) - c'(a) \geq 0, \quad (\gamma)
\]

\[
\int_{s \in S} u(w(s))dF(s; a) - c(a) \geq U_0, \quad (\mu)
\]

where $\mathbb{S}_1 = \{s \in S; \pi(s) - w(s) \geq 0\}$ and $\mathbb{S}_2 = \{s \in S; \pi(s) - w(s) < 0\}$.
The first-order conditions in this case are given by:

\[
(1 - \theta)v'[(1 - \theta)(\pi(s) - w(s))] \div w'(w(s)) = \mu + \gamma \frac{f_a(s; a)}{f(s; a)}, \forall s \in S_1
\]

(13)

\[
v'(\pi(s) - w(s)) \div w'(w(s)) = \mu + \gamma \frac{f_a(s; a)}{f(s; a)}, \forall s \in S_1
\]

(14)

The principal’s and the agent’s relative prudence, measured by the factor \( k \), will now have to be considered with respect to a given taxation policy. Proceeding as before yields our last result.

**Proposition 4** Assume that the principal is more prudent than the agent by a factor \( k \), that a constant tax rate \( \theta \) applies to the principal’s positive net profit, and that the principal’s after-tax net profit \((1 - \theta)(\pi(s) - w(s))\) is nondecreasing in \( s \). Then:

(i) the optimal wage schedule is \((\delta(k), 0)\)-concave when net profits are negative, where the number \( \delta(k) \) gets smaller with \( k \) and tends to 0 as \( k \) grows to infinity;

(ii) the optimal wage schedule is \((\delta(k, \theta), 0)\)-concave when net profits are positive, where the number \( \delta(k, \theta) \) gets smaller with \( k \) and tends to 0 as \( k \) grows to infinity;

(iii) \( \delta(k, \theta) \) increases with \( \theta \), so the higher the tax rate \( \theta \) on corporate profits the cruder the pay-performance convexity.

Statements (i) and (ii) coincide with Theorem 1’s conclusion. Part (iii) raises again the possibility of convexified incentive schemes. Recall that a prudent or downside risk averse principal worries more about the variability of net profits \((\pi(s) - w(s))\) in bad states than in good states. Taxation on positive net profits, however, makes the good states no longer as good as before. The principal then becomes more sensitive to net profit variations in the now not-so-good states. Temptation to have the agent bear more of these variations grows and might finally prevail under very high taxes. This rationale (meant here to be normative, however) can be supported empirically. Jensen and Murphy (2004, p. 30), for example, report that:

In 1994, the Clinton tax act (the Omnibus Budget Reconciliation Act of 1993) defined non-performance related compensation in excess of $1 million as “unreasonable” and therefore not deductible as an ordinary business expense for corporate income tax purposes.

Ironically, although the populist objective was to reduce “excessive” CEO pay levels, the ultimate outcome of the controversy (…) was a significant increase in executive compensation, driven by an escalation in option grants that satisfied the new IRS regulations and allowed pay significantly in excess of $1 million to be tax deductible to the corporation. (emphasis added)
When corporate profits are taxed, in sum, a prudent board might nevertheless prefer to grant the CEO more and more revenue as the firm’s performance gets better and better, especially if this motivates him further without changing the expected tax bill by much.

6 Concluding remarks

Assuming the principal is not risk neutral is somewhat unusual in the corporate finance literature. Yet, prudence for example - now understood and formally defined in economics as aversion to downside risk (Menezes et al. 1980; Eeckhoudt and Schlesinger 2006) - corresponds to well-documented behavioral characteristics of investors (Harvey and Siddique 2000; Ang et al. 2006). It also seems to capture fairly well the requirement that corporate boards exercise a fiduciary duty of care to protect their corporation’s interest (Clark 1985; Blair and Stout 1999; Gutierrez 2003; Lan and Herakleous 2010). Taking stock of these observations and analyses, this paper examined what should happen to an agent/CEO’s incentive compensation under a prudent principal/corporate board. We found that the CEO’s contract should then be approximately concave (in the sense of Páles 2003) in the performance signal. As the board exhibits greater prudence relative to its top manager, moreover, the pay-performance relationship should lean more towards real concavity.

When the principal is prudent, the optimal contract trades off downside risk and incentives. An approximately concave incentive scheme clearly shifts some downside risk upon the agent, as remuneration follows the pattern shown in Figure 1 and is on the whole more sensitive to improved performance at the lower levels than across the highpoints. By doing so, a prudent principal increases the agent’s motivation to keep the firm away from the worst.

Interestingly, several government policies that were adopted in the aftermath of the 2008 financial debacle can be viewed as surrogates for having prudent corporate boards. The 2010 Dodd-Frank Wall Street Reform and Consumer Protection Act in the United States, for instance, contains three key measures to ‘concavify’ the incentive compensation of CEOs and top executives.16 First, empirical work so far suggests that providing shareholders with a regular vote on pay, as the ‘Say-on-Pay’ clause does, is likely to increase the sensitivity of CEO remuneration to poor accounting performance (see Ferri and Maber 2012). Second, mandatory ‘hedging policies’ - which ask companies to disclose whether or not employees or directors are allowed to use financial instruments to offset a fall in the market value of equity securities granted as compensation - will certainly deter some means to avoid downside risk which would otherwise tend to ‘convexify’ the pay-performance relationship (as shown in subsection 4.1 above). Third, asking firms to both disclose total payments made to the CEO and the median of overall compensation awarded

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16 For a summary and some comments on these measures, see Murphy (2012, p. 47-51) and Equilar (2010).
to all employees may well put an implicit cap on the growth rate of CEO rewards. To see if this truly happens is left to future empirical work, of course. Whether such regulations are more effective and efficient than fostering fiduciary duties and good governance in the first place also remains to be seen.\textsuperscript{17}

To be sure, however, not all public policies that preceded or followed financial crises have been supportive of exposing CEOs to more downside risk. As mentioned in the previous subsection, the 1993 \textit{Omnibus Budget Reconciliation Act} in the U.S., for instance, which made CEO gains above $1 million non-deductible if not performance-based, is well-known to have encouraged companies to ‘convexify’ incentive pay by granting more stock options (Murphy 2011). As Proposition 2 shows, firm rescue programs such as TARP tend similarly to support sheltering CEOs from downside risk. Proposition 3’s prescription, finally, suggests that beefing up the tax rate on high incomes, as the French government is now doing, may well lead even prudent corporate boards to go for convex incentive schemes.\textsuperscript{18}

Exploring the tradeoff between downside risk and incentives is only beginning, of course. Approximately concave incentive schemes have their supporters (e.g., Benmelech et al. 2010) and their detractors. Siding with the latter, Murphy and Jensen (2011) point out, for instance, that managers subject to concave bonuses are encouraged to smooth performance across periods, thereby hiding superior results at one time in order to use them later when facing harsher circumstances. The process of setting incentive contracts for CEOs is indeed a complex one. It involves negotiations and third-party inputs (from consultants, other employees, etc.) which are open to manipulations, power struggles and conflicts of interest. Shareholders, directors and other stakeholders can also be quite heterogeneous, so it might not be clear whose risk preferences should be taken into account after all. The choice of an incentive scheme is furthermore subject to other criteria, such as attracting and retaining talented people. This paper thus represents only one additional step towards building a complete, integrated and operational normative framework for the design of executive incentive contracts.

\textbf{APPENDIX}

\textbf{Proof of Lemma 1.}

Risk aversion of at least one player is sufficient to obtain that \( w'(s) \geq 0 \). Indeed we have, with

\textsuperscript{17} The matter of regulatory efficiency seems particularly relevant nowadays, considering the recent sharp declines in initial public offerings (IPOs) and the number of public companies. Reporting on this phenomenon in its May 19\textsuperscript{th} 2012 edition, \textit{The Economist} (p. 30) contends that: “Public companies have always had to put up with more regulation than private ones because they encourage ordinary people to risk their capital. But the regulatory burden has become heavier, especially after the 2007-08 financial crisis.”

\textsuperscript{18} While the French government’s intention clearly seems to tackle what are deemed to be illegitimate pay levels in times of crisis, the upshot will be to restore the incentive structure that might have contributed to the current situation.
\[ v(\pi(s) - w(s)) \text{ denoted as } v(\cdot) \text{ and } u(w(s)) \text{ denoted as } u(\cdot): \]

\[
\frac{\partial}{\partial s} \left( \frac{v'(\pi(s) - w(s))}{u'(w(s))} \right) = \frac{w'(s) \cdot (v''(\cdot) \cdot u'(\cdot) - v'(\cdot) \cdot u''(\cdot))}{(u'(\cdot))^2} = -w'(s) \cdot (v''(\cdot) \cdot u'(\cdot) + v'(\cdot) \cdot u''(\cdot)) + \pi'(s) \cdot v''(\cdot) \cdot u'(\cdot) \]

The latter must be positive, by Assumption 1, in order to satisfy equation (3). A necessary condition for this is \( w'(s) \geq 0 \). ♦

**Proof of Theorem 1.**

Let us now compute the second derivative of the left-hand side term in equation (3). Assumption 1 entails it must be negative.

\[
\frac{\partial^2}{\partial s^2} \left( \frac{v'(\pi(s) - w(s))}{u'(w(s))} \right) = \frac{1}{\left( u'(\cdot) \right)^3} \cdot \left\{ -w''(\cdot) \cdot (v'' u' + v' u'') - w' \cdot \left[ (\pi' - w') v'' u' + w' v'' u' + (\pi' - w') v'' u'' + w' v''' \right] + \pi'' v' u' + \pi' \cdot \left[ \left( \pi' - w' \right) v''' u' + v'' w' u'' \right] \right\} u'^2 + 2w' u'. w'. \left[ w'(v'' u' + v' u'') - \pi' v'' u' \right] \]

\[
= \frac{1}{u'^3} \cdot \left\{ -w''(\cdot) \cdot (v'' u' + v' u'') + (\pi' - w')^2 v''' u' + (\pi' - w') w' v'' u'' - w'^2 v' u''' + \pi'' v' u' \right\} u'
- \left( \pi' - w' \right) v'' v' u' u^2 (u' + 2) + 2w'^2 v^2 v'
\]

\[
= \frac{1}{u'^3} \cdot \left\{ -w''(\cdot) \cdot (v'' u' + v' u'') + (\pi' - w')^2 v''' u' - w'^2 v' w''' + \pi'' v' u' \right\} u'
- 2w'^2 v'' \cdot \left( (\pi' - w') v'' u' - v' v' w' \right)
\]

\[
= \frac{1}{u'^2} \cdot \left[ -w''(\cdot) \cdot (v'' u' + v' u'') + (\pi' - w')^2 v''' u' - w'^2 v' w''' + \pi'' v' u' \right]
+ 2w' R_u \cdot \frac{\left( (\pi' - w') v'' u' - v' v' w' \right)}{u'^2}
\]
The last term here is in fact \( \frac{\partial^2}{\partial s^2} \left( \frac{\pi'(s) - w(s)}{u'(w(s))} \right) \), and it is positive by Lemma 1. Then:

\[
\frac{\partial^2}{\partial s^2} \left( \frac{\pi'(s) - w(s)}{u'(w(s))} \right) = 2w''R_w \frac{\partial}{\partial s} \left( \frac{\pi'(s) - w(s)}{u'(w(s))} \right) + \frac{1}{w^2} \left[ w''(u' + v'u'') + (\pi' - w')^2v''u' - w^2v'u'' + \pi''v' \right] \\
= 2w''R_w \frac{\partial}{\partial s} \left( \frac{\pi'(s) - w(s)}{u'(w(s))} \right) + \frac{v''}{w} \left[ w''(R_v + R_u) + (\pi' - w')^2P_vR_v - w'^2P_vR_v - \pi''R_v \right] \\
= 2w''R_w \frac{\partial}{\partial s} \left( \frac{\pi'(s) - w(s)}{u'(w(s))} \right) + \frac{v''}{w} \left[ w''.(R_v + R_u) - \pi''R_v + (\pi' - w')^2P_vR_v - w'^2P_vR_v \right]
\]

The sign of this last expression depends on the sign of \((\pi' - w')^2P_vR_v - w'^2P_vR_v\). Two cases are possible. Either

\[
\left( \frac{\pi'(s) - w'(s)}{w'(s)} \right)^2 \geq 1 \frac{k}{k}
\]
and it is then necessary that \(w''(s) < 0\) so \(w\) is concave at \(s\), or

\[
\left( \frac{\pi'(s) - w'(s)}{w'(s)} \right)^2 < 1 \frac{k}{k}
\]

In the latter situation, let \(M = \max_{s \in S} w'(s)\), which is a finite positive real number since \(w'\) is positive and differentiable (hence continuous) on the compact set \(S\), and take \(\delta(k) > 0\) so that \((\frac{\delta(k)}{4M})^2 = \frac{1}{k}\). The latter inequality is equivalent to

\[
(\pi'(s) - w'(s))^2 < \left( \frac{\delta(k)}{4M} \right)^2 (w'(s))^2.
\]

For all \(x \in S, x \neq s\), then:

\[
| (\pi'(s) - w'(s))(x - s) | < \left( \frac{\delta(k)}{4M} \right) w'(s) | x - s |
\]

\[
< \frac{\delta(k)}{4} | x - s |
\]

so

\[
w'(s)(x - s) < \pi'(s)(x - s) + \frac{\delta(k)}{4} | x - s |
\]
Now, since \(w\) is differentiable at \(s\), we have (by definition) that
\[
w(x) = w(s) + w'(s)(x - s) + r(x)
\]
with the residual \(r(x)\) such that \(\lim_{x \to s} \frac{r(x)}{x - s} = 0\). The last inequality entails that
\[
w(x) < w(s) + \pi'(s)(x - s) + \left( \frac{r(x)}{x - s} + \frac{\delta(k)}{4} \right) |x - s|
\]
\[
\leq w(s) + \pi'(s)(x - s) + \frac{\delta(k)}{2} |x - s|
\]
if \(x\) is sufficiently close to \(s\). Since \(\pi'(s)\) is decreasing in \(s\) by assumption, the latter and Lemma 2 mean that \(w(s)\) is \((\delta(k), 0)\)-concave on a subinterval of \(S\) that contains \(s\). Since this is be true at any point \(s\), keeping the same number \(\delta(k)\), then \(w^*(s)\) is \((\delta(k), 0)\)-concave on \(S\). ♦

**Proof of Proposition 1.**

First consider the states in which the optimal wages are strictly positive. In all these states, we have \(\lambda(s) = 0\) and the optimal reward function is identical to the one obtained in the previous section. It is then increasing and approximately concave in \(s\), as in Theorem 1.

Let \(s_0\) be the signal value for which a zero wage is optimal. Since the reward function is increasing on the positive subspace, this signal is unique. Moreover, the limited liability constraint is binding for all signals lower than \(s_0\). This proves a).

To show b), consider the following two subsets
\[
\overline{S}_0 = \{ s \in S; s > s_0 \}
\]
\[
\underline{S}_0 = \{ s \in S; s \leq s_0 \},
\]
and let us compare \(a^*\) and \(a^{**}\). The agent subject to limited liability computes the following program for any reward schedule \(w(s)\):
\[
\max_a U = \int_{s \in \overline{S}_0} u(w(s))dF(s; a) + u(0).F(s_0; a) - c(a)
\]
The first-order condition is given by
\[
a^{**} / \int_{s \in \overline{S}_0} u(w(s))dF_a(s; a^{**}) + u(0).F_a(s_0; a^{**}) - c'(a^{**}) = 0 \tag{15}
\]
Recall now the first-order condition under unlimited liability:
\[
a^* / \int_{s \in S} u(w(s))dF(s; a^*) - c'(a^*) = 0 \tag{16}
\]
From Assumption 1, we have that \( F_a(s_0; a) < 0 \). The utility function \( u(\cdot) \) being increasing, it is also true that \( u(0) \geq u(w(s)) \) for any negative \( w(s) \), with at least one strict inequality. Hence, comparing (16) and (15), we can conclude that \( a^* > a^{**} \) for any given reward function \( w(s) \).

\[ \star \]

**Proof of Proposition 2.**

The Lagrangian function for this problem is given by

\[
\mathcal{L} = \int_{s \in S_1} \left( v(\pi(s) - w(s))dF(s; a) + v(0).F(s_1; a) + \gamma \left( \int_{s \in S} u(w(s))dF_a(s; a) - c'(a^*) \right) + \mu \left( \int_{s \in S} u(w(s))dF(s; a) - c(a) - U_0 \right) \right)
\]

Two expressions must be considered when verifying the first-order conditions for maximizing \( \mathcal{L} \). If \( s \in S_1 \), we have

\[
\frac{v'(\pi(s) - w(s))}{u'(w(s))} = \mu + \gamma \frac{f_{a}(s; a)}{f(s; a)}, \quad \forall s \in S_1
\]  

and, in particular,

\[
\lim_{s \to s_1} \frac{v'(\pi(s) - w(s))}{u'(w(s))} = \lim_{s \to s_1} \left( \mu + \gamma \frac{f_{a}(s; a)}{f(s; a)} \right)
\]

For \( s \in S_1 \), we have

\[
\frac{\partial \mathcal{L}}{\partial w(s)} = u'(w(s)).f(s; a) \left( \mu + \gamma \frac{f_{a}(s; a)}{f(s; a)} \right), \quad \forall s \in S_1
\]  

Expression (17) is identical to (3) in the unlimited case. Thus, the reward schedule corresponds to the one described in (i) if the principal is ‘more prudent’ than the agent.

From (18) we have that \( \mu + \gamma \frac{f_{a}(s; a)}{f(s; a)} > 0 \). This expression being continuous and non decreasing in \( s \) (Assumption 1), there exists a state \( \hat{s} < s_1 \) such that \( \mu + \gamma \frac{f_{a}(s; a)}{f(s; a)} > 0 \) \( \forall s \in [\hat{s}, s_1] \) and \( \mu + \gamma \frac{f_{a}(s; a)}{f(s; a)} \leq 0 \) \( \forall s \leq \hat{s} \). With \( \frac{\partial^2 \mathcal{L}}{\partial w(s)^2} = u''(w(s)).f(s; a) \left( \mu + \gamma \frac{f_{a}(s; a)}{f(s; a)} \right) \), this implies that

\[
\frac{\partial \mathcal{L}}{\partial w(s)} > 0 \quad \text{and} \quad \frac{\partial^2 \mathcal{L}}{\partial w(s)^2} < 0, \quad \forall s \in [\hat{s}, s_1]
\]

\[
\frac{\partial \mathcal{L}}{\partial w(s)} \leq 0 \quad \text{and} \quad \frac{\partial^2 \mathcal{L}}{\partial w(s)^2} > 0, \quad \forall s \leq \hat{s}
\]

The Lagrangian function \( \mathcal{L} \) being concave in \( w(s) \) on \( [\hat{s}, s_1] \) and convex otherwise, the optimal revenue is the maximum possible one for any \( s \leq s_1 \). More precisely, \( w(s) \) being continuous and increasing on \( S_1 \), the reward schedule satisfies \( w^*(s) = w(s_1) = \pi(s_1) = 0 \) for any \( s \leq s_1 \). \( \star \)
Proof of Proposition 3.

Consider condition (10). Let \( A = \frac{\partial}{\partial s}((1 - \tau(s))w(s)) \), which is assumed to be positive. We have that

\[
\frac{\partial}{\partial s} \left( \frac{v'(\pi(s) - w(s))}{(1 - \tau(s))u'(1 - \tau(s))w(s)} \right) = \frac{(\pi' - w')v''(1 - \tau)u' - v'(u'A(1 - \tau) - \tau'u')}{(1 - \tau)^2u^2}
\]

Now, write \( B = \frac{\partial}{\partial s}((\pi(s)/u'(1 - \tau))) = \frac{\pi''(1 - \tau) + 2\pi'}{(1 - \tau)^2} \). With \( \pi'' = 0 \) by assumption, the latter simplifies to \( B = \frac{2\pi'}{(1 - \tau)^2} \). Then:

\[
\frac{\partial^2}{\partial s^2} \left( \frac{v'(\pi(s) - w(s))}{(1 - \tau(s))u'(1 - \tau(s))w(s)} \right) = \frac{1}{(1 - \tau)u'} \left[ (\pi'' - w'')v'' + (\pi' - w')^2v''
+ v''(\pi' - w')(R_uA + \tau'/(1 - \tau)) + v'(R_uA + R_uA' + B) \right]
- \frac{1}{(1 - \tau)^2u^2} \left[ ((1 - \tau)Au'' - \tau'u') ((\pi' - w')v'' + v'(R_uA + \tau'/(1 - \tau))) \right]
\]

\[
= \frac{1}{(1 - \tau)u'} \left[ (\pi'' - w'')v'' + (\pi' - w')^2v''
+ v''(\pi' - w')(R_uA + \tau'/(1 - \tau)) + v'(R_uA + R_uA' + B)
+ (AR_u + \tau'/(1 - \tau)) ((\pi' - w')v'' + v'(R_uA + \tau'/(1 - \tau))) \right]
\]

Note that \( A' = (1 - \tau)w'' - 2\tau'w' \). The latter expression becomes

\[
\frac{\partial^2}{\partial s^2} \left( \frac{v'(\pi(s) - w(s))}{(1 - \tau(s))u'(1 - \tau(s))w(s)} \right) = \frac{1}{(1 - \tau)u'} \left[ ((\pi''v'' + w''v'(R_u(1 - \tau) - v'')) + (\pi' - w')^2v''
+ v''(\pi' - w')(R_uA + \tau'/(1 - \tau)) + v'(R_uA - 2R_u\tau'w' + B)
+ (AR_u + \tau'/(1 - \tau)) ((\pi' - w')v'' + v'(R_uA + \tau'/(1 - \tau))) \right]
\]

The first term in line (22), namely \( \pi''v'' \), is positive by assumption. The sign of the second term depends
on the sign of \( w''\). Lines (23) and (24) now remain to be signed. The sum of these lines can be written:

\[
(\pi' - w')^2 w'' + v''(\pi' - w') (R_u A + \pi'/(1 - \tau)) + v'(R'_u A - 2R_u \pi' w' + B) \\
+ (AR_u + \pi'/(1 - \tau)) ((\pi' - w') v'' + v' (R_u A + \pi'/(1 - \tau)))
\]

\[
= v'. [(\pi' - w')^2 d_v + R'_u A - R_u (\pi' - w') (R_u A + \pi'/(1 - \tau)) - 2R_u \pi' w' + B] \\
+ (AR_u + \pi'/(1 - \tau)) (R_u A + \pi'/(1 - \tau) - (\pi' - w').R_u)]
\]

With \( R_u = \frac{\partial R}{\partial s} = AR_u (R_u - P_u) = AR^2_u - A.d_u, \) and \( B = \frac{2\pi'}{(1-\tau)\tau}, \) it becomes:

\[
v'. [(\pi' - w')^2 d_v - (1 - \tau)^2 .w'^2 .d_u \\
+ A^2 R^2_u - R_u (\pi' - w') (R_u A + \pi'/(1 - \tau)) - 2R_u \pi' w' + B] \\
+ (AR_u + \pi'/(1 - \tau)) (R_u A + \pi'/(1 - \tau) - (\pi' - w').R_u)]
\]

\[
= v'. [(\pi' - w')^2 d_v - (1 - \tau)^2 .w'^2 .d_u \\
+ A^2 R^2_u + (R_u A + \pi'/(1 - \tau))^2 \\
- 2. (R_u \pi' w' + R_u (\pi' - w') (R_u A + \pi'/(1 - \tau)) - \tau'/(1 - \tau)^2))]
\]

(25)

If the Agent is risk neutral \( R_u = 0 \), and the tax rate is constant \( \tau'(s) = 0 \), the latter reduces to:

\[
v'. [(\pi' - w')^2 d_v - (1 - \tau)^2 .w'^2 .d_u]
\]

(26)

In this case, the sign of \( \frac{\partial^2}{\partial s^2} \left( \frac{v'(\pi(s) - w(s))}{(1 - \tau(s))w(s)(1 - \tau(s)w(s))} \right) \) depends on the sign of \( (\pi' - w')^2 d_v - (1 - \tau)^2 .w'^2 .d_u. \)

By assumption, the net revenue of the agent is non decreasing, which implies that \( 0 < \tau < 1. \) Thus we have:

\[
\left( \frac{\pi'(s) - w'(s)}{1 - \tau(s).w'(s)} \right)^2 > \left( \frac{\pi'(s) - w'(s)}{w'(s)} \right)^2
\]

If the principal is more prudent than the agent by a factor \( k, \) we have \( d_u/d_u < 1/k. \) Then if \( \left( \frac{\pi'(s) - w'(s)}{w'(s)} \right)^2 \geq \frac{1}{k} \) we have always \( \left( \frac{\pi'(s) - w'(s)}{1 - \tau(s).w'(s)} \right)^2 > \frac{1}{k} \). Thus, the function \( w^*(s) \) is \((\delta(k), 0)-\)concave. Besides, the agent being risk-neutral, \( k \) tends towards infinity and, from Theorem 1, \( \delta(k) \) tends towards zero. The incentive wage \( w^*(s) \) is then \((0, 0)-\)concave, so concave. This demonstrates (i).

Now, consider a risk-averse, or risk-neutral, agent \( R_u \geq 0 \) and a linear, non-decreasing and non-constant tax function: it satisfies \( \tau' > 0 \) and \( \tau'' = 0. \) Now, expression (25) holds. It is immediate to see that its sign does no longer only depend on the sign of \( (\pi' - w')^2 d_v - (1 - \tau)^2 .w'^2 .d_u. \) Thus despite the fact that \( (\pi' - w')^2 d_v - (1 - \tau)^2 .w'^2 .d_u > 0, \) it is no longer sufficient that the principal be more prudent than the agent as defined in Definition 1 to obtain the \((\delta(k), 0)\)-concavity of \( w''(s). \) This is Point ii).
Proof of Proposition 4.

First write \( C(s) = \frac{\partial}{\partial s} [(1-\theta)(\pi(s) - w(s))] = (1-\theta)(\pi'(s) - w'(s)) \) We have for any \( s \in S_1 \)

\[
\frac{\partial}{\partial s} \left( \frac{v'((1-\theta)(\pi(s) - w(s)))}{u'(w(s))} \right) = \frac{Cu'' + v'w''}{u'} = v'(R_u w' - C R_v)
\]

Net profits after taxation are nondecreasing in \( s \) by assumption, so \( C \geq 0 \). Thus \( w'(s) > 0 \) is a necessary condition for \( \frac{\partial}{\partial s} \left( \frac{v'((1-\theta)(\pi(s) - w(s)))}{u'(w(s))} \right) > 0 \).

Computation of the second derivative now gives

\[
\frac{\partial^2}{\partial s^2} \left( \frac{v'((1-\theta)(\pi(s) - w(s)))}{u'(w(s))} \right) = \frac{\left( Cu'' + R_u v' w'' \right) u''w'}{(u')^2} = \frac{\left( Cu'' + R_u v' w'' \right) u''w'}{(u')^2}
\]

\[
= \left( C' v'' + C'' v''' + (R'_u v' + R_u C v'') \right) w' + R_u v' w'' + (C' v'' + R_u v' w') R_u w'
\]

\[
= \left( u' \right) \left[ R_u (C^2 P_v - C') + (R'_u - R_u C R_v) w' + R_u w'' + (R_u w' - C R_v) R_u w' \right] \quad (27)
\]

where \( \frac{dR_u}{ds} = R'_u = -w'(d_u - R_u^2). \)

We must analyze the sign of the three components in the brackets in Equation (27). With \( C = (1-\theta)(\pi' - w') \), we have:

\[
R_u (C^2 P_v - C') + (R'_u - R_u C R_v) w' + R_u w'' + (R_u w' - C R_v) w'
\]

\[
= C^2 d_v - C' R_v - w^2 d_u + w^2 R_u^2 - R_u C R_v w' + R_u w'. (R_u w' - C R_v) + R_u w''
\]

\[
= ((1-\theta)^2(\pi' - w')^2 d_v - w^2 d_u) + 2w'. R_u. (R_u w' - C R_v) - (1-\theta) \pi'' R_v + ((1-\theta) R_v + R_u). w''
\]

The second term in (28) is equal to \( \frac{2R_u w'(s) u'(s)}{v'(s)} \frac{\partial}{\partial s} \left( \frac{v'((1-\theta)(\pi(s) - w(s)))}{u'(w(s))} \right) \) and is positive. The third term is positive by assumption. Then the sign of \( w''(s) \) for any \( s \in S_1 \) depends on the sign of the first term, namely \( (1-\theta)^2(\pi' - w')^2 d_v - w^2 d_u \).

As for the proof of Theorem 1, two cases are possible. Either

\[
\left( \frac{(1-\theta). (\pi'(s) - w'(s))}{w'(s)} \right)^2 \geq \frac{1}{k},
\]
and, since $\frac{1}{k} > \frac{d_u}{d_v}$, the expression $(1 - \theta)^2(\pi' - w')^2.d_v - w'^2 d_u$ is positive. It is then necessary that $w''(s) < 0$ so $w$ is concave at $s \in S_1$, or

$$
\left( \frac{(1 - \theta). (\pi'(s) - w'(s))}{w'(s)} \right)^2 < \frac{1}{k} .
$$

It can also be written

$$
\left( \frac{\pi'(s) - w'(s)}{w'(s)} \right)^2 < \frac{1}{k \cdot (1 - \theta)^2} .
$$

In the latter situation, let $M = \max_{s \in S} w'(s)$, which is a finite positive real number since $w'$ is positive and
differentiable (hence continuous) on the compact set $S$, and take $\delta(k, \theta) > 0$ so that

$$
\left( \frac{\delta(k, \theta)}{4M} \right)^2 = \frac{1}{k \cdot (1 - \theta)^2} .
$$

The latter inequality is equivalent to

$$(\pi'(s) - w'(s))^2 < \left( \frac{\delta(k, \theta)}{4M} \right)^2 (w'(s))^2 .$$

For all $x \in \overline{S_1}, x \neq s$, then:

$$| (\pi'(s) - w'(s))(x - s) | < \left( \frac{\delta(k, \theta)}{4M} \right) w'(s) | x - s |$$

$$\leq \frac{\delta(k, \theta)}{4} | x - s | ,$$

so

$$w'(s)(x - s) < \pi'(s)(x - s) + \frac{\delta(k, \theta)}{4} | x - s | .$$

Now, since $w$ is differentiable at $s$, we have (by definition) that

$$w(x) = w(s) + w'(s)(x - s) + r(x)$$

with the residual $r(x)$ such that $\lim_{x \to s} \frac{r(x)}{x - s} = 0$. The last inequality entails that

$$w(x) \leq w(s) + \pi'(s)(x - s) + \left( \frac{r(x)}{| x - s |} + \frac{\delta(k, \theta)}{4} \right) | x - s |$$

$$\leq w(s) + \pi'(s)(x - s) + \frac{\delta(k, \theta)}{2} | x - s |$$

if $x$ is sufficiently close to $s$. Since $\pi'(s)$ is non-decreasing in $s$ by assumption, the latter and Lemma 2 mean that $w(s)$ is $(\delta(k, \theta), 0)$-concave in $s \in \overline{S_1}$. Concerning the states $s \in S_1$ no taxation applies because
net profits are negative, we have $\delta(k, 0) = \delta(k)$ (see (29) above). Thus $w(s)$ is $(\delta(k), 0)$-concave in $s \in S_1$.

This proves assertion (i).

Point (ii) is immediate from (29): $\delta(k, \theta)$ is increasing in $\theta$. ♦
References


Figure 1. The function $g: X \to R$ is $(\delta,0)$-concave on $X$. 