A Century of Eating: 
Revealed preferences for nutrients and foods in the United States

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Abstract

The USDA’s food assistance program, SNAP, uses shadow prices for nutrients, derived using linear programming to evaluate optimal benefits. We use linear programming, least squares and maximum entropy to estimate consumer shadow prices for 18 nutrients and 21 food taste values from 1910-2006. Assumptions about taste preferences explain differences across these econometric tools. This study explains correlations between taste and nutrient shadow prices with demographic composition of the U.S., which trends may unveil intuition behind unhealthy eating habits. Our estimated taste values and demographic relationships reveal misalignments of SNAP benefits with its overall goal of nutrition supplementation.

JEL Classification: C61, D11, H53, I38, Q18

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Introduction

The Supplemental Nutrition Assistance Program (SNAP) constitutes the largest federal food assistance program. In 2006, an average of 26.5 million participants received SNAP benefits. The number of recipients grew to 47.6 million by May 2013 (ERS 2013). Talk of a new Farm Bill mentions a possible cost of $375 billion over the next five years for SNAP.

SNAP nationally launched as the Food Stamp Program in 1974 (FNS 2013). In late 2008, the Food Stamps Program became SNAP, with a new mission to increase nutrition availability to low-income residents (Brownell et al. 2011). SNAP bases household benefits on a per-capita, consumer valuation for nutrients. The USDA uses linear programming to calculate the nutrient values as shadow prices, which represents what a consumer would be willing to pay for an additional unit of a nutrient – provided that consumers pay no attention to the taste of food. This “taste-free” valuation method for uncovering consumers’ shadow prices for nutrients is used to determine the household benefits.

The objective of this study is to examine the tool used by the U.S. Government to determine food assistance spending, and compare this tool to other methods that take taste and preferences into account. We estimate shadow prices for 18 nutrients and taste values for 21 different food products from 1910-2006 using three econometric methods. This study tracks these shadow prices and various taste values over time, and explains the relationships between these findings and U.S. demographic variables.

The USDA uses linear programming to determine the least-cost food bundles, given constraints requiring certain levels of nutrients for different age and gender groups. Each age and gender group has a unique recommended daily allowance of nutrients, from which linear programming can optimize the least-cost food bundles for each group. The cost of each group’s
food bundle exactly determines the amount of benefits allocated to these SNAP recipients. Historically the linear programming approach has focused on nutrition characteristics of the foods, but disregarded taste and preferences between food products.

Often, food valuation based strictly on nutrition attributes does not accurately characterize consumer preferences. Lancaster (1966) suggests that consumers purchase product characteristics, which are bundled into goods. Consumers may not follow SNAP purchasing recommendations based solely on nutrient values, which results in suboptimal nutrition. For example, SNAP bases the benefits for a child\(^1\) between nine and eleven years on a least-cost food bundle consisting of low fat milk, low-cost meats, unpopular fruits, general mixed vegetables and legumes. If the child's diet varies from this low-cost food plan, he or she consumes a different level of nutrients than intended by the program. The lack of food preferences in this model may lead to suboptimal consumption of nutrients according SNAP guidelines. Participants will buy cheap and tasty food, rather than cheap and nutrition food, which does not meet the SNAP objectives of providing beneficiaries with a nutritious diet.

Several methods exist to calculate nutrient valuation, each with unique assumptions about consumer behavior and taste preferences. Some methods assume that each food product has a core taste quality that the consumer values, while other methods ignore these unique product taste values. While it may seem more intuitive to consider unique taste values for each food product, this approach increases the cost of feeding all the nation’s low- and no-income residents. Moreover, identifying the unique product taste value proves analytically difficult.

The analytic methods we compare in this study include linear programming, least squares and information theory. Each model applies restrictions on consumers’ utility derived from food. While linear programming does not consider utility for anything aside from nutrients, least

\(^1\) Children comprised 45% of SNAP participants in 2011 (ERS 2013).
squares estimates both the nutrient shadow prices as well as the utility derived from eating. The least squares model does not explicitly distinguish utility derived from specific food products, but we can infer specific taste values. Finally, maximum entropy directly estimates both the nutrient shadow prices as well as the utility derived from each individual food product. The three models impose very different restrictions upon taste assumptions by distinguishing total taste aside from nutrients or tastes for specific food products.

The literature investigating nutrient valuation began when Stigler (1945) developed a theory of linear programming for cost minimization under essential nutrition constraints. His approach was least-cost food rationing in response to World War II. The linear programming model estimates the least cost bundle of food products to meet the nutritional criteria. Stigler constrained the nutrients to levels above the minimum recommended dietary allowances. Silberberg's (1985) work utilized linear programming also, but constrained nutrients above the observed levels of consumption to minimize expenditures under revealed and consistent nutrition habits. In this study, we modify the Silberberg approach by holding the minimum nutrition constraint equal to the observed, average consumption level. While Silberberg constrained nutrients to meet a minimum, observed level, we constrain nutrients to exactly equal the observed level. This allows consumers to purchase nutrients above a recommended level, and replicates SNAP's benefit determination process.

Gorman (1956) discusses factors other than nutrients, which play a role in determining the demand in food products. Ladd and Suvannunt (1976) used least squares to find the hedonic prices of these non-nutrient characteristics. We build from their model and include a collective “eating” factor of demand in addition to that for individual nutrients. This allows us to clearly identify why consumers purchase food apart from physical health requirements. We can infer the
taste value for individual food products by adding the product’s regression residuals to the felicity of eating, which remains constant across food products.

The information theory model allows us to treat the product price as a function of both characteristic prices and product-specific taste value, where consumers make trade-offs between nutrition and taste. Adding product-specific taste preferences increases the number of variables without adding degrees of freedom, which leads to more unknown parameters than equations. With such an ill posed problem, standard econometric modeling will not properly estimate the parameters (Mittelhammer, Judge and Miller 2000). Beatty (2007) uses an entropy-based econometric approach to find shadow prices for nutrients and food product taste. We extend the theory by using continuous probability distribution functions (PDFs). We smooth these continuous PDFs over a support, which we carefully define. These smoothed PDFs then aid in finding the expected nutrient taste shadow price, as well as the expected food product taste value. These findings suggest why food assistance programs are highly correlated with unhealthy eating habits (Beatty et al. 2012). Since consumers value food products such as sodas and other non-nutritious items, they purchase these with some of their food assistance benefits.

We study the relationships between demographic composition with nutrient shadow prices and tastes for food. Understanding the direction of American diversity and income inequality can provide policymakers insight into the future trends of food preferences, perhaps encouraging more efficient SNAP allocations as well as compatible incentives toward healthy eating.

This paper continues in the following manner. The upcoming section discusses theoretical assumptions and restrictions in a utility maximization setting, in which nutrient composition can explain food prices. The basic theoretical discussion will follow with the three models - the processes of linear programming, least squares, and information theory through
maximum entropy. Next, we discuss the data, as well as the empirical results from the models. The final section consists of policy implications.

**Development of Taste and Nutrient Shadow Prices**

We begin with a utility maximization problem where consumers make purchasing decisions over food products and all other goods. As Gorman (1956) describes, consumers gain utility from each of the nutrients and the food product itself. The optimality problem proceeds as follows:

\[
\begin{align*}
\max & \quad u(a_i(x_1, \ldots, x_j), \ldots, a_J(x_1, \ldots, x_j); x_1, \ldots, x_j; y_1, \ldots, y_N) \\
n & \quad \text{s.t. } \sum_{i=1}^{I} (p_i x_i) + \sum_{n=1}^{N} (q_n y_n) = m \\
& \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad 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2 For the sake of clear utility maximization, this paper assumes independence of nutrients. While some nutrients may help augment or deplete absorption of other nutrients, this is not consistent for across consumption levels for these nutrients. Additionally, consumers can buy food products with the information of nutrient makeup in mind, but the information of nutrient links between one another is not available on the product.
(3) \( I = u(\bar{a}_j(x_1, \ldots, x_j); x; y_n) + \lambda \left( m - \sum_{i=1}^{l} p_i x_i - \sum_{m=1}^{M} q_n y_n \right) + \mu_j \left( \sum_{i=1}^{l} a_{ij} x_i - \bar{a}_j \right) \)

with first-order Kuhn-Tucker conditions³:

\[
\begin{align*}
\frac{\partial I}{\partial x_i} &= \frac{\partial u}{\partial x_i} - \lambda p_i + \sum_{j=1}^{J} a_{ij} \mu \leq 0, \quad x_i \geq 0, \quad x_i \frac{\partial I}{\partial x_i} = 0; \\
\frac{\partial I}{\partial y_n} &= \frac{\partial u}{\partial y_n} - \lambda q_n \leq 0, \quad y_n \geq 0, \quad y_n \frac{\partial I}{\partial y_n} = 0; \\
\frac{\partial I}{\partial \bar{a}_j} &= \frac{\partial u}{\partial \bar{a}_j} - \mu_j \leq 0, \quad \bar{a}_j \geq 0, \quad \bar{a}_j \frac{\partial I}{\partial \bar{a}_j} = 0; \\
\frac{\partial I}{\partial \lambda} &= m - \sum_{i=1}^{l} (p_i x_i) - \sum_{m=1}^{M} (q_n y_n) = 0; \\
\frac{\partial I}{\partial \mu_j} &= \sum_{i=1}^{l} a_{ij} x_i - \bar{a}_j = 0.
\end{align*}
\]

Summing the first three conditions, while substituting in the last two equality constraints, yields

(5) \( x_i \frac{\partial I}{\partial x_i} + y_n \frac{\partial I}{\partial y_n} + \bar{a}_j \frac{\partial I}{\partial \bar{a}_j} = x_i \frac{\partial u}{\partial x_i} + y_n \frac{\partial u}{\partial y_n} + \bar{a}_j \frac{\partial u}{\partial \bar{a}_j} - \lambda m = 0. \)

Combining equations (4) and (5), we arrive at our following, hedonic pricing model:

(6) \( p_j = \frac{\partial u}{\partial x_i} / (x_i \frac{\partial u}{\partial x_i} + y_n \frac{\partial u}{\partial y_n} + \bar{a}_j \frac{\partial u}{\partial \bar{a}_j}) + \sum_{j=1}^{l} \left[ \frac{\partial u}{\partial \bar{a}_j} / (x_i \frac{\partial u}{\partial x_i} + y_n \frac{\partial u}{\partial y_n} + \bar{a}_j \frac{\partial u}{\partial \bar{a}_j}) \right], \)

where \( \frac{\partial u}{\partial x_i} / (x_i \frac{\partial u}{\partial x_i} + y_n \frac{\partial u}{\partial y_n} + \bar{a}_j \frac{\partial u}{\partial \bar{a}_j}) \) represents the marginal dollar utility for food products,

and \( \frac{\partial u}{\partial \bar{a}_j} / (x_i \frac{\partial u}{\partial x_i} + y_n \frac{\partial u}{\partial y_n} + \bar{a}_j \frac{\partial u}{\partial \bar{a}_j}) \) the marginal dollar utility for nutrients, or shadow prices for taste and nutrition, respectively.

³ Note that we treat the constraints as equalities, and monotonicity of \( u(\cdot) \) implies \( \lambda > 0 \) and \( \mu_j > 0 \).
We write equation (6) more succinctly as

\begin{equation}
(7) \quad p_i = r_i + \sum_{j=1}^{J} d_{ij} \pi_j .
\end{equation}

Here, \( r_i \) represents the shadow price of food product \( i \), otherwise known as the taste for this food product. Additionally, \( \pi_j \) represents the shadow price for each nutrient, \( j \). Price is now a function of the taste and nutrient shadow prices.

**Empirical Models**

We compare three different approaches to generating shadow prices – linear programming, least squares, and information theory. We consider how each model describes the price of food products as a function of taste and nutrient prices, as given in equation (7).

**Linear Programming**

In the linear programming model, only nutrient content and shadow prices determine product prices. We use Silberberg's (1985) model, but constrain nutrients at the level observed.

We write the minimization problem for each year as:

\begin{equation}
\begin{aligned}
\min_{\{x_i\}} & \quad f_i = \sum_{i=1}^{I} p_{ii} x_{ii} \\
\text{s.t.} & \quad p_{ii} = \sum_{j=1}^{J} \pi_{ij} a_{ij} \\
& \quad \sum_{i=1}^{I} a_{ij} x_{ii} = \bar{a}_{ij} \quad \forall \ j \in J \text{ and } t \in T \\
& \quad x_{ii} \geq 0 \quad \forall \ i \in I \text{ and } t \in T.
\end{aligned}
\end{equation}
We minimize food expenditure, \( f_t \), while holding each level of nutrient consumption \( \sum_{i=1}^{f} a_{ij} x_{ij} \) at the observed nutrient consumption for each year, \( \bar{a}_{tj} \). We find a minimum level of expenditure for each year without a change in observed nutrient consumption. The optimized level of \( x_{ti} \) helps us to derive each nutrient shadow price. Price in equation (8) consists of nutrient values only and excludes the taste value from equation (7).

After obtaining the minimized food expenditure, we compare the results to the observed level of food expenditure. Subtracting the minimum expenditures needed to obtain the observed nutrients from the observed expenditure levels yields the maximum possible shadow price for an individual product's taste value.

**Least Squares**

The model of least squares, as motivated by Ladd and Suvannunt (1976), assumes a positive shadow price for taste, \( \frac{\partial u}{\partial X} > 0 \). We consider a generalized least squares model for the hedonic price equation:

\[
\pi_{nj} = \sum_{j=1}^{j} \pi_{nj} a_{nj} + r_{t} + \varepsilon_{nj}
\]

Equation (9) determines the shadow price for each nutrient, \( \pi_{nj} \), as well as the marginal value of utility from eating, denoted as \( r_{t} \). Here, \( r_{t} \) measures the felicity of eating, not a taste value specific to any product. When we add this value from eating with each food product's error term, we infer the individual products' taste values. This provides us with an estimate of the food taste value \( r_{t} \) from equation (7).

We allow both positive and negative values for the nutrient shadow prices \( \pi_{nj} \). Consumers might rationally consume where \( \pi_{nj} \) is negative if the food product’s taste value \( r_{ti} \),
outweighs the disutility of nutrient \( j \) (Leathers 1979). Consumers may consume more of any nutrient than they would if they could separate the nutrients from products they enjoy. Diminishing marginal utility most likely exists for all nutrients.

**Information Theory**

We would like to directly measure the shadow prices for each nutrient and taste for each food product, directly estimating each element of equation (7). However, including a product-specific variable results in more unknown parameters than equations, which results in an under-identified system of equations and hence an ill posed problem.

We turn to a maximum entropy approach, which minimizes the average logarithmic height of the probability density functions (PDFs) of the shadow prices and food-specific taste attributes. These PDFs have a pre-specified support. We introduce the distributions of unknowns through moment conditions for each product. We transform our hedonic pricing model to

\[
\begin{equation}
\begin{aligned}
p_{ti} &= \sum_{j=1}^{J} a_{tij} E(\Pi_{tj}) + E(R_{ti}) \quad \forall \, i \in I \text{ and } t \in T. \\
\end{aligned}
\end{equation}
\]

Here, \( \Pi_{tj} \) and \( R_{ti} \) represent random variables for the nutrient shadow prices and food taste values, respectively. Price in equation (10) is a function of the mean of these random variables, \( E(\Pi_{tj}) \) and \( E(R_{ti}) \).

The support of \( R_{ti} \) is straightforward. The value \( r_{ti} \) can take any value up to the price of food \( i \). Hence, \( r_{ti} \in [0, p_{ti}] \) \( \forall \, i \in I \text{ and } t \in T \). We allow for the possibility of consuming at a negative \( \pi_{tj} \), as we did for least squares. The support is \( \pi_{tj} \in [-\bar{\pi}_{tj}, \bar{\pi}_{tj}] \), bounded symmetrically around zero, where

\[
\begin{equation}
\begin{aligned}
\bar{\pi}_{tj} &= \min_{\{i : a_{tij} > 0\}} \frac{p_{ti}}{a_{tij}}, \quad \forall \, j \in J \text{ and } t \in T.
\end{aligned}
\end{equation}
\]

We assume no nutrient can have a marginal value to consumers that exceeds the market price of
that food. Otherwise, the consumption side of the market would tend to bid up the price of the
food, leading to a higher market equilibrium price.

With the supports for both \( \Pi_{ij} \) and \( R_{ti} \), we can apply maximum entropy. Since we do not
know the distributions of these two random variables, then we need to set up a distribution with
largest entropy, or largest uncertainty. Jaynes introduced this idea in 1957. He stated that to
maximize entropy in a distribution, one must maximize the negative integral of the PDF
multiplied by the logarithm of the PDF. We write the maximum entropy equation as:

\[
\max - \sum_{i=1}^{l} \int_{0}^{p_{ti}} f_{ti}(r) \ln f_{ti}(r) \, dr - \sum_{j=1}^{J} \int_{-\pi_{tij}}^{\pi_{tij}} g_{tij}(\pi) \ln g_{tij}(\pi) \, d\pi
\]

\[
\text{s.t.} \quad \int_{0}^{p_{ti}} f_{ti}(r) \, dr = 1 \ \forall \ i \in I \text{ and } t \in T
\]

\[
\int_{-\pi_{tij}}^{\pi_{tij}} g_{tij}(\pi) \, d\pi = 1 \ \forall \ j \in J \text{ and } t \in T
\]

\[
p_{ti} = \int_{0}^{p_{ti}} r f_{ti}(r) \, dr + \sum_{j=1}^{J} a_{tij} \int_{-\pi_{tij}}^{\pi_{tij}} \pi g_{tij}(\pi) \, d\pi \ \forall \ i \in I.
\]

The Lagrangian is

\[
\mathcal{L} = - \sum_{i=1}^{l} \int_{0}^{p_{ti}} f_{ti}(r) \ln f_{ti}(r) \, dr - \sum_{j=1}^{J} \int_{-\pi_{tij}}^{\pi_{tij}} g_{tij}(\pi) \ln g_{tij}(\pi) \, d\pi + \\
\sum_{i=1}^{l} \lambda_{ti} \left[ 1 - \int_{0}^{p_{ti}} f_{ti}(r) \, dr \right] + \sum_{j=1}^{J} \mu_{tij} \left[ 1 - \int_{-\pi_{tij}}^{\pi_{tij}} g_{tij}(\pi) \, d\pi \right] + \\
\sum_{i=1}^{l} \gamma_{ti} \left[ p_{ti} - \int_{0}^{p_{ti}} r f_{ti}(r) \, dr + \sum_{j=1}^{J} a_{tij} \int_{-\pi_{tij}}^{\pi_{tij}} \pi g_{tij}(\pi) \, d\pi \right].
\]

We obtain the following first-order conditions:

\[
-\ln f_{ti}(r) - \lambda_{ti} - \gamma_{ti} r_{ti} - 1 = 0 \ \forall \ r_{ti} \in [0, p_{ti}] \text{ and } i \in I
\]

\[\text{footnote}{\text{It is known that problems such as equation (13), which are called isoperimetric calculus of variations problems (Clegg 1968), are solved by maximizing the Lagrangian point-wise with respect to each } \partial f_{ti}(r_{t}) \text{ and } \partial g_{tij}(\pi_{tij}) \text{ for each } (i, j, t) \text{ and all } (r, \pi) \text{ in their respective supports (Seierstad and Sydsæter 1987).}}\]
\[-\ln g_{tj}(\pi) - \mu_{tj} - \sum_{i=1}^{l} \gamma_{ti} \alpha_{tij} \pi_{tj} - 1 = 0 \]
\[\forall \pi_{tj} \in [-\tilde{\pi}_{tj}, \tilde{\pi}_{tj}] \text{ and } j \in J.\]

Simplifying equation (14) into exponential form yields

\[(15a) \quad f_{tj}(r) = e^{-(\lambda_{ti} + 1)} e^{-\gamma_{ti} r_{ti}} \quad \forall r_{ti} \in [0, p_{ti}] \text{ and } i \in I \quad \text{and} \]
\[(15b) \quad g_{tj}(\pi) = e^{-(\mu_{tj} + 1)} e^{-\delta_{tj} \pi_{tj}} \quad \forall \pi_{tj} \in [-\tilde{\pi}_{tj}, \tilde{\pi}_{tj}] \text{ and } j \in J, \]

where \(\delta_{tj} = \sum_{i=1}^{l} \gamma_{ti} \alpha_{tij}\). We define \(\delta_{tj}\) as a convenient reduction in notation.

Using \(\int_{0}^{p_{ti}} f_{tj}(r) dr = 1\) to eliminate \(e^{-(\lambda_{ti} + 1)}\) and \(\int_{-\tilde{\pi}_{tj}}^{\tilde{\pi}_{tj}} g_{tj}(\pi) d\pi\) to eliminate \(e^{-(\mu_{tj} + 1)}\), we extend equations (15) through the following taste shadow price density functions,

\[1 = e^{-(\lambda_{ti} + 1)} \int_{0}^{p_{ti}} e^{-\gamma_{ti} r_{ti}} dr = e^{-(\lambda_{ti} + 1)} \left[ -\frac{1}{\gamma_{ti}} e^{-\gamma_{ti} p_{ti}} \bigg|_{0}^{p_{ti}} \right] = e^{-(\lambda_{ti} + 1)} \left[ -\frac{1}{\gamma_{ti}} (e^{-\gamma_{ti} p_{ti}} - 1) \right] \]
\[\Rightarrow e^{-(\lambda_{ti} + 1)} = \frac{\gamma_{ti}}{1 - e^{-\gamma_{ti} p_{ti}}}, \text{ and further with the nutrient shadow density function,} \]
\[1 = e^{-(\mu_{tj} + 1)} \int_{-\tilde{\pi}_{tj}}^{\tilde{\pi}_{tj}} e^{-\delta_{tj} \pi_{tj}} d\pi \]
\[= e^{-(\mu_{tj} + 1)} \left[ -\frac{1}{\delta_{tj}} e^{-\delta_{tj} \tilde{\pi}_{tj}} \bigg|_{-\tilde{\pi}_{tj}}^{\tilde{\pi}_{tj}} \right] \]
\[= e^{-(\mu_{tj} + 1)} \left[ -\frac{1}{\delta_{tj}} (e^{-\delta_{tj} \tilde{\pi}_{tj}} - e^{-\delta_{tj} (-\tilde{\pi}_{tj})}) \right] \]
\[\Rightarrow e^{-(\mu_{tj} + 1)} = \frac{\delta_{tj}}{e^{\delta_{tj} \tilde{\pi}_{tj}} - e^{-\delta_{tj} \tilde{\pi}_{tj}}}. \]

Plugging these statements into equations (15a) and (15b), we arrive at the following:

\[(16a) \quad f_{tj}(r) = \frac{\gamma_{ti} e^{-\gamma_{ti} r_{ti}}}{1 - e^{-\gamma_{ti} p_{ti}}} \quad \forall r_{ti} \in [0, p_{ti}] \text{ and } i \in I \quad \text{and} \]
\[(16b) \quad g_{tj}(\pi) = \frac{\delta_{tj} e^{-\delta_{tj} \pi_{tj}}}{e^{\delta_{tj} \tilde{\pi}_{tj}} - e^{-\delta_{tj} \tilde{\pi}_{tj}}} \quad \forall \pi_{tj} \in [-\tilde{\pi}_{tj}, \tilde{\pi}_{tj}] \text{ and } j \in J. \]
From these equations, we use L’Hôpital’s rule and integration by parts to find our expected value for each random variable, \( E(\Pi_{tj}) \) and \( E(R_{ti}) \). Appendix A contains the detailed calculations to determine the expected values of the shadow prices. The expected values are

\[
(17a) \quad E(R_{ti}) = p_{ti} \left[ \frac{1}{1-e^{-\gamma_{ti}p_{ti}}} \right] + \frac{1}{\gamma_{ti}} \quad \text{and}
\]
\[
(17b) \quad E(\Pi_{tj}) = \bar{n}_{tj} \left[ \frac{1+e^{2\delta_{tj}\gamma_{tj}}}{1-e^{-2\delta_{tj}\gamma_{tj}}} \right] + \frac{1}{\delta_{tj}}.
\]

Substituting \( \delta_{tj} = \sum_{i=1}^{l} a_{tij} \gamma_{ti} \) back into our hedonic price constraint from equation (10), \( p_{ti} = \sum_{j=1}^{J} a_{tij} E(\Pi_{tj}) + E(R_{ti}) \), we obtain

\[
(18) \quad p_{ti} = \sum_{j=1}^{J} a_{tij} \left[ \bar{n}_{tj} \left[ \frac{1+e^{2\delta_{tj}\gamma_{tj}}}{1-e^{-2\delta_{tj}\gamma_{tj}}} \right] + \frac{1}{\delta_{tj}} \right] + p_{ti} \left[ \frac{1}{1-e^{-\gamma_{ti}p_{ti}}} \right] + \frac{1}{\gamma_{ti}}
\]
\[
= \sum_{j=1}^{J} a_{tij} \left[ \bar{n}_{tj} \left[ \frac{1+e^{2\delta_{tj}\gamma_{tj}}}{1-e^{-2\delta_{tj}\gamma_{tj}}} \right] + \frac{1}{\sum_{i=1}^{l} \gamma_{ti} a_{tij}} \right] + p_{ti} \left[ \frac{1}{1-e^{-\gamma_{ti}p_{ti}}} \right] + \frac{1}{\gamma_{ti}}
\]

\( \forall i \in I \) and \( t \in T \).

The resulting price equation has 21 unknowns of \( \gamma_{ti} \) for each food product and 21 equations. Strict concavity implies a unique, optimal solution for \( \gamma_{ti}^* \). Since we know that \( \gamma_{ti} = 0 \) violates this price constraint, we can assume that \( \gamma_{ti} \neq 0 \). After solving for \( \gamma_{ti}^* \), we explicitly find \( \delta_{tj}^* = \sum_{i=1}^{l} a_{tij} \gamma_{ti}^* \) \( \forall i \in I, j \in J \) and \( t \in T \).

**Data**

We use an aggregate annual time series from 1910 through 2006. The per capita consumption of the 21 food products, nutrients and average retail prices come from USDA and U.S. Bureau of Labor Statistics sources (LaFrance 1999a). The food consumption data is from the USDA’s *Food Consumption, Prices and Expenditures* and measures the data as food *disappearance* as opposed to direct food consumption. That is, the difference between food
available (the sum of production, beginning inventories and imports) and nonfood use (exports, farm use and industrial consumption). Table 1 contains a detailed list of the 21 individual food products. We provide a complete list of the 18 nutrients in Table 2.\textsuperscript{5}

Tables 3 and 4 provide summary statistics about the consumption of these nutrients and food products, respectively. We have 97 years of observations for nutrients and food products, each of which scales down to daily consumption. Tables 3 and 4 present the mean, standard deviation, minimum and maximum of daily intake of each nutrient and food product, respectively.

The demographic data consists of three different categories – family, income and economic health, and food policy. The family category includes average family size, population proportions of ethnicities, and population proportion of age groups. Ethnicity categories are white, black, and other. The category “other” consists of all races other than white or black, and this comprised less than one percent of the total population until 1963. Age groups consist of population proportion who is under 15 years old, who is 15 to 54 years old, and who is 55 years and older. The income category includes per capita disposable income in real 1967 dollars, budget share spent on food at home and food away from home, and the unemployment rate. The food policy category consists of mandatory pasteurization of milk in 1917, the fortification of vitamin D in milk in 1932, the Social Security Act of 1935, the Food Stamps Program nationwide in 1974, and the mandatory labeling of Nutrition Facts in 1992. We also used dummy variables for during and post-World War II periods. Summary statistics for the demographic data are located in Table 5.

\textsuperscript{5} We do not include carbohydrates, due to the linear relationship of calories with protein, carbohydrates and fats. We remove carbohydrates to avoid singularity without loss of generality and have checked for robustness.
**Empirical Results**

We analyze the 18 nutrient shadow prices for each model and the 21 food taste values for least squares and maximum entropy. Appendices B and C contain graphs for all of these shadow prices and taste values. These are readily available online. Tables of all shadow prices and taste values are also available online.\(^6\)

**Linear Programming**

We find positive nutrient shadow prices from this model. The shadow prices generally move parallel to each other, with some variation of individual nutrients. Typically, the shadow price for fat is higher than the shadow prices of all the other nutrients. Instability occurs in all shadow prices following the Great Depression, with a drop in all shadow prices in 1932. We also see an increase in most shadow prices during World War II, with a spike around 1945-1947, due to the increased consumption of many nutrients during this time. With recession and wartime variation in prices and consumption, this instability during the 1930s and 1940s is expected. A high degree of variation exists in the shadow prices during the 1990s, and they move in a parallel pattern during this period.

The difference between the calculated minimum expenditure and actual expenditure can provide a maximum possible taste value aside from nutrition, which represents food expenditure not devoted to nutrition. This represents the sum of maximum possible food taste values. These values should grow as income rises, due to diminishing marginal utility of nutrients and food products. The time trend of this sum of all maximum food shadow prices is in Figure 1. From the early 1920s onward, the values exhibit a general increasing trend until about the 1970s. We especially see increases in this value during periods of economic growth, such as the 1920s

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\(^6\) These are located online at [http://cahnrs-cms.wsu.edu/ses/gradstudents/shrader/](http://cahnrs-cms.wsu.edu/ses/gradstudents/shrader/). Further information is available upon request from the corresponding author at Rebekah.Shrader@wsu.edu.
(followed by a drop in this value during the Great Depression), the mid- to late-1940s due to World War II, and the early 2000s.

**Least Squares**

The coefficient estimates from least squares represent point estimates of nutrient shadow prices\(^7\). We estimate the hedonic price equation from equation (9). The constant estimate plus each product's error term comprises that food's taste value.

We find some negative shadow prices. Many empirical hedonic studies have found negative shadow prices, such as with certain vegetable characteristics (Ladd and Suvannunt, 1976). Here, negative shadow nutrient prices likely reveal diminishing marginal utility in nutrient consumption. At this point, it would be rational to suspend consumption in that nutrient. Since these nutrients come bundled with other factors, such as taste for a food product, people may consume nutrients at a negative marginal utility.

Conversely, nutrients that many would discourage for health reasons may have positive shadow prices. For example, cholesterol yields almost exclusively positive shadow prices throughout the decades. This may occur if foods with higher cholesterol taste better.

Sodium's estimated shadow price seems to follow a decreasing trend most of the century, starting at around .04¢ per milligram. In the mid 1970s, sodium's shadow price turned negative. This could be due to the effect of decreasing marginal utility from nutrient consumption, since sodium consumption has increased over time. In the past century, per capita sodium intake has increased by about a third, from just over 300,000 milligrams per year, reaching over 460,000 milligrams per year. This could also occur as health information suggests less sodium is better for one’s health.

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\(^7\) Many of the shadow prices are not statistically significantly different from zero; however, since we are allowing both positive and negative shadow prices, including zero, this statistical significance is not relevant to our study.
The food taste values, as calculated by adding the constant term to individual food product's errors, also span between positive and negative values. Many products move in parallel fashion, such as milk, butter and cheese. The sweeping negative food tastes in the early- and mid-1950s correlates with a higher level of volatility with nutrient shadow prices. If all nutrient shadow prices peak, the food taste values will simultaneously decrease, *ceteris paribus*.

The results from least squares may appear random, but one must consider the marginal nature of these prices. That is, as consumption of a nutrient increases, the marginal value of the next unit will be decreasing. For example, the shadow price of protein is positive with a dip in the 1940s, until turning negative in the 1970s. Examining the graph of protein consumption, one can see a sudden increase in protein consumption in the 1940s, as well as a steady increase in consumption starting in the 1970s. Similar correlations of shadow price values with the relative nutrient consumption trend occur with other nutrients, such as cholesterol and fat.

**Maximum Entropy**

Similar to linear programming, the nutrient and food product shadow prices from maximum entropy are all positive, even though we allowed for the possibility of negative values. All positive shadow prices come from the theory that people purchase the whole bundle of a good, including all its nutrients and its taste value. We still see diminishing marginal utility with decreasing trends in shadow prices, correlating with increasing consumption of the respective nutrient or food product.

Unlike the results from least squares, nutrient shadow prices from maximum entropy do not appear to follow a pattern with one another. This indicates that each nutrient and food product is distinguishable from one another. On average, we find smoother maximum entropy shadow prices over time than the other two models, and the values tend to be midrange to the
other two extremes (i.e. linear programming had small shadow prices, and least squares had large and varying shadow prices).

While sodium as measured from least squares yields decreasing and ultimately negative shadow prices, maximum entropy yields decreasing, yet positive, shadow prices for sodium after roughly WWII. This matches our theory that increasing consumption of a nutrient corresponds with a decreasing shadow price. Figure 2 presents the consumption trend for sodium over time.

Comparison between Models

**Nutrient Shadow Prices**

The first obvious difference in nutrient shadow prices is the variation of shadow prices. Least squares yields much more volatile nutrient shadow prices. We expect this result, since the least squares provides point estimates with wide confidence intervals, while linear programming yields least cost nutrient values, and maximum entropy yields shadow prices based from smoothed PDFs on a tight bound. Figure 3 demonstrates the three models’ shadow prices for fat. This graph is representative of the volatility differences between the models’ estimates. Linear programming yields smaller nutrient shadow prices than the other models. This follows from the least-cost food rationing aspect of linear programming. Note that the model yielding the smallest nutrient values is the model that the USDA uses to allocate SNAP benefits. Only if the participants were to consume the calculated food bundle would they obtain ideal nutrition levels.

Another characteristic to observe is whether a model yields positive and/or negative shadow prices. Linear programming yields all positive shadow prices, which follows from the fact that this model excludes the possibility of a separate food taste value. While both maximum entropy and least squares allows for negative shadow prices, maximum entropy only yields positive results. This results from the fact that consumers express some level of positive value
upon paying for goods with these nutrients. That is, there is some positive market price for these nutrients if consumers are willing to pay for them. Vitamin C experiences negative shadow prices from least squares, while the other two models estimate only positive shadow prices. This is presented in Figure 4.

Since linear programming estimates the lowest shadow prices between the three models, it provides the USDA with a low cost program. However, this also constrains SNAP recipients to purchase foods solely for nutrition content, lest they attain suboptimal nutrition. Least squares would not be ideal for calculating SNAP allocations, since it sometimes yields negative shadow prices and relatively volatile estimates. Maximum entropy, however, would be more incentive compatible for SNAP beneficiaries, as food taste is present when estimating nutrient shadow prices. These shadow prices are always positive, which is also compatible with positive market prices for food.

Using the maximum entropy shadow prices increases the cost of the SNAP program. The linear programming model determines the least-cost bundles of food. However, the goal of SNAP to supplement nutrition differs from the least-cost food-rationing goal of the Food Stamps Program. If the USDA were to use a more taste compatible plan based on maximum entropy shadow prices, participants might meet the recommended nutrient intake more often. However, a maximum entropy based program would undoubtedly increase costs.

**Food Taste Shadow Prices**

Similar to the differences in nutrient shadow prices between models, the least squares food taste values are most volatile. The food taste values are often larger from the maximum entropy model than from the least squares model. This may follow from the tight supports for the nutrient shadow prices we imposed for maximum entropy, allowing for larger food taste values.
Interestingly, the food taste values from maximum entropy only sometimes move with a similar pattern as the corresponding taste value from least squares. For example, the taste values estimated from maximum entropy for fresh noncitrus fruit seem to be a smoothed version of the least squares estimates, but beef seems to be a situation where they move in a almost paralleling fashion (see Figures 5 and 6). These two select examples are representative of the rest of the food shadow prices; either the maximum entropy trend is a smooth alternative to that of least squares, or the maximum entropy trend somewhat parallels that of least squares.

Maximum entropy yields positive taste values, while many taste values from least squares become negative a few times over the century. All of the taste values from least squares become negative in the early 1950s, which most likely represents a post-war effect.

Economic theory of diminishing returns holds with many of the estimates from maximum entropy, as seen in Figures 7 and 8. Frozen dairy consumption increased from under five to above 25 pounds per year. Similarly, maximum entropy estimates decreasing taste values. The average American would pay less for one more pound of ice cream, since the marginal utility from the next pound of ice cream is not as valuable at a higher consumption level.

Another factor to consider when looking at the food taste estimates is the changing prices of that product. For example, poultry costs much less than it used to, due to more efficient production. Figure 9 represents the price trend of pound per poultry in real 1967 dollars. In this graph, one can see the drastically decreasing price from 1945 until the 1980s, when the price plateaus. The maximum entropy shadow price for poultry parallels the market price of this good very closely, as can be seen in Figure 10. Least square continues its usual, volatile trend of food taste values.
Explaining Shadow Prices with Demographic Variables

In order to approximate the relationships between demographic variables with our estimated nutrient and food product shadow prices, we use a seemingly unrelated regression (SUR) for each model. The results vary slightly across the three models. In general, the SUR models predict better fitting estimates based on R-squared values for the maximum entropy shadow prices than for the other models\(^8\).

**Theoretical Model**

The SUR model is one that comprises multiple regression equations, of which each has a separate dependent variable. The dependent variables are the nutrient shadow prices and the food taste values we estimated. For each of the three models, we run a SUR model separately for nutrient shadow prices and food taste values, totaling to six SUR models.

Beginning with the nutrient shadow prices as the dependent variables, consider a system of \(J\) seemingly unrelated equations, so that

\[
\pi_j = X_j \beta_j + \varepsilon_j, \quad j = 1, ..., 18, \tag{19}
\]

where \(X_j\) is our matrix of independent variable vectors.\(^9\) In matrix notation,

\[
X_j = \begin{bmatrix}
X_1 & 0 & \ldots & 0 \\
0 & X_2 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & X_{18}
\end{bmatrix}.
\] \tag{20}

Let \(\Omega = \Sigma_j \otimes I\), where \(\Sigma_j\) is the \(J \times J\) covariance matrix of disturbances, and \(\otimes\) is the Kronecker multiplier. The efficient GLS estimator is then

---

\(^8\) Results from our SUR models are located online at [http://cahnrs-cms.wsu.edu/ses/gradstudents/shrader/](http://cahnrs-cms.wsu.edu/ses/gradstudents/shrader/). Other information is available upon request from the corresponding author at Rebekah.Shrader@wsu.edu.

\(^9\) Each of the variables in Equation (19) is a matrix of \(J\) vectors spanning the 97 years of estimates.
(21) \[ \hat{\beta} = [X'\Omega^{-1}X]^{-1}X'\Omega^{-1}\pi. \]

This SUR model has a similar form when estimating the food taste values as the dependent variables. In this case, we consider a system of I equations, such that

(22) \[ r_i = X_i\theta + \epsilon_i, \quad i = 1,\ldots,21. \]

Letting \( \Psi = \Sigma \otimes I, \) where \( \Sigma \) is the \( I \times I \) covariance matrix of disturbances. The efficient GLS estimator is similarly

(23) \[ \hat{\theta} = [X'\Psi^{-1}X]^{-1}X'\Psi^{-1}r. \]

**Empirical Results**

As to minimize unnecessary complexity, two tables present representative nutrient shadow price and food taste value SUR coefficients. Respectively, these are tables 6 and 7. The representative nutrient shadow prices are those for calories, protein, fat, and calcium. The representative food taste values are those for milk, beef, poultry, processed non-citrus fruit, sugar and caloric sweeteners, and coffee.

**Nutrient Shadow Prices**

Considering table 6, the positive coefficient for real per capita disposable income for most of the models’ nutrient shadow prices suggests normal goods. As income increases, consumers are willing to pay a higher price for the next unit of nutrient, pushing the optimal quantity upward. The models providing these statistically significant, positive coefficients are linear programming and maximum entropy. Only in the SUR for our least squares shadow prices do we see statistically significant, negative coefficients for real per capita disposable income for vitamin A, riboflavin and magnesium. (Note that these three aforementioned nutrients are not in the representative four of table 6.)
The variable average family size has a negative coefficient for protein shadow prices under both least squares and maximum entropy models. As the average family size increases by one person, the least squares’ model of protein shadow price should decrease by about 11 cents a gram, according to our least squares model. A larger family values the next amount of protein less, likely because they will switch away from expensive protein to cheaper foods in order to feed more people. This coefficient for the maximum entropy protein shadow price is negative .05 cents - substantially smaller than that for least squares. We expect a larger coefficient for least squares due to more volatile shadow prices than those estimated through maximum entropy.

For some nutrients, a population with a higher proportion of non-white and non-black ethnicities would indicate a higher shadow price. For example, population proportions of black and white dummy variables have negative coefficients when explaining the shadow price for calories using maximum entropy. This is also the case for many other nutrients under each of the three models. As one can see from table 6, both least squares and maximum entropy suggests non-white and non-black population to value calcium more as well. However, linear programming’s calcium, iron, and fat shadow prices have positive black and white coefficients. This is the same case for calcium under maximum entropy as well as iron and potassium under least squares. Clearly, each of the three models implies a unique relationship between nutrient shadow prices and race.

**Food Taste Shadow Prices**

Consider now table 7, representative of demographic coefficients explaining food tastes. Average family size has positive coefficients for many least squares taste values. For example, the least squares, milk taste value coefficient is positive, implying that larger families value milk more so than smaller families. The only statistically significant, negative coefficients for average
family size are for maximum entropy’s taste values of some meats and coffees, teas and spices. As with the negative coefficients for protein, this may occur if larger families move away from meats to other, cheaper foods. Larger families may value coffee less, since children drink coffee at a lower rate than adults.

The category of non-white, non-black ethnicities often had a positive influence on nutrients, it has a mixed influence on most food products. For example, the non-white, non-black ethnicities are estimated to value milk, beef and coffee relatively less under maximum entropy, but value processed, non-citrus fruits relatively higher. It is possible that this portion of the population values nutrients relative to food tastes more than the white and black populations. However, some foods that are positively related to the non-white, non-black ethnicity group are fish, fruit, vegetables, as well as coffee, teas and spices. Since a substantial portion of this ethnicity category is Asian, it is intuitive that fish and teas would have higher values due to cultural habits, tastes and preferences.

An interesting policy variable is the dummy variable for Women, Infants, and Children (WIC) becoming a national health and nutrition program in 1975. This variable has a negative coefficient for milk and beef, but a positive coefficient for coffee and butter, using the maximum entropy model. Most WIC benefits go to mothers with infants. These mothers may not buy liquid milk if they use powder formulas and other baby foods. These are merely suggestive reasons for such coefficients. The dummy variable based on this may represent other things happening at this time. For example, the negative coefficient of WIC to maximum entropy’s beef taste value may follow the national trend away from beef consumption.
Conclusion

The shadow prices of nutrients under each model describe how much the consumer would pay for one more unit of each nutrient. These models utilize a linear hedonic price function, which represents the equilibrium of individuals' preferences with suppliers' cost functions.

While the USDA utilizes linear programming to estimate consumers’ nutrient shadow prices, this paper has demonstrated that nutrient shadow prices will be substantially different when estimated alongside food taste values. SNAP uses the USDA’s estimated nutrient shadow prices to determine the amount of assistance provided to beneficiaries. The goal of SNAP is for participants to meet recommended nutritional needs. Unfortunately, the USDA’s food bundle does not represent consumers’ eating habits. Individual consumers will make purchasing decisions with taste values in mind, and they will purchase very different food bundles if the USDA does not account for these taste values.

The objective of this study is to examine SNAP’s current method of linear programming to determine food assistance spending. We then compare linear programming to alternative methods, estimating shadow prices for 18 nutrients and taste values for 21 food products under three different models from 1910-2006. We then explain relationships between shadow prices with U.S. demographic variables.

Linear programming, least squares, and maximum entropy were the tools to calculate these shadow prices and taste values. Linear programming estimates only nutrient shadow prices. However, least squares also estimates a minimum taste value, common across food products. Summing food specific error terms from least squares to this common taste value finds the inferred, food specific taste values. Maximum entropy estimates all nutrient shadow prices as
well as food taste values directly. Maximum entropy is able to handle what was ill posed under least squares. Further, this study extended theory to a new application of maximum entropy.

The assumptions driving these estimates are whether to include food shadow prices in the utility for eating. Of the three models, maximum entropy best models reality when considering how consumers make choices influenced by tastes and preferences. However, this is not the model used to determine the allocation of SNAP benefits.

For example, SNAP currently models a food bundle for men 20 to 50 years old consuming only $0.54 of coffee each month, and absolutely no soft drinks (Wilde et al.). When considering the 2006 shadow price for coffee, teas, cocoa and spices, one can see the $0.57 taste value for the next pound is higher than shadow prices of most other foods. Consumers value coffee highly, yet SNAP only models a tiny portion of the food bundle dedicated to coffee. Evidently, this does not align to consumer tastes and preferences.

Since maximum entropy considers food taste values alongside nutrient shadow prices, it aligns with realistic consumer behavior and nutrient valuation. While the USDA is minimizing costs, the goal of the program is no longer least-cost food rationing as linear programming calculates. The goal of the USDA’s largest food assistance program is in its name - to supplement consumers with proper nutrition. Estimating nutrient shadow prices without simultaneously considering food tastes does not accurately model how people choose to eat the way they do.
References


Shapiro, I. 2005. New IRS Data Show Income Inequality is Again on the Rise. *Center on Budget and Policy Priorities*.


Wilde, P., J. Llobrera, and F. Campbell. The Thrifty Food Plan Calculator. *Academic and Working Papers* Gerald J. and Dorothy R. Friedman School of Nutrition Science and Policy, Tufts University
Figures

Figure 1. Sum of Maximum Food Taste Values
Figure 2. Sodium Consumption

[Graph showing the consumption of sodium (mg) from 1910 to 2003, with a steady increase followed by fluctuations and a slight decrease towards the end.]
Figure 3. Fat Shadow Prices

Cents per gram of fat

LP Fat (g)
ME Fat (g)
LS Fat (g)
Figure 4. Vitamin C Shadow Prices

Cents per microgram of C

LP C (mg)
ME C (mg)
LS C (mg)
Figure 5. Fresh Noncitrus Fruit Shadow Prices
Figure 6. Beef Shadow Prices

Cents per pound of beef

- ME Beef
- LS Beef
Figure 7. Frozen Dairy Shadow Prices
Figure 8. Frozen Dairy
Figure 9. Poultry Price
Figure 10. Poultry Shadow Prices
### Tables

#### Table 1. Food Items Used in Estimation

<table>
<thead>
<tr>
<th>Food Item</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Milk</td>
</tr>
<tr>
<td>2. Butter</td>
</tr>
<tr>
<td>3. Cheese</td>
</tr>
<tr>
<td>4. Frozen Dairy (Ice Cream)</td>
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<tr>
<td>5. Other Dairy (Canned and Dry Milk)</td>
</tr>
<tr>
<td>6. Beef</td>
</tr>
<tr>
<td>7. Pork</td>
</tr>
<tr>
<td>8. Other Red Meat</td>
</tr>
<tr>
<td>9. Fish</td>
</tr>
<tr>
<td>10. Poultry</td>
</tr>
<tr>
<td>11. Fresh Citrus Fruit</td>
</tr>
<tr>
<td>12. Fresh Non-citrus Fruit</td>
</tr>
<tr>
<td>13. Fresh Vegetables</td>
</tr>
<tr>
<td>14. Potatoes</td>
</tr>
<tr>
<td>15. Processed Fruit</td>
</tr>
<tr>
<td>16. Processed Vegetables</td>
</tr>
<tr>
<td>17. Fats and Oils</td>
</tr>
<tr>
<td>18. Eggs</td>
</tr>
<tr>
<td>19. Cereals</td>
</tr>
<tr>
<td>20. Sugars and Caloric Sweeteners</td>
</tr>
<tr>
<td>21. Coffee, Tea and Cocoa</td>
</tr>
</tbody>
</table>

#### Table 2. Nutritional Element vs. Unit of Measurement

<table>
<thead>
<tr>
<th>Nutritional Element</th>
<th>Unit of Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Food Energy</td>
<td>kcal</td>
</tr>
<tr>
<td>2. Protein</td>
<td>g.</td>
</tr>
<tr>
<td>3. Fats</td>
<td>g.</td>
</tr>
<tr>
<td>4. Cholesterol</td>
<td>mg.</td>
</tr>
<tr>
<td>5. Vitamin A</td>
<td>IU</td>
</tr>
<tr>
<td>6. Vitamin B12</td>
<td>mg.</td>
</tr>
<tr>
<td>7. Vitamin B6</td>
<td>mg.</td>
</tr>
<tr>
<td>8. Vitamin C</td>
<td>mg.</td>
</tr>
<tr>
<td>9. Niacin</td>
<td>mg.</td>
</tr>
<tr>
<td>10. Riboflavin</td>
<td>mg.</td>
</tr>
<tr>
<td>11. Thiamin</td>
<td>mg.</td>
</tr>
<tr>
<td>12. Calcium</td>
<td>mg.</td>
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<tr>
<td>13. Iron</td>
<td>mg.</td>
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<tr>
<td>14. Magnesium</td>
<td>mg.</td>
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<tr>
<td>15. Phosphorous</td>
<td>mg.</td>
</tr>
<tr>
<td>16. Potassium</td>
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<td>17. Sodium</td>
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<td>18. Zinc</td>
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#### Table 3. Nutrient Consumption

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<td>Kcal/day</td>
<td>3334.021</td>
<td>220.2681</td>
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<td>Protein g/day</td>
<td>96.90722</td>
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<td>Fat g/day</td>
<td>138.6804</td>
<td>15.25174</td>
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<td>Cholesterol mg/day</td>
<td>444.8454</td>
<td>32.63157</td>
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<td>Vitamin A IU/day</td>
<td>7395.361</td>
<td>1575.714</td>
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<td>Vitamin B12 mg/day</td>
<td>8.227835</td>
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<td>Vitamin B6 mg/day</td>
<td>2.004124</td>
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<td>Vitamin C mg/day</td>
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<td>Niacin mg/day</td>
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<td>Riboflavin mg/day</td>
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<td>Thiamin mg/day</td>
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<td>Calcium mg/day</td>
<td>902.7835</td>
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<td>Iron mg/day</td>
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<td>Magnesium mg/day</td>
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<td>Phosphorus mg/day</td>
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<td>Potassium mg/day</td>
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<td>Sodium mg/day</td>
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<td>139.5188</td>
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<td>Zinc mg/day</td>
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Table 4. Food Consumption

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<tr>
<td>Milk lb/day</td>
<td>0.78593</td>
<td>0.11648</td>
<td>0.57614</td>
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<td>Butter lb/day</td>
<td>0.02721</td>
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<td>Cheese lb/day</td>
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<td>Frozen Dairy lb/day</td>
<td>0.05638</td>
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<td>Other Dairy lb/day</td>
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<td>Beef lb/day</td>
<td>0.18009</td>
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<td>Pork lb/day</td>
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<td>0.01837</td>
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<td>Other Red Meat lb/day</td>
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<td>Fish lb/day</td>
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<td>Poultry lb/day</td>
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<td>Fresh Citrus Fruit lb/day</td>
<td>0.08292</td>
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<td>Fresh Noncitrus Fruit lb/day</td>
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<td>Fresh Veggies lb/day</td>
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<td>Potatoes lb/day</td>
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<td>Processed Fruit lb/day</td>
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<td>Processed Veggies lb/day</td>
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<td>Fats and Oils lb/day</td>
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<td>Eggs lb/day</td>
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<td>Cereals lb/day</td>
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<td>Sugars and Sweeteners lb/day</td>
<td>0.31945</td>
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<td>Coffee, Tea and Cocoa lb/day</td>
<td>0.03779</td>
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Table 5. Demographic Variables

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<td>Average Family Size</td>
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<td>Population Proportion 15-54</td>
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<td>Population Proportion Black</td>
<td>0.109896</td>
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<td>Population Proportion White</td>
<td>0.872775</td>
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<td>Population Proportion Other</td>
<td>0.017327</td>
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<td>Per Capita Disposable Income</td>
<td>2668.5</td>
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<td>Unemployment</td>
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Table 6. Coefficients for Demographic Variables relating to Nutrient Shadow Prices

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Table 7.
Coefficients for Demographic Variables relating to Food Tastes

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Appendix A, Estimated Shadow Prices using Maximum Entropy

From equation (16a), we can use L'Hôpital's rule to obtain:

\[ f_{ti}(r) = \lim_{\gamma_{ti} \to 0} \left[ \frac{\gamma_{ti} e^{-\gamma_{ti} r_{ti}}}{1 - e^{-\gamma_{ti} r_{ti} p_{ti}}} \right] = \lim_{\gamma_{ti} \to 0} \frac{e^{-\gamma_{ti} r_{ti} (1 - \gamma_{ti} p_{ti})}}{1 + \gamma_{ti} r_{ti} p_{ti}} = \frac{(1 - 0) + 1}{p_{ti} + 1} = \frac{1}{p_{ti}}. \]

This implies that \( f_{ti}(r) = \frac{1}{p_{ti}} \), which is a uniform distribution over the support \([0, p_{ti}]\). From this, \( \gamma_{ti} = 0 \) if and only if \( E(R_{ti}) = \frac{1}{2}p_{ti} \), and further that \( \gamma_{ti} > 0 \) if and only if \( E(R_{ti}) < \frac{1}{2}p_{ti} \), and similarly that \( \gamma_{ti} < 0 \) if and only if \( E(R_{ti}) > \frac{1}{2}p_{ti} \), depending on the sign of \( \gamma_{ti} \).

Let us consider the case where \( \gamma_{ti} \neq 0 \). We want to find \( E(R_{ti}) \). We know that

\[ E(R_{ti}) = \int_0^{p_{ti}} r f_{ti}(r) \, dr = \int_0^{p_{ti}} r \frac{\gamma_{ti} e^{-\gamma_{ti} r_{ti}}}{1 - e^{-\gamma_{ti} r_{ti} p_{ti}}} \, dr \]

To solve equation (b), we will integrate by parts, setting

\[ u = r_{ti}, \quad v = \frac{-e^{-\gamma_{ti} r_{ti}}}{1 - e^{-\gamma_{ti} r_{ti} p_{ti}}}, \quad u' = 1, \quad \text{and} \quad v' = \frac{\gamma_{ti} e^{-\gamma_{ti} r_{ti}}}{1 - e^{-\gamma_{ti} r_{ti} p_{ti}}}. \]

Now, we transform equation (c) into

\[ E(R_{ti}) = -r \frac{e^{-\gamma_{ti} r_{ti}}}{1 - e^{-\gamma_{ti} r_{ti} p_{ti}}} \bigg|_0^{p_{ti}} - \int_0^{p_{ti}} \frac{e^{-\gamma_{ti} r_{ti}}}{1 - e^{-\gamma_{ti} r_{ti} p_{ti}}} \, dr \]

\[ = -r \frac{e^{-\gamma_{ti} r_{ti}}}{1 - e^{-\gamma_{ti} r_{ti} p_{ti}}} \bigg|_0^{p_{ti}} - \frac{e^{-\gamma_{ti} r_{ti}}}{\gamma_{ti} [1 - e^{-\gamma_{ti} r_{ti} p_{ti}}]} \bigg|_0^{p_{ti}} = -p_{ti} \frac{e^{-\gamma_{ti} r_{ti} p_{ti}}}{1 - e^{-\gamma_{ti} r_{ti} p_{ti}}} + \frac{1}{\gamma_{ti}} \]

\[ = p_{ti} \left[ \frac{1}{1 - e^{-\gamma_{ti} r_{ti} p_{ti}}} \right] + \frac{1}{\gamma_{ti}}. \]

Defining \( \delta_{tj} = \sum_{i=1}^J a_{tij} Y_{ti} \) \( \forall j \in J \), we know that the Lagrange multipliers for the nutrient shadow prices, \( \delta_{tj} \), are determined by those of the taste attributes, \( \gamma_{ti} \). Accordingly, if and only if \( \delta_{tj} = 0 \), then \( E(\Pi_{tj}) = 0 \), if and only if \( \delta_{tj} > 0 \), then \( E(\Pi_{tj}) < 0 \), and similarly, if and only if \( \delta_{tj} < 0 \), then \( E(\Pi_{tj}) > 0 \), by the nature of Lagrangian multipliers. If \( \gamma_{ti} = 0 \) \( \forall i \in I \), then
\[ E(R_{ti}) = \frac{1}{2}p_{ti} \quad \forall \, i \in I \] and \[ E(\Pi_{tj}) = 0 \quad \forall \, j \in J \]. Thus, we know that we cannot have \[ \gamma_{ti} = 0 \quad \forall \, t \in T \] and \( i \in I \), since this would imply implies that \( p_{ti} = \frac{1}{2}p_{ti} \quad \forall \, i \in I \), which is a statement of contradiction. We obtain the expression of the mean \( E(\Pi_{tj}) \neq 0 \) by integrating by parts our estimated function of equation (16b). Allow

\[
\begin{align*}
    (e) \quad u = \pi_{tj} \quad \text{and} \quad v' &= \frac{\delta_{tj}e^{-\delta_{tj}\pi_{tj}}}{e^{\delta_{tj}\pi_{tj}} - e^{-\delta_{tj}\pi_{tj}}} \quad \text{so that} \\
    u' &= 1 \quad \text{and} \quad v = -\frac{e^{-\delta_{tj}\pi_{tj}}}{e^{\delta_{tj}\pi_{tj}} - e^{-\delta_{tj}\pi_{tj}}}.
\end{align*}
\]

we obtain the following, recalling that \( \delta_{tj} = \sum_{i=1}^{I} a_{tij} \gamma_{ti} \):

\[
\begin{align*}
    (f) \quad E(\Pi_{tj}) &= \int_{-\pi_{tj}}^{\pi_{tj}} \pi \, g(\pi) \, d\pi = \int_{-\pi_{tj}}^{\pi_{tj}} \pi \, \left[ \frac{\delta_{tj}e^{-\delta_{tj}\pi}}{e^{\delta_{tj}\pi_{tj}} - e^{-\delta_{tj}\pi_{tj}}} \right] d\pi \\
    &= -\pi \left. \frac{e^{-\delta_{tj}\pi}}{e^{\delta_{tj}\pi_{tj}} - e^{-\delta_{tj}\pi_{tj}}} \right|_{-\pi_{tj}}^{\pi_{tj}} - \int_{-\pi_{tj}}^{\pi_{tj}} \frac{e^{-\delta_{tj}\pi}}{e^{\delta_{tj}\pi_{tj}} - e^{-\delta_{tj}\pi_{tj}}} \, d\pi \\
    &= -\pi \left. \frac{e^{-\delta_{tj}\pi_{tj}}}{e^{\delta_{tj}\pi_{tj}} - e^{-\delta_{tj}\pi_{tj}}} \right|_{-\pi_{tj}}^{\pi_{tj}} - \int_{-\pi_{tj}}^{\pi_{tj}} \frac{e^{-\delta_{tj}\pi}}{e^{\delta_{tj}\pi_{tj}} - e^{-\delta_{tj}\pi_{tj}}} \, d\pi \\
    &= -\pi \left[ \frac{e^{-\delta_{tj}\pi_{tj}}}{e^{\delta_{tj}\pi_{tj}} - e^{-\delta_{tj}\pi_{tj}}} \right] - \int_{-\pi_{tj}}^{\pi_{tj}} \frac{e^{-\delta_{tj}\pi}}{e^{\delta_{tj}\pi_{tj}} - e^{-\delta_{tj}\pi_{tj}}} \, d\pi \\
    &= \pi \left[ \frac{e^{-\delta_{tj}\pi_{tj}}}{e^{\delta_{tj}\pi_{tj}} - e^{-\delta_{tj}\pi_{tj}}} \right] - \frac{1}{\delta_{tj}} \left[ \frac{e^{-\delta_{tj}\pi_{tj}}}{e^{\delta_{tj}\pi_{tj}} - e^{-\delta_{tj}\pi_{tj}}} \right] \\
    &= \pi \left[ \frac{e^{-\delta_{tj}\pi_{tj}}}{e^{\delta_{tj}\pi_{tj}} - e^{-\delta_{tj}\pi_{tj}}} \right] + \frac{1}{\delta_{tj}} = \pi \left[ \frac{1 + e^{2\delta_{tj}\pi_{tj}}}{1 - e^{2\delta_{tj}\pi_{tj}}} \right] + \frac{1}{\delta_{tj}}.
\end{align*}
\]

With these findings, we can substitute \( \delta_{tj} = \sum_{i=1}^{I} a_{tij} \gamma_{ti} \) back into our hedonic price constraint from equation (10).